

# HEAT TRANSFER ANALYSIS FOR UNSTEADY HIGH VELOCITY PIPE FLOW

Part I On Minimization of Temperature Distortion in the Thermocouple Cavity

Part II Improved Accuracy in the Prediction of Surface Heat Flux and Temperature by Intrinsic Thermocouple

Part III Prediction of Transient Surface Heat Flux and Temperature on a Hollow Cylinder

by

Ching Jen Chen, Jenq Shing Chiou, Peter Li  
and Hsai Yin Lee

Sponsored by

U.S. Army Research Office  
Grant Number DAA-G29-76-G-0123

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IIHR Report No. 195

Iowa Institute of Hydraulic Research  
The University of Iowa  
Iowa City, Iowa

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## ABSTRACT

The work done for the project is reported here in three parts. The first is the analysis for minimization of the temperature distortion due to the thermocouple cavity. The error is minimized or reduced to zero by optimizing a combination of cavity diameter and depth and the thermocouple material and size. The second is the refinement of the presently available computer programs for prediction of the surface temperature and heat flux at the inner surface of the pipe by inverting the temperature measured by an interior probe close to the heated surface. The refinement is achieved by using the double precision format in the program and adapting the dimensionless formulation. The third is to study the inversion solution for a large time duration of a time dependent surface heat flux. The solution is obtained by the method of Laplace transform and the convolution integral. Each of the above three subjects is reported as a part of the present report.

## ACKNOWLEDGEMENTS

The investigators would like to thank Mr. D.M. Thomsen of General Rodman Laboratory, Rock Island Arsenal, Rock Island, Illinois for his constant participation in the course of the research.

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PART I ON MINIMIZATION OF TEMPERATURE DISTORTION  
IN THE THERMOCOUPLE CAVITY

I. INTRODUCTION

A direct measurement of transient surface temperature and heat flux is often difficult. For example, a surface involves two modes of heat transfer, say, radiative and convective heat transfer. In this case if the measuring probe has a different radiative property than that of the surface, erroneous measurements will result. A piston or projectile sliding over the cylindrical surface is another case where the direct measurement at the surface is difficult. A surface involving melting or ablation is also difficult to make direct measurement. Therefore, indirect estimation by inverting the temperature history inside the heat conducting solid as measured by a thermocouple is often used for prediction of the surface temperature and heat flux. Beck [1], Hernning and Parker [2], Frank [3], Imber and Khan [4], Stolz [5], Chen and Thomsen [6] have developed different inversion solutions for this purpose. All of these solutions assumed that the cavity drilled into the solid does not distort the true temperature distribution. Thus, it is important that the temperature measurement by an interior probe is accurate and involves least distortion or error. Theoretically Beck [7], Masters and Stein [8], Burnett [9], and Chen and Li [10] studied the distortion of the temperature field in the presence of thermocouple and its cavity. Experimentally Chen and Danh [11] showed that appreciable distortion, say 10%, of temperature field may exist for a normal implant of the thermocouple into a solid body. From studies of Chen and Li [10] and Beck [7] they found that with a proper combination of the thermocouple cavity diameter, its depth, and the thermocouple material and its diameter

the distortion of temperature field with respect to space or time can be minimized if not eliminated. In this report we study the optimum combination of these parameters such that at a given situation one knows what is the best combination to use and what is the magnitude of the temperature distortion.

## II. FORMULATION OF PROBLEM

In the present study we consider a disk depicted in Figure 1 which has a thickness  $D$  and is drilled a cavity of a diameter to a depth of  $\epsilon$  distance from the heating surface. The heat flux  $Q$  is assumed to be constant and the upper surface of the disk is assumed to be insulated. A thermocouple of a diameter  $d_t$  is then welded on the cavity base. Furthermore, the disk may be thought to approximate a cut out from a hollow cylinder if the radius of the disk is small compared with the radius of the cylinder. The diameter of the disk is chosen to be  $2D$  outside which the temperature distortion due to the thermocouple cavity becomes negligible. For this to be true one needs to restrict the ratio of cavity diameter to the disk diameter  $d/2D$  be small. The unfilled cavity can be air or insulating material.

The basic idea to minimize or to eliminate the temperature distortion is based on a proper choice of the thermocouple size and the material which has a higher thermal conductivity than that of the disk so as to conduct more heat away at the cavity base balancing the insulation effect of the insulator in the cavity.

Let  $X$  and  $Y$  be respectively the coordinate along and normal to the heated disk surface and the  $Y$  axis coincide with the axis of the cavity.

The thermal conductivity is assumed to be constant and the temperature distribution is axisymmetrical. The governing equations for the transient heat conduction in dimensionless form are:

for the disk (subscript "1", see Figure 2)

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial x^2} + \frac{1}{x} \frac{\partial \theta}{\partial x} + \frac{\partial^2 \theta}{\partial y^2} \quad (1)$$

for the insulating material in the cavity (subscript "2")

$$\frac{\partial \theta}{\partial \tau} = \frac{\alpha_2}{\alpha_1} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{1}{x} \frac{\partial \theta}{\partial x} + \frac{\partial^2 \theta}{\partial y^2} \right) \quad (2)$$

for the thermocouple (subscript "3")

$$\frac{\partial \theta}{\partial \tau} = \frac{\alpha_3}{\alpha_1} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{1}{x} \frac{\partial \theta}{\partial x} + \frac{\partial^2 \theta}{\partial y^2} \right) \quad (3)$$

where  $\tau = \alpha_1 t / D^2$  is the dimensionless time,  $x = X/D$  the dimensionless radial coordinate,  $y = Y/D$ , the dimensionless distance normal to the heated surface.  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$  are respectively the thermal diffusivity of the disk, insulating material, and the thermocouple. The dimensionless temperature  $\theta$  is defined as  $T \kappa_1 / QD$  where  $T$  is the temperature above the initial, uniform temperature.

The initial temperature of the disk is then

$$\theta(x, y, 0) = 0 \quad (4)$$

The boundary conditions, see Figure 2, are:

a constant heat flux at the lower surface,

$$y = 0 \quad \left. \frac{\partial \theta}{\partial y} \right|_{y=0} = 1 \quad (5)$$

the insulation at the upper surface,

$$y = 0 \quad \left. \frac{\partial \theta}{\partial y} \right|_{y=0} = 0 \quad (6)$$

the zero temperature distortion at the edge of the disk,

$$x = 1 \quad \left. \frac{\partial \theta}{\partial x} \right|_{x=1} = 0 \quad (7)$$

and the axisymmetric condition at the cavity axis

$$x = 0 \quad \left. \frac{\partial \theta}{\partial x} \right|_{x=0} = 0 \quad (8)$$

The condition (7) of the zero temperature distortion was verified by Chen and Li [10] in their early calculation when the cavity diameter is one tenth of the disk diameter. In addition to the above boundary condition the temperature and heat flux at the interface of the disk, thermocouple, and insulating material are taken to be continuous.

### III ANALYSIS

There are five parameters that can be varied for the present analysis. They are (a) the dimensionless distance from the base of the cavity to the heated surface  $\epsilon/D$ , (b) the size of the cavity  $d/D$ , (c) the ratio of the thermocouple diameter to that of the cavity,  $d_t/d$ .



(d) the thermal conductivity ratio  $\kappa_2/\kappa_1$ , and  $\kappa_3/\kappa_1$  which comes from the continuity of heat flux at interfaces. (e) the ratio of the product of density and specific heat  $\rho_3 c_3/\rho_1 c_1$  (or equivalently to the ratio of thermal diffusivity  $\alpha_3/\alpha_2$  or  $\alpha_2/\alpha_1$ ). The subscripts 1, 2 and 3 denote the disk, insulation and thermocouple materials.

Because of the complexity of the geometry and the multicity of material the method of finite element technique as discussed by Wilson and Nikel [12] is adapted with the aid of a computer program developed by Wilson [13]. The present problem is subdivided into finite element as required by the method, see Figure 2. The dimensionless pie section is subdivided into 121 finite elements with 12 dividing lines on both coordinates. Each element is defined by four nodal points where nodal points are denoted by intersections of the dividing lines and numbered as shown in Figure 2. The material property corresponding to each element is then assigned to the program developed by Wilson [13]. The solution at each nodal with respect to time is then obtained.

For numerical calculation three typical values of the distance from heated surface to the base of the cavity  $\epsilon/D$  are chosen to be 0.04, 0.1 and 0.2. The cavity diameter is fixed at one tenth of the disk diameter. The thermocouple to cavity diameter ratio  $d_t/d$  is made to vary 0, 0.2, 0.4, 0.6, 0.8 and 1.0. Regarding the range of the ratio of the thermal conductivity and the ratio of the product of density and specific heat we surveyed these ratios for the commonly used thermocouples in Table 1 [14] and plotted in Figure 3. Figure 3b shows the ratio of

thermocouple conductivity to that of the disk  $\kappa_3/\kappa_1$  versus the ratio of density - specific heat product  $\rho_3 c_3/\rho_1 c_1$  where the value of the conductivity is taken to be the average value between 200 and 800°K. One sees that for most practical situations the ratio of density - specific heat product  $\rho_3 c_3/\rho_1 c_1$  is approximately constant at 0.7 except when the conductivity ratio  $\kappa_3/\kappa_1$  is small. Thus the value of  $\rho_3 c_3/\rho_1 c_1$  and  $\kappa_3/\kappa_1$  for calculation are chosen, as shown in triangular symbols of Figure 3b, to cover the practical range. The corresponding value for  $\rho_2 c_2/\rho_1 c_1$  and  $\kappa_2/\kappa_1$  for the insulation material are chosen to be fixed at 0.5 and 0.005 which is a typical value for Teflon insulating material and is also approximately the order of magnitude for air.

#### IV. RESULTS AND DISCUSSIONS

Numerical results of the calculations are presented in Tables 2 to 4 and Figures 4 to 9. The percentage error of temperature is defined as the distorted temperature divided by a reference temperature defined by  $QD/\kappa_1$ . Tables 2 to 4 give the percentage error of temperature distortion as a function of time for different values of the parameters  $d_t/d$  (0 to 1.0),  $\kappa_3/\kappa_1$  (0.5 to 10),  $\rho_3 c_3/\rho_1 c_1$  (0.5 to 1.8) and  $\epsilon/D$  (0.02 to 0.1).

Figures 4, 5 and 6 show the three typical temperature distributions in the steel disk near the thermocouple junction for the case  $\epsilon/D = 0.06$   $d/2D = 0.1$  at the time  $\tau = 0.08$ . Figure 4 is the temperature distribution when the cavity is filled entirely with the insulation material (Teflon "2"  $\kappa_2/\kappa_1 = 0.005$ ,  $\rho_2 c_2/\rho_1 c_1 = 0.5$ ). The dimensionless isotherms

$T\kappa_1/QD$  shows the distortion of temperature distribution. One sees that the insulation effect on the heat transfer near the cavity base not only creates a much higher junction temperature of  $T\kappa_1/QD = 0.342$  than the undistorted one of 0.255 at the edge of the disk giving an error of 8.7% but also causes a hot spot at the heating surface with a higher temperature of  $T\kappa_1/QD = 0.374$  over the undistorted one of 0.311. On the other hand the temperature distribution in the insulation material is much lower than the true temperature creating a large temperature gradient at the base of the cavity. Figure 5, contrary to Figure 4, is the temperature distribution when the cavity is completely filled with thermocouple material whose thermal conductivity is ten times larger than the disk material e.g., copper versus steel. Now over-conduction of heat by the thermocouple has created a cold spot at the base of the cavity giving,  $T\kappa_1/QD$  of 0.17 versus the undistorted one of 0.255 with an error of 8.5% as well as at the heating surface with  $T\kappa_1/QD$  of 0.236 versus 0.311. The temperature distribution in the thermocouple now becomes higher than the undistorted one in the disk. By properly choosing the ratio of thermocouple diameter to that of the cavity one may minimize these distortions of the temperature response at the base of the cavity. This is shown in Figure 6 where the thermocouple diameter  $d_t$  is chosen to be 0.4 of the cavity diameter  $d$  with  $\kappa_3/\kappa_1 = 10$ , e.g. copper-steel combination. Figure 6 shows that distortions at the base of the cavity and at the heating surface are almost eliminated giving the temperature  $T\kappa_1/QD$  of 0.245 and 0.320 with respectively the error of 1% and 0.9%.

In order to examine the details of the distorted temperature response at the base of the cavity we tabulated the results in Tables 2, 3 and 4 and plotted the error percentage as function of the ratio of the thermocouple diameter to that of the cavity for different cavity depth, time, and thermal conductivity in Figures 7, 8 and 9. In these Figures the  $\rho c$  ratio ranges from 0.7 to 1.3 which covers most of the practical application.

In the case when the  $\rho c$  ratio is equal to or less than one the error in temperature response at the base of the cavity in these figures are all positive or overheat for  $\kappa_3/\kappa_1 \leq 1$ . This is because the thermal conductivity of the thermocouple is less than that of the disk and the heat capacity  $\rho c$  of the thermocouple is also small. Therefore, no extra conduction of heat can be achieved by the thermocouple to compensate for the blocking of the heat transfer by the insulation material in the cavity. On the other hand, if  $\kappa_3/\kappa_1 > 1$  the error of temperature varies from positive value for  $d_t/d = 0$  to some negative value as  $d_t/d$  approaches 1. Thus for  $\kappa_3/\kappa_1 > 1$  a properly chosen combination of thermocouple and insulation material can minimize the error. For example, in Figure 7 a combination of  $\kappa_3/\kappa_1 = 10$ ,  $\rho_3 c_3/\rho_1 c_1 = 0.75$ ,  $\epsilon/D = 0.1$  and  $d_t/d = 0.5$  produces almost negligible error. This combination shown to be optimum at  $\tau = 1.0$  in Figure 7 is also optimum for other time periods (see Table 2). Therefore once an optimum combination of parameters is chosen it is valid throughout the entire transient period of an experiment. From Figures 7, 8 and 9 one can also see that the optimum ratio of  $d_t/d$  which gives zero temperature error decreases as the  $\kappa_3/\kappa_1$  ratio increases.

This implies that for the thermocouple with a larger thermal conductivity a smaller diameter is sufficient to eliminate the temperature distortion. The result shown in Figures 7, 8 and 9 can in general be adapted for use in practical application to choose the size of thermocouple and cavity, the thermocouple material and the depth of the cavity to be drilled.

As mentioned earlier that when  $\kappa_3/\kappa_1 \leq 1$  and  $\rho_3 c_3/\rho_1 c_1 \leq 1$  the error of the temperature response at the base of the cavity are all positive. However we found (see Tables 2.2, 3.2 and 4.2) that if  $\rho_3 c_3/\rho_1 c_1$  ratio is made large enough during the transient period the error of temperature response at the base of the cavity may indeed become negative even when  $\kappa_3/\kappa_1 \leq 1$ . Physically although the thermal conductivity of the thermocouple  $\kappa_3$  is smaller than that of the disk material, but with a larger heat capacitance  $\rho_3 c_3/\rho_1 c_1 > 1$  the thermocouple is still capable of absorbing extra heat flux and hence eliminates the temperature distortion at the cavity base during the transient period. To illustrate this fact we examine Figure 7 (or see Table 4.2) for the data of  $\kappa_3/\kappa_1 = 1$  and  $\rho_3 c_3/\rho_1 c_1 = 1.3$ . One sees that when  $d_t/d = 1$  the temperature distortion can indeed be negative. Therefore, if  $d_t/d$  are chosen between 0.8 and 1 the error can be minimized. However one must keep in mind that the elimination of error by heat capacitance can work during the transient period only for once a steady state conduction is established the heat capacity  $\rho c$  will no longer have any effect and over heating at the cavity base eventually will develop. This can be seen best from the governing equation (1) that at steady state the unsteady term which contains  $\rho c$  product is zero and is not a parameter affecting the distortion.

Another important fact that should be mentioned is that in general the optimum choice of  $d_t/d$  ratio for given  $\kappa_3/\kappa_1$  and  $\rho c$  ratio does not vary very much with the variation of  $\epsilon/D$  ratio. The insensitivity of the optimum  $d_t/d$  ratio to the  $\epsilon/D$  ratio ranging from 0.02 to 0.1 means that the distortion of the temperature is insensitive to the cavity depth or the thickness of the disk. This fact was already pointed out by Chen and Danh [11] in their experiment that the temperature distortion at the base of the cavity is more sensitive to the variation of the cavity diameter than the depth of the cavity drilled.

As an example of a practical application, let us consider a measurement of the transient temperature response of an engine block made of aluminum. From Figure 3b we know that aluminum has high thermal conductivity. Therefore copper-constantan thermocouple which has a higher thermal conductivity than aluminum should be chosen. For this material combination we have  $\kappa_3/\kappa_1 = 1.69$   $\rho_3 c_3/\rho_1 c_1 = 1.3$ . Now if the thermocouple cavity is drilled such that  $\epsilon/D = 0.1$  then from Figure 7 interpolating between  $\kappa_3/\kappa_1 = 2$  and 1 for  $\rho_3 c_3/\rho_1 c_1 = 1.3$  we find that the optimum  $d_t/d$  for  $\kappa_3/\kappa_1 = 1.69$  is approximately 0.7

One disadvantage of invoking finite element analysis is that the result does not give a clear functional relation among the parameters involved. In an attempt to obtain a simple and useful relation to relate the various parameters we note the following fact and result:

(a) the optimum  $d_t/d$  ratio for zero temperature distortion is a strong function of  $\kappa_3/\kappa_1$  and  $\rho c$  ratio but is relatively insensitive to the  $\epsilon/D$  ratio, (b) from the theoretical reasoning the  $d_t/d$  ratio is independent

of  $\rho c$  ratio if the problem is steady state. A simple steady one dimensional analysis in which the thermocouple and the insulation material in the cavity is made to conduct the same amount of heat that would be transferred without the cavity gives the relation

$$d_t/d = \sqrt{(\kappa_1 - \kappa_2)/(\kappa_3 - \kappa_2)} \quad (9)$$

Using the above equation as a base we find that for the transient heat conduction as calculated by the finite element method the following equation (10) correlates very well with the optimum  $d_t/d$  ratio.

$$d_t/d = (\rho_3 c_3 / \rho_1 c_1)^{0.3} \sqrt{(\kappa_1 - \kappa_2)/(\kappa_3 - \kappa_2)} \quad (10)$$

Equation (10) gives an error or distortion of no more than two percentage points. In practice equation (10) may be used as a rule of thumb.

## V. CONCLUSION

An analysis of the temperature distortion caused by the cavity drilled into a disk to accommodate the thermocouple has been studied. The calculation is carried out for the case of constant heat flux. It is shown that the temperature at the base of the cavity distorted from that without a cavity can be eliminated by a properly chosen combination of the ratio of the thermocouple diameter to the cavity diameter,  $d_t/d$  and the thermocouple material  $\kappa_3/\kappa_1$ . The optimum ratio of  $d_t/d$  can be found from Figures 7, 8 and 9 or Tables 2, 3 and 4, or approximately from equation (10). As a rule the thermocouple must be chosen

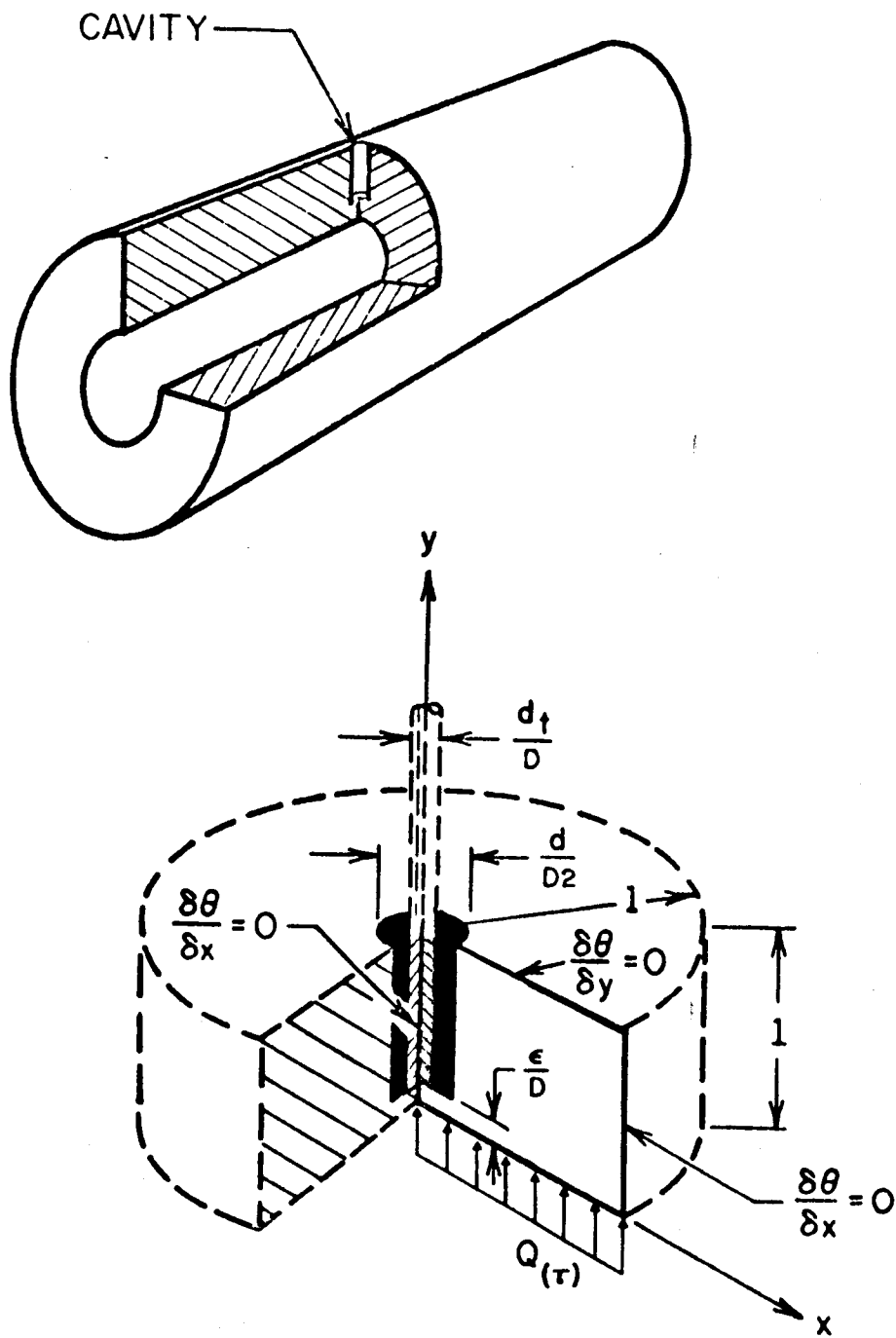
to have a higher thermal conductivity than that of the heat conducting solid. The cavity diameter should be as small as practically possible. For the case of time dependent surface heat flux the present result may be also used as a general guide.

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-  Steel
-  Thermocouple
-  Air Insulation

Figure 1 Geometric Representation of Problem

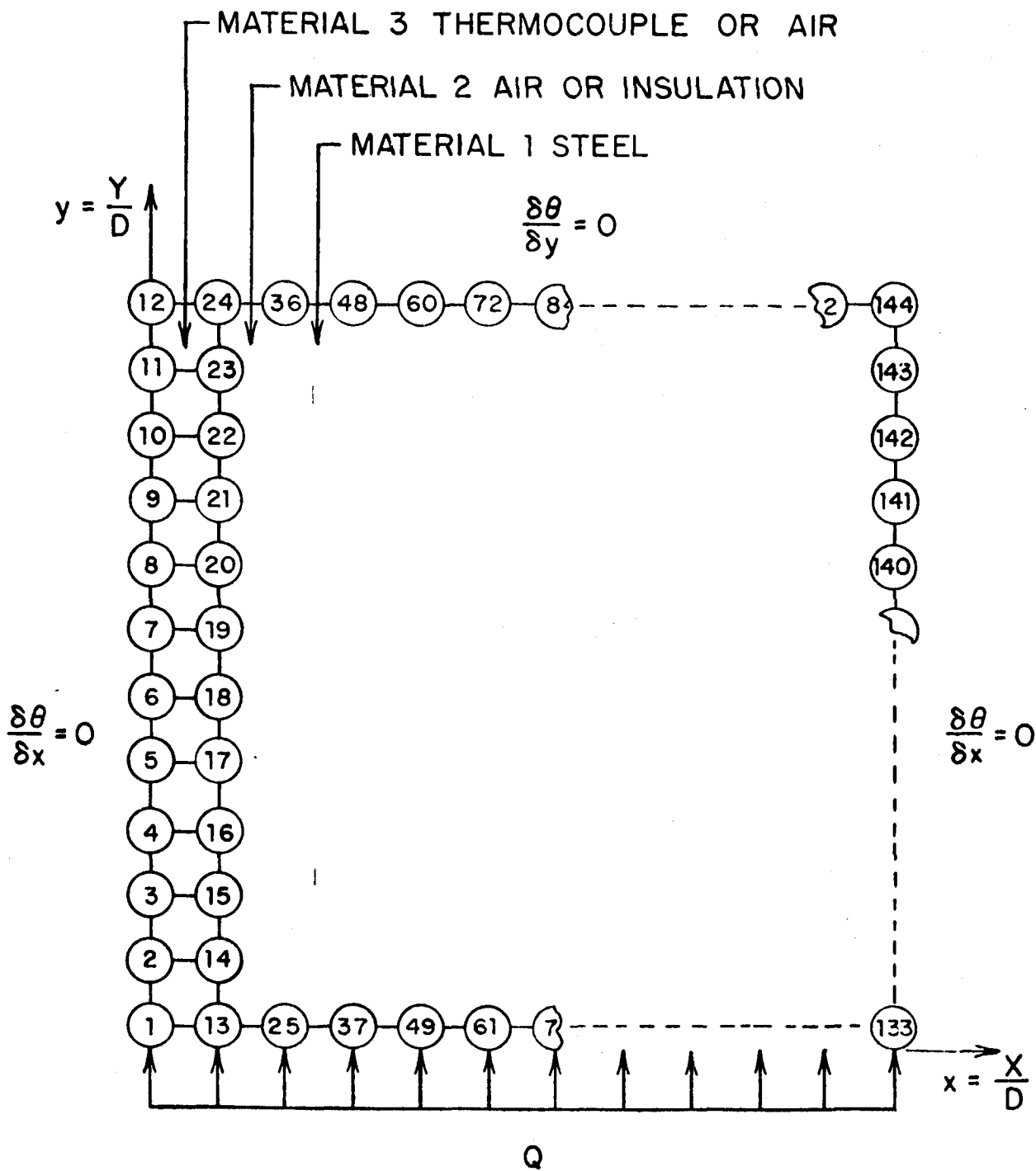


Figure 2 Finite Element Idealization

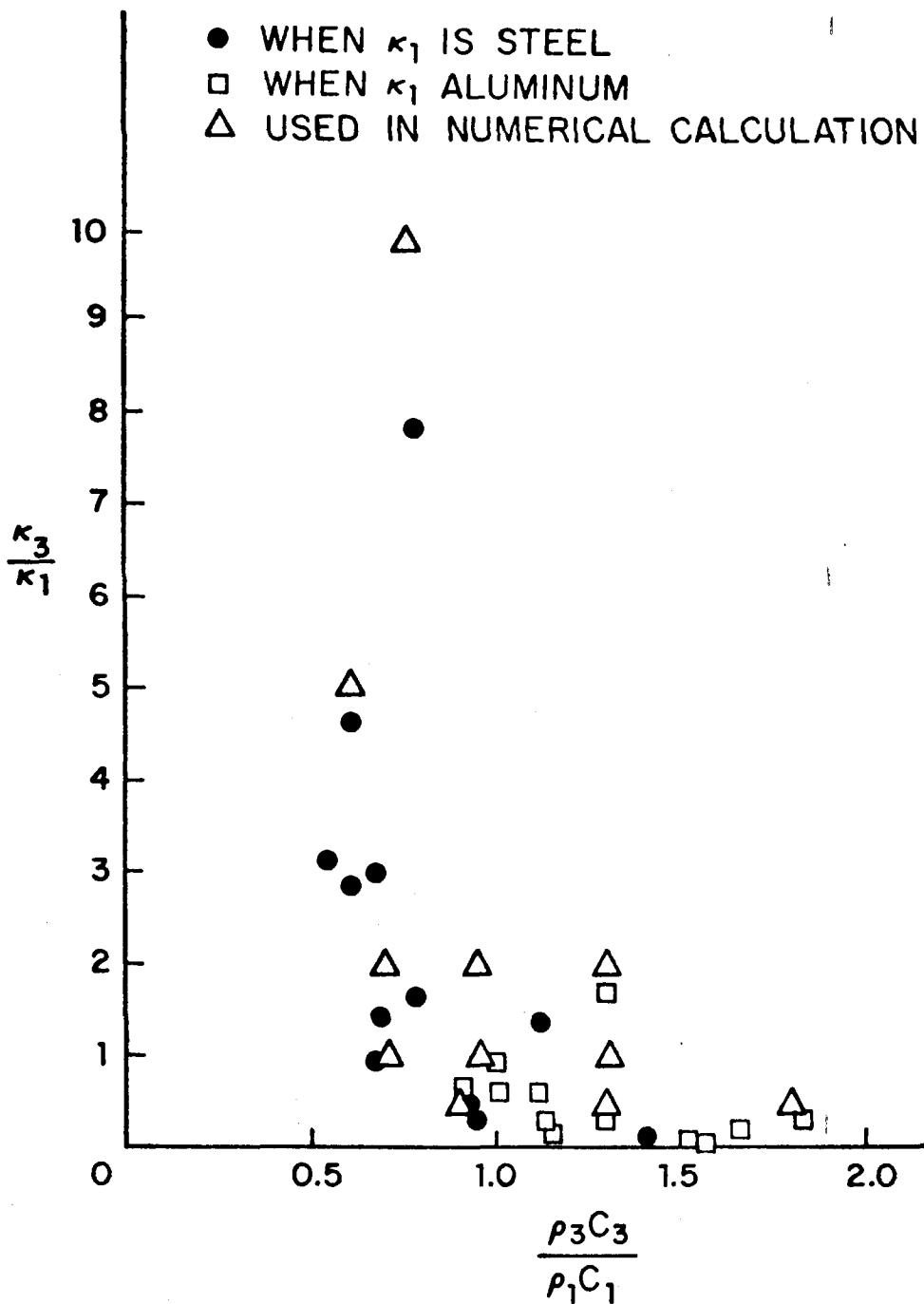


Figure 3b Variation of Thermal Conductivity Ratio and Density - Specific Heat Ratio

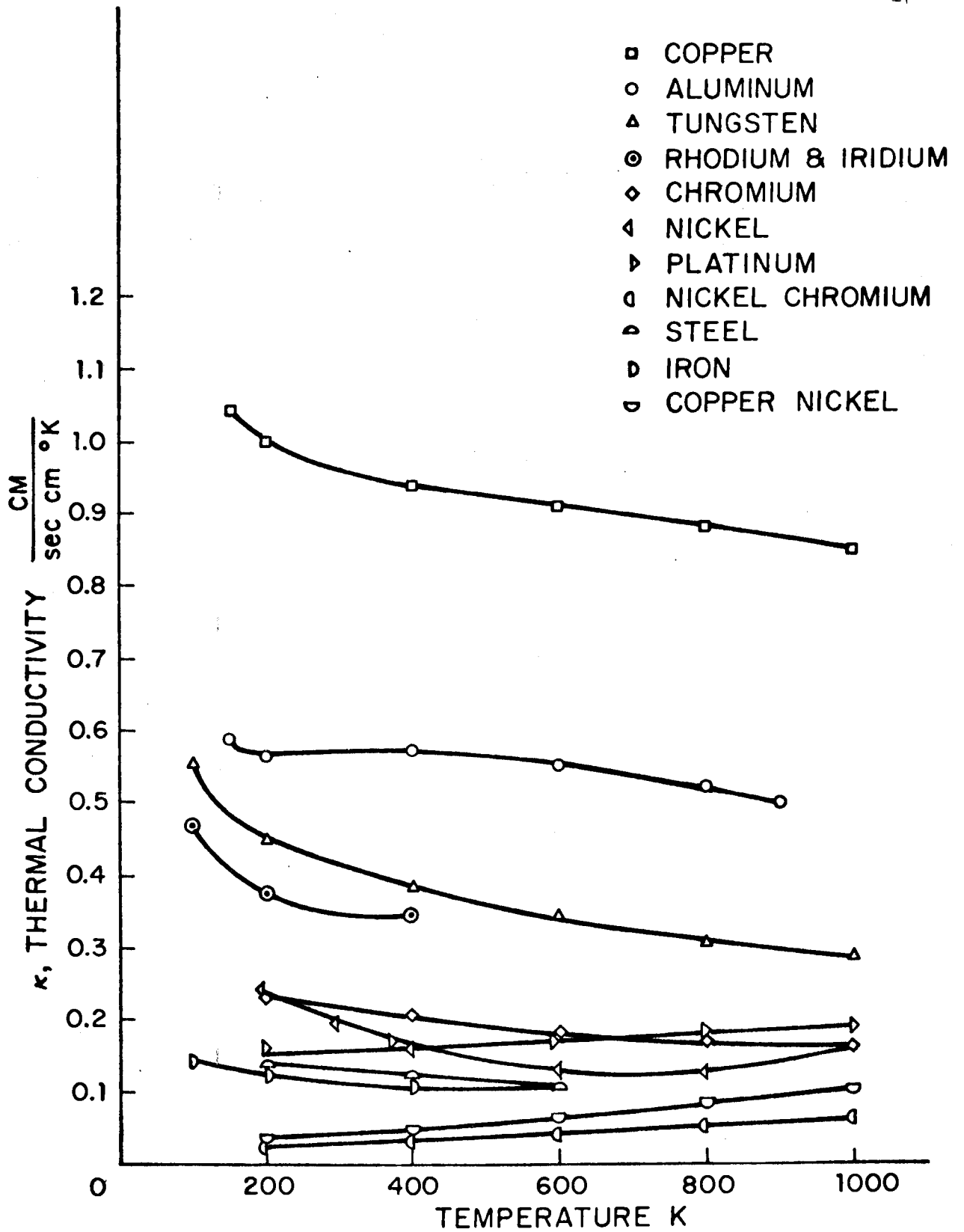


Figure 3a Thermal Conductivity of Thermocouple Materials

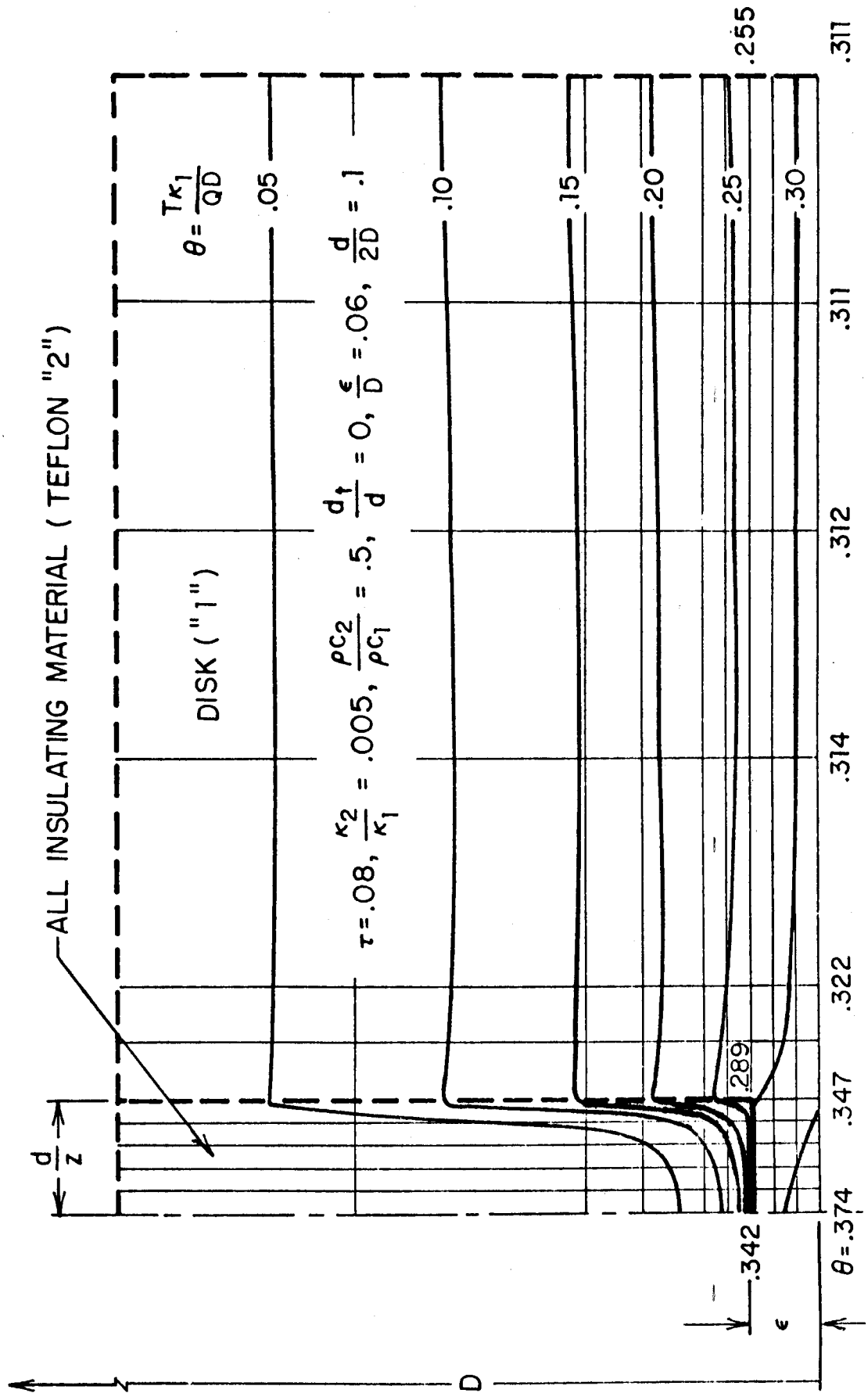


Figure 4 Temperature Distribution with Insulation Material Filled the Cavity

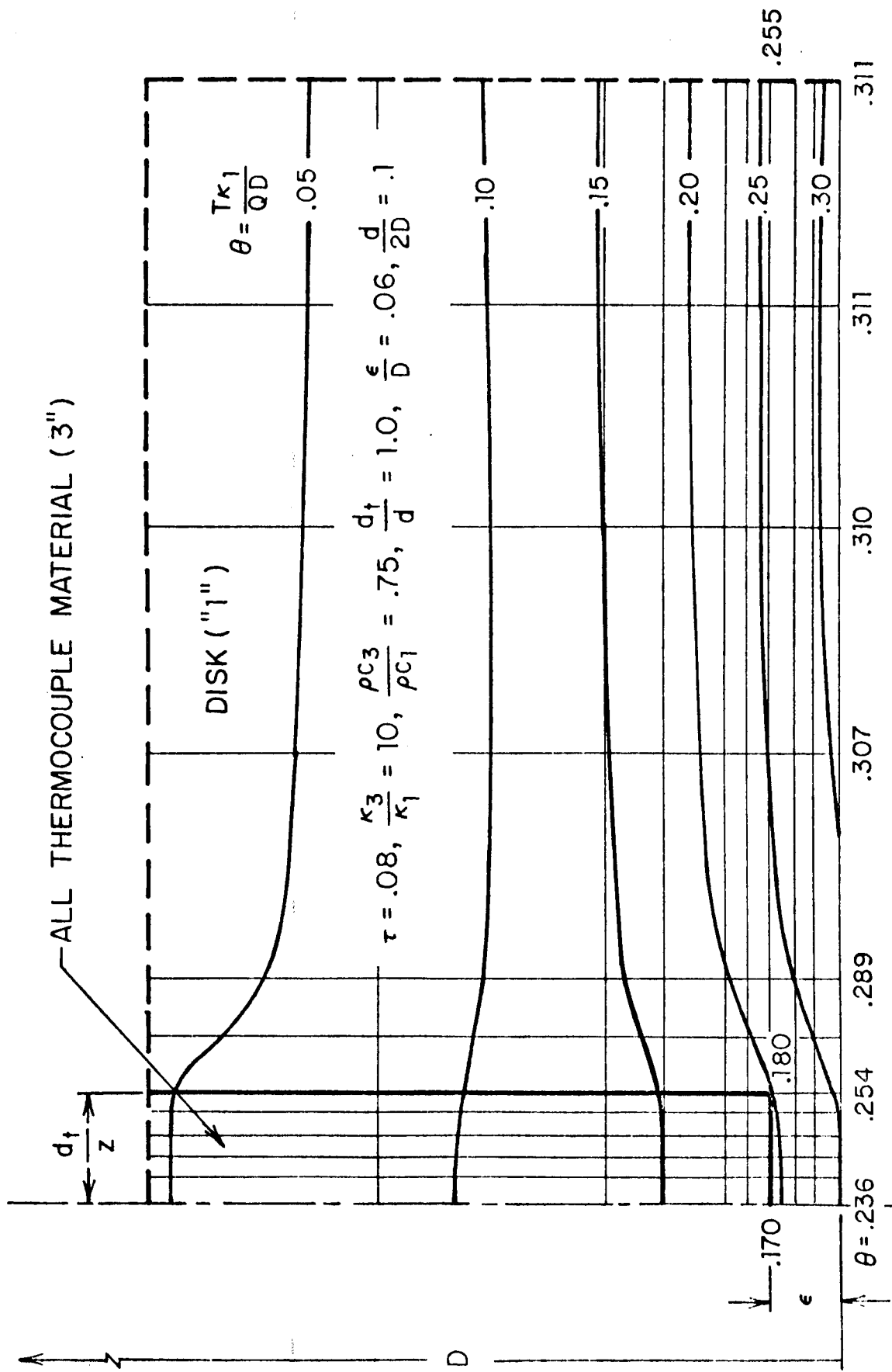


Figure 5 Temperature Distribution with Thermocouple Material Filled the Cavity

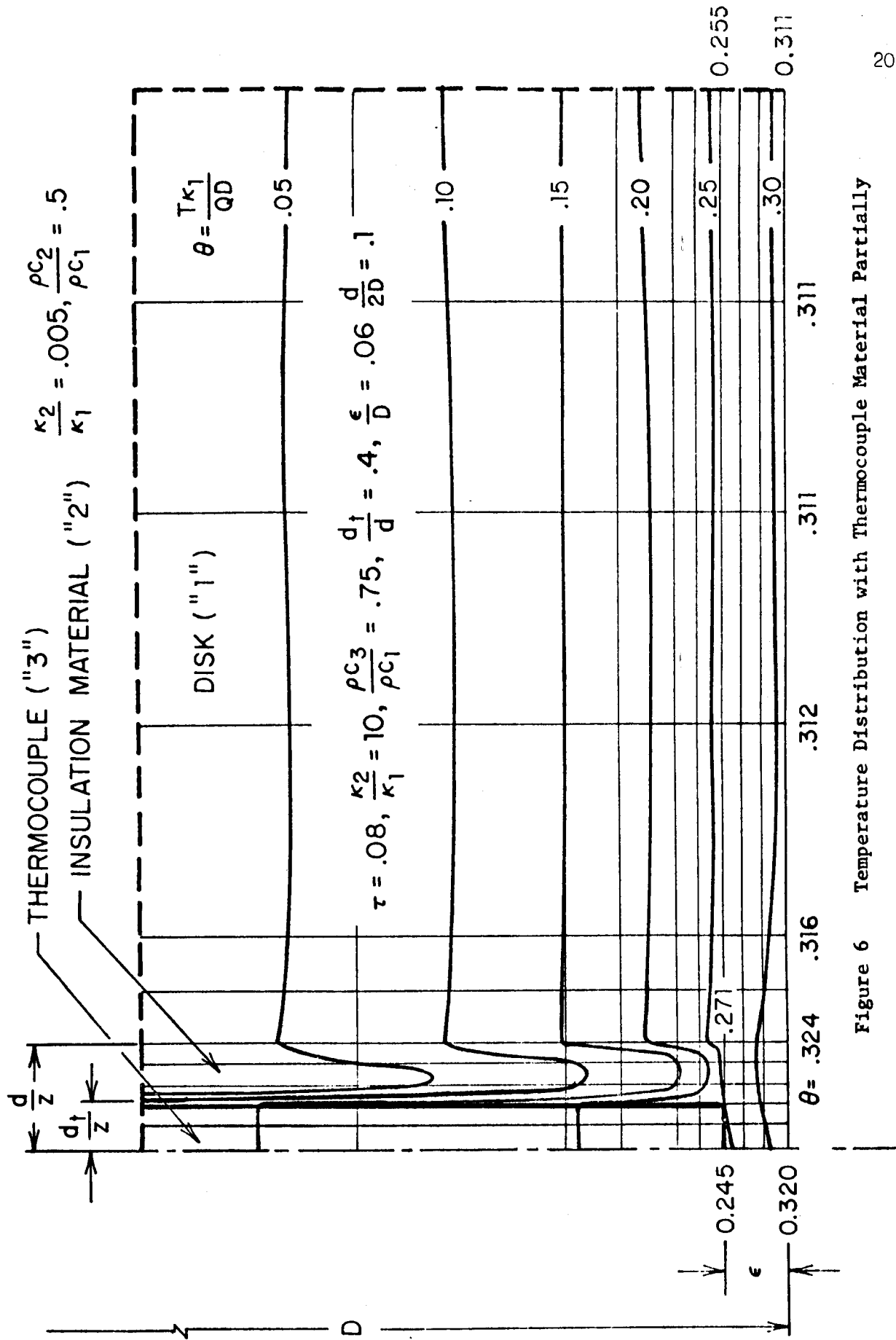


Figure 6 Temperature Distribution with Thermocouple Material Partially Filled the Cavity



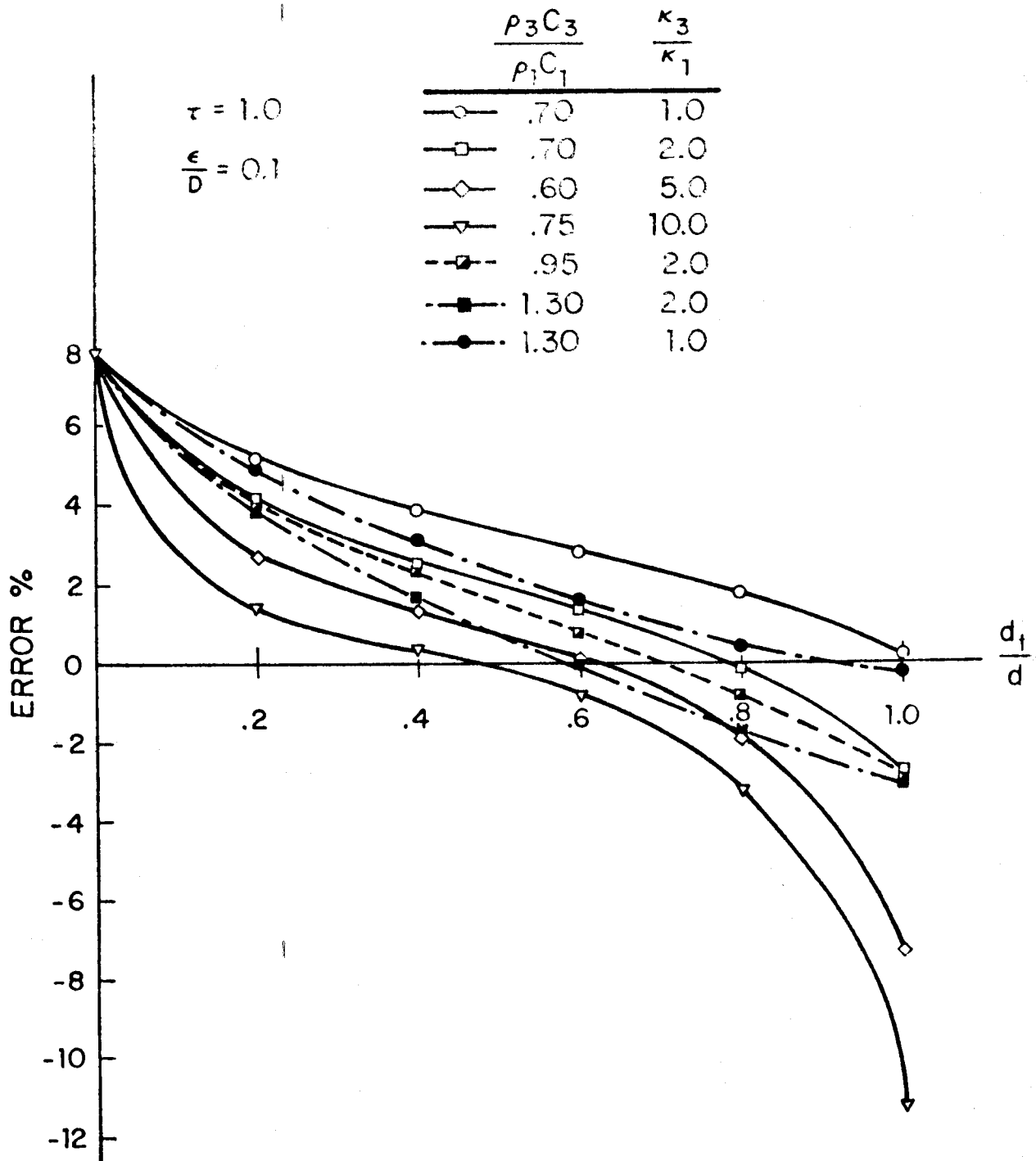


Figure 7 Percentage Error vs.  $d_t/d$  Ratio for Various  $\kappa_3/\kappa_1$ , and  $\rho C$  Ratios  $\epsilon/D = 0.1$

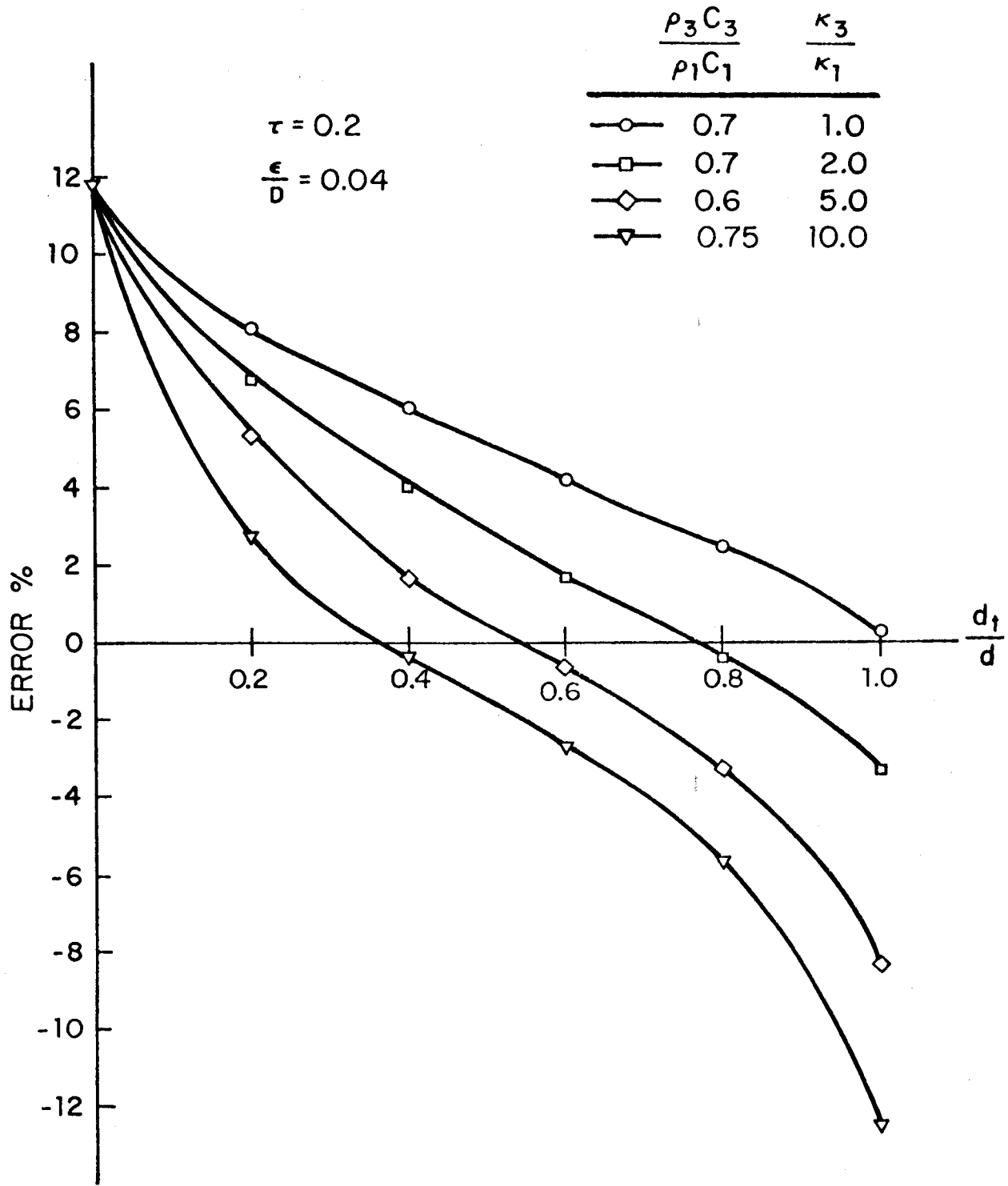


Figure 8 Percentage Error vs.  $d_t/d$  Ratio for Various  $\kappa_3/\kappa_1$  and  $\rho C$  Ratios  $\epsilon/D = 0.04$

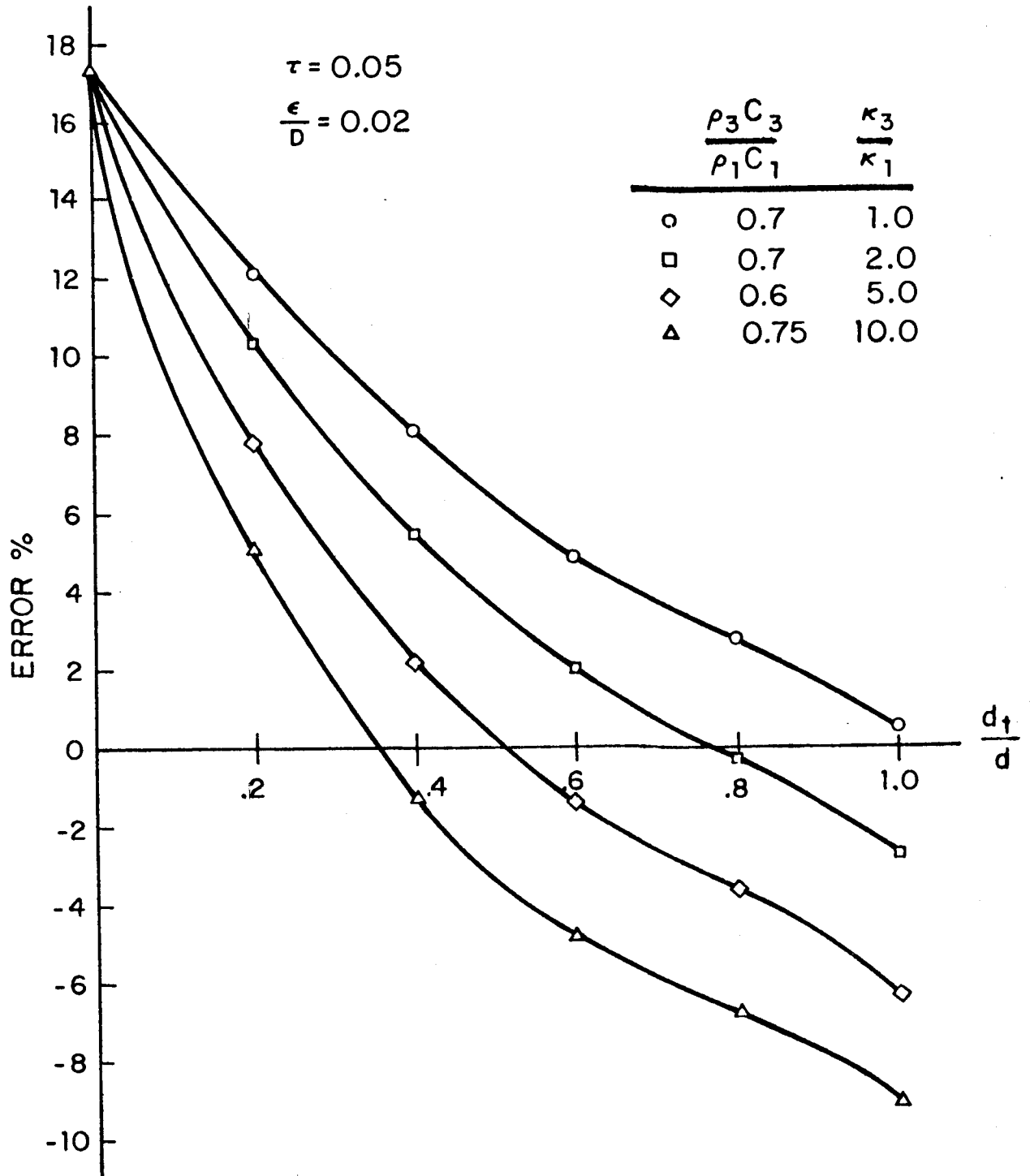


Figure 9 Percentage Error vs.  $d_t/d$  Ratio for Various  $\kappa_3/\kappa_1$  and  $\rho_c$  Ratios  $\epsilon/D = 0.02$

Table 1a  
Commonly Used Thermocouples

Types of Thermocouples	Temperature Ranges	
	°F	°C
Copper/Constantan	-300/750	-148/398
Iron/Constantan	-300/1600	-148/871
Chromel/Alumel	-300/2300	-148/1260
Chromel/Constantan	32/1800	0/982
Platinum 10% Rhodium/Platinum	32/2800	0/1537
Platinum 13% Rhodium/Platinum 6% Rh	100/3270	37/1798
Platinel 1813 Platinel 1503	32/2372	0/1300
Iridium/Iridium 60% Rhodium	2552/3326	1400/1830
Tungsten 3% Rhenium/Tungsten 25% Rhenium	50/4000	10/2204
Tungsten/Tungsten 26% Rhenium	60/5072	15/2800
Tungsten 5% Rhenium/Tungsten 26% Rhenium	32/5000	0/2760

Table 1b

## Thermal Properties of Thermocouple Materials

Metal, Insulator (subscript 3)	$K_3$	$\frac{\text{cal}}{\text{sec. cm } \kappa}$	$\frac{\kappa_3}{\kappa \text{ steel}}$	$\frac{\kappa_3}{\kappa \text{ aluminum}}$	$\rho_3 c_3$	$\frac{\text{cal}}{\text{cm}^3}$	$\frac{\rho_3 c_3}{\rho c \text{ steel}}$	$\frac{\rho_3 c_3}{\rho c \text{ aluminum}}$
Aluminum	0.554	4.66	1.00	0.65	0.60	1.0		
Copper	0.935	7.87	1.69	0.84	0.78	1.30		
Chromium	0.201	1.70	0.36	0.84	0.78	1.30		
Nickel	0.160	1.35	0.29	1.20	1.12	1.86		
Platinum	0.174	1.46	0.31	0.74	0.69	1.14		
Steel	0.119	1.0	0.21	1.08	1.00	1.66		
Tungsten	0.373	3.14	0.67	0.59	0.55	0.92		
Iridium	0.345	2.91	0.62	0.66	0.61	1.01		
Rhodium	0.360	3.03	0.65	0.72	0.67	1.12		
Rhenium	0.111	0.94	0.20	0.73	0.68	1.14		
Nickel-Chromium	0.040	0.34	0.07	1.01	0.94	1.56		
Copper-Nickel	0.059	0.50	0.11	0.99	0.92	1.52		
Teflon	$6 \times 10^{-4}$	0.005	0.001	0.54	0.5	0.83		
Air	$1.26 \times 10^{-4}$	0.001	0.0002	0.53	0.49	0.82		

Note: The value quoted is the averaged value over 200 to 800 °K whenever data are available.

Table 2

Percentage Error of Temperature at Cavity Base ( /D = 0.1)

Table 2.1		Table 2.2		Table 2.3			
$\rho_3 c_3 / \rho_1 c_1$	$\kappa_3 / \kappa_1 = 0.5$	$\rho_3 c_3 / \rho_1 c_1$	$\kappa_3 / \kappa_1 = 1.0$	$\rho_3 c_3 / \rho_1 c_1$	$\kappa_3 / \kappa_1 = 1.0$		
$\tau \times 10^2$	$d_t/d$	$\tau \times 10^2$	$d_t/d$	$\tau \times 10^2$	$d_t/d$		
Error %		Error %		Error %			
0.9	1	1.9	1.4	1.0	0.8	0.6	0.5
	2	3.4	2.5	1.9	1.4	1.1	1.0
	4	5.1	3.8	2.9	2.2	1.7	1.5
	8	6.4	4.7	3.7	2.9	2.3	1.9
	12	6.9	5.1	4.1	3.3	2.6	2.2
	20	7.4	5.5	4.5	3.7	2.9	2.4
	40	7.8	5.9	4.8	4.0	3.2	2.6
	60	7.9	6.0	4.9	4.0	3.2	2.6
	80	7.9	6.0	4.8	3.9	3.2	2.6
	100	7.9	6.0	4.8	3.9	3.2	2.6
1.3	1	1.3	0.9	0.6	0.4	0.3	0.3
	2	2.4	1.7	1.1	0.8	0.6	0.6
	4	3.7	2.6	1.8	1.2	1.1	1.1
	8	4.6	3.4	2.4	1.6	1.5	1.5
	12	5.0	3.8	2.7	1.9	1.8	1.8
	20	5.4	4.2	3.1	2.2	2.0	2.0
	40	5.8	4.5	3.4	2.5	2.3	2.3
	60	5.9	4.5	3.4	2.5	2.3	2.3
	80	5.9	4.5	3.3	2.5	2.3	2.3
	100	5.8	4.5	3.3	2.5	2.3	2.3
1.8	1	1.2	0.7	0.4	0.1	0.1	0.1
	2	2.3	1.4	0.8	0.4	0.3	0.3
	4	3.6	2.3	1.4	0.7	0.6	0.6
	8	4.5	3.0	1.9	0.9	1.0	1.0
	12	4.9	3.4	2.1	1.1	1.2	1.2
	20	5.3	3.8	2.5	1.5	1.6	1.6
	40	5.6	4.1	2.8	1.7	1.9	1.9
	60	5.7	4.1	2.7	1.7	1.9	1.9
	80	5.7	4.1	2.6	1.6	1.9	1.9
	100	5.7	4.0	2.6	1.6	1.9	1.9

0.7

0.95

1.3

1.3

1.3

Table 3

Percentage Error of Temperature at Cavity ( $\epsilon/D = 0.04$ )

Table 2.3		Table 3.1		Table 2.4	
$\rho_3^c / \rho_1^c$	$\rho_3^c / \rho_1^c$	$\rho_3^c / \rho_1^c$	$\rho_3^c / \rho_1^c$	$\rho_3^c / \rho_1^c$	$\rho_3^c / \rho_1^c$
$\kappa_3 / \kappa_1 = 5.0$		$\kappa_3 / \kappa_1 = 0.5$		$\kappa_3 / \kappa_1 = 10$	
$d_t/d$	$\epsilon/D = 0.1$	$d_t/d$	$\epsilon/D = 0.04$	$d_t/d$	$\epsilon/D = 0.1$
$\tau \times 10^2$	Error %	$\tau \times 10^2$	Error %	$\tau \times 10^2$	Error %
0.6	1	0.7	0.1	0.7	0.1
	2	1.3	0.1	1.3	0.1
	4	2.1	0.5	2.1	0.5
	8	2.5	0.6	2.5	0.6
	12	2.5	0.7	2.5	0.7
	20	2.6	0.9	2.6	0.9
	40	2.6	1.2	2.6	1.2
	60	2.7	1.4	2.7	1.4
	80	2.7	1.4	2.7	1.4
	100	2.7	1.4	2.7	1.4
	1	0.7	0.1	0.7	0.1
	2	1.3	0.1	1.3	0.1
	4	2.1	0.5	2.1	0.5
	8	2.5	0.6	2.5	0.6
	12	2.5	0.7	2.5	0.7
	20	2.6	0.9	2.6	0.9
	40	2.6	1.2	2.6	1.2
	60	2.7	1.4	2.7	1.4
	80	2.7	1.4	2.7	1.4
	100	2.7	1.4	2.7	1.4
	1	0.7	0.1	0.7	0.1
	2	1.3	0.1	1.3	0.1
	4	2.1	0.5	2.1	0.5
	8	2.5	0.6	2.5	0.6
	12	2.5	0.7	2.5	0.7
	20	2.6	0.9	2.6	0.9
	40	2.6	1.2	2.6	1.2
	60	2.7	1.4	2.7	1.4
	80	2.7	1.4	2.7	1.4
	100	2.7	1.4	2.7	1.4
	1	0.7	0.1	0.7	0.1
	2	1.3	0.1	1.3	0.1
	4	2.1	0.5	2.1	0.5
	8	2.5	0.6	2.5	0.6
	12	2.5	0.7	2.5	0.7
	20	2.6	0.9	2.6	0.9
	40	2.6	1.2	2.6	1.2
	60	2.7	1.4	2.7	1.4
	80	2.7	1.4	2.7	1.4
	100	2.7	1.4	2.7	1.4

Table 3.4

Table 3.3

$\rho_3 c_3 / \rho_1 c_1$        $\kappa_3 / \kappa_1 = 5$        $\epsilon / D = 0.04$

$d_t / d$	$\tau \times 10^2$	Error %	0.4	0.6	0.8	1.0
0.2	0.3	-0.1	-0.3	-0.4	-0.4	-0.6
0.4	0.9	0.0	-0.5	-0.8	-1.1	-1.1
0.8	2.0	0.4	-0.6	-1.2	-2.1	-2.1
1.6	3.4	0.9	-0.7	-1.8	-3.2	-3.2
2.4	4.1	1.2	-0.8	-2.1	-4.0	-4.0
4.0	4.6	1.4	-0.9	-2.5	-5.0	-5.0
8.0	4.7	1.4	-1.1	-3.0	-6.4	-6.4
12.0	4.7	1.4	-1.0	-3.2	-7.3	-7.3
16.0	4.7	1.6	-0.8	-3.2	-7.9	-7.9
20.0	4.7	1.7	-0.6	-3.2	-8.3	-8.3

Table 3.3

$\rho_3 c_3 / \rho_1 c_1$        $\kappa_3 / \kappa_1 = 10$        $\epsilon / D = 0.04$

$d_t / d$	$\tau \times 10^2$	Error %	0.4	0.6	0.8	1.0
0.2	0.0	-0.4	-0.6	-0.7	-0.9	-0.9
0.4	0.3	-0.6	-1.1	-1.4	-1.7	-1.7
0.8	1.0	-0.7	-1.7	-2.3	-3.0	-3.0
1.6	2.1	-0.8	-2.5	-3.5	-4.6	-4.6
2.4	2.6	-0.8	-2.9	-4.2	-5.7	-5.7
4.0	2.9	-0.9	-3.4	-5.0	-7.2	-7.2
8.0	2.9	-1.0	-3.7	-5.8	-9.5	-9.5
12.0	2.8	-0.8	-3.4	-5.8	-11.0	-11.0
16.0	2.8	-0.5	-3.1	-5.7	-11.9	-11.9
20.0	2.8	-0.3	-2.7	-5.6	-12.5	-12.5

Table 3.2

$\rho_3 c_3 / \rho_1 c_1$        $\kappa_3 / \kappa_1 = 1.0$        $\epsilon / D = 0.04$

$d_t / d$	$\tau \times 10^2$	Error %	0.4	0.6	0.8	1.0
0.2	0.8	0.5	0.3	0.2	0.1	0.1
0.4	1.8	1.1	0.7	0.4	0.3	0.3
0.8	3.4	2.2	1.4	0.8	0.4	0.4
1.6	5.4	3.6	2.2	1.3	0.4	0.4
2.4	6.3	4.3	2.7	1.6	0.4	0.4
4.0	7.1	5.0	3.2	1.9	0.4	0.4
8.0	7.6	5.5	3.7	2.2	0.4	0.4
12.0	7.9	5.8	4.0	2.4	0.4	0.4
16.0	8.0	5.9	4.1	2.4	0.4	0.4
20.0	8.1	6.1	4.2	2.5	0.3	0.3

Table 3.4

$\rho_3 c_3 / \rho_1 c_1$        $\kappa_3 / \kappa_1 = 10$        $\epsilon / D = 0.04$

$d_t / d$	$\tau \times 10^2$	Error %	0.4	0.6	0.8	1.0
0.2	0.7	0.4	0.2	0.1	0.0	0.0
0.4	1.6	0.9	0.5	0.2	0.0	0.0
0.8	3.3	1.9	1.0	0.4	0.1	0.1
1.6	5.2	3.3	1.8	0.7	0.1	0.1
2.4	6.1	3.9	2.2	0.9	0.1	0.1
4.0	6.9	4.6	2.6	1.1	0.1	0.1
8.0	7.5	5.1	3.1	1.4	0.1	0.1
12.0	7.7	5.4	3.3	1.6	0.1	0.1
16.0	7.9	5.6	3.5	1.6	0.1	0.1
20.0	8.0	5.7	3.6	1.7	0.1	0.1

Table 3.4

$\rho_3 c_3 / \rho_1 c_1$        $\kappa_3 / \kappa_1 = 1.0$        $\epsilon / D = 0.04$

$d_t / d$	$\tau \times 10^2$	Error %	0.4	0.6	0.8	1.0
0.2	0.6	0.3	0.0	-0.1	-0.1	-0.1
0.4	1.5	0.7	0.2	-0.1	-0.2	-0.2
0.8	3.1	1.6	0.6	-0.1	-0.3	-0.3
1.6	5.0	2.8	1.2	0.1	-0.4	-0.4
2.4	5.9	3.5	1.5	0.2	-0.4	-0.4
4.0	6.7	4.1	1.0	0.3	-0.4	-0.4
8.0	7.3	4.6	2.2	0.4	-0.4	-0.4
12.0	7.6	4.9	2.5	0.5	-0.4	-0.4
16.0	7.7	5.1	2.7	0.6	-0.4	-0.4
20.0	7.9	5.2	2.8	0.7	-0.3	-0.3





PART II IMPROVED ACCURACY IN THE PREDICTION OF SURFACE HEAT FLUX  
AND TEMPERATURE BY AN INTRINSIC THERMOCOUPLE

1. INTRODUCTION

In the study of transient heat transfer, many experimental difficulties may arise if heat flux sensors or thermocouples are installed direct at the surface of a body. For example, a probe may be damaged by a piston or a projectile sliding over a cylinder or barrel. A probe on a melting and ablative surface of heat shield can be easily destroyed because of high temperature. Furthermore a surface probe exposed to both radiative and convective environment may measure an erroneous surface heat flux and temperature if the probe has a different radiative property from that of the measured surface. In these circumstances, calculation of the transient surface heat flux and the surface temperature can be achieved by inverting a temperature history measured at some location inside the body.

In general, the prediction of a surface heat flux and temperature by the measured data at some location interior to a body is known as the "inverse problem". Many configurations, such as spheres, cylinders, and slabs, had been studied by many workers and many methods such as numerical, graphical, series, convolution integral, and Laplace transforms were used. Stolz [1], Beck [2] and Williams and Curry [3], considered the numerical inversion of the integral solution for semi-infinite and other bodies. In this method, care is required in selecting a time interval in order to achieve a stable solution. Carslaw and Jaeger [4], Burggraf [5], Koveryanov [6], and Shumakov [7], respectively considered different series approaches in which generally the local heat flux at an interior location and their higher derivatives are required. However, it is difficult to measure experimentally or to process the measured data for the derivative

of the temperature. Sparrow, Haji-Sheikh, and Lundgren [8], Imber and Kahn [9], Imber [10], Sabherwal [11], Masket and Vastano [12], Deverall and Channapragada [13] and Chen and Thomsen [14] applied the transform method. In these works, the solution is represented in either an integral form after some manipulation of the contour integral from the inverse transform, or in a series form after an expansion of the solution for small and large times. Using Laplace transformation Chen and Thomsen [14] introduced a polynomial in terms of an error function to represent the response of thermocouple measurement and the inversion is accomplished for any transient surface heat flux at the inner surface of a cylindrical tube. In their study, the cylindrical thickness was assumed to be relatively thick such that the temperature at a large distance from the heating surface remains constant. Therefore, only one interior temperature response near the surface was needed in the experimental measurement. Their inversion solution however was valid only for a short duration due to the asymptotic expansion of the modified Bessel function in the inverse Laplace transform. Chen and Chiou [15] studied the inversion problem for the case of a semi-infinite slab or a thick slab using a Laplace transformation. The exact solution was obtained from the inverse Laplace transform for any time interval. It was then shown that their analysis may be approximately applied to the case of the hollow cylinder if the interior temperature response is measured at a location close to the inner wall.

This report presents (a) the improved numerical solution of the inversion solution reported by Chen and Chiou [15] and (b) a further demonstration of the capability of the solution.

The theoretical analysis of Chen and Chiou [15] is recapitulated in Appendix I in which the surface heat flux and temperature is predicted by inverting a temperature history measured at some location inside the solid body. The inversion solution is obtained by invoking Laplace transformation. Both the surface heat flux and temperature are given by Eqs. (19) and (20) in Appendix I.

It was thought that the accuracy of the computer program generated for the solution in the previous report by Chen and Chiou [15] can be improved further for the following reasons. First, the coefficients  $b_n$  (see Appendix Eq. (11)) in the previous formulation has a dimension of temperature. Therefore the determination of the coefficients depends on the temperature range of each particular experiment. It was found that the absolute value of the coefficients  $b_n$  in some cases can become as large as an order of  $10^{44}$ . Therefore during the subsequently numerical manipulation in the computer program error due to round off and the standard fixed up when an underflow occurred may become appreciable. To remedy this difficulty the dimensionless formulation is introduced in the analysis (Appendix I) in which the coefficient  $b_n$  is also made dimensionless. As a result the magnitude of the coefficient  $b_n$  can be greatly reduced. Secondly, the double precision format was not used throughout the previous computer program. It is felt that further accurate results may be obtained if the double precision format is adopted in the program.

In Section II the new computer solution is shown to be indeed more accurate. Also in Section III the solution is shown to be capable of predicting a case involving a periodic surface heat flux or periodic temperature variation.

## II. RESULTS OF THE IMPROVED COMPUTER PROGRAM

The previous computer program of Chen and Chiou [15] was recasted in dimensionless form and written in the double precision format. The new computer program is listed in Appendix II. The results predicted by the new and previous computer program are given in Appendix III and shown in Figure 1 for the case of the constant surface heat flux. This is the case in which a steel slab initially at a uniform temperature is suddenly subjected to a constant heat flux  $Q$  at one of the surfaces and kept at the initial temperature on the other surface. Figure 1 shows the solution predicted by inverting the temperature response at an interior of the slab from the new and previous computer program. This solution predicted by the new and previous programs used the ten term representation for the thermocouple response. The comparison clearly shows the improvement of the new solution over the previous one. Except for the short time duration the solution with the new program reduces the error to only one half of the error of the previous program i.e., an error of less than one percent. In the short time period the solution exhibits a Gibbs phenomenon\* because of the discontinuity of the surface temperature gradient occurred at initial condition. The solution shows a 17% of initial overshoot of heat flux and then a 7.8% of undershoot before the solution approaches the constant heat flux. It should be remarked that Gibbs phenomenon is artificially

\* Gibbs phenomena [3]: for a sequence of transformation  $T_n(t)$ ,  $n = 1, 2, \dots$  of a function  $q(t)$  (here  $q(t) = \text{constant}$ ) if the interval  $[\lim_{t \rightarrow t_0} \inf_{n \rightarrow \infty} T_n(t), \lim_{t \rightarrow t_0} \sup_{n \rightarrow \infty} T_n(t)]$  contains points outside the interval  $[\lim_{t \rightarrow t_0} \inf q(t), \lim_{t \rightarrow t_0} \sup q(t)]$  then the sequence is said to exhibit a Gibbs phenomena.

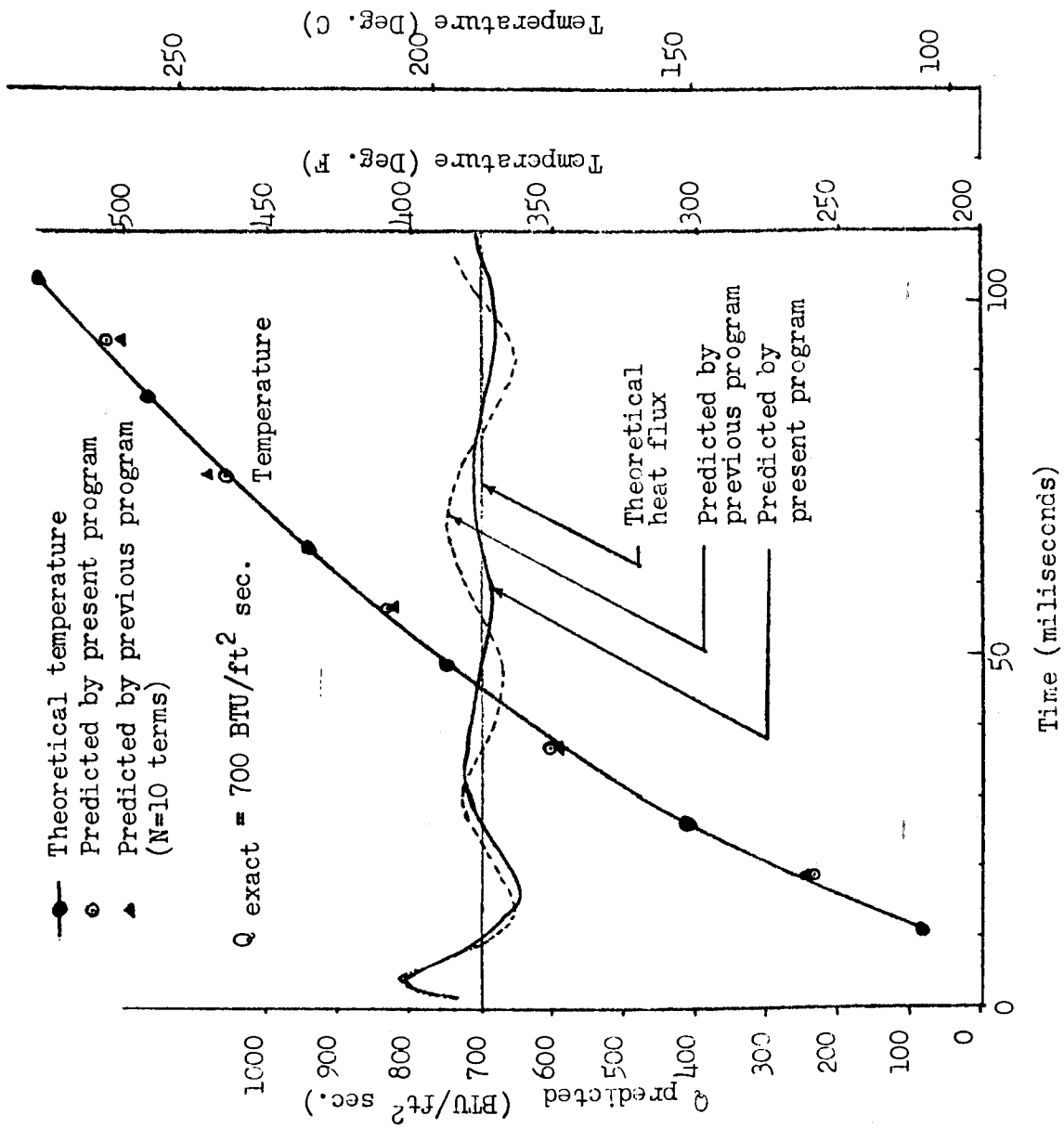


Figure 1 Comparison of Previous and Present Programs

introduced due to the idealization of the initial condition. In most practical situations the surface heat flux will be continuous. Therefore, Gibbs phenomenon will not appear.

Figure 2 (see Table 1 also) shows the comparison between the solutions for the constant heat flux case with 10 and 20 term representation for the thermocouple response. One sees that the solution with 20 term representation after the initial Gibbs phenomenon quickly approaches the expected constant heat flux solution with a negligible error of less than 0.14 percent. This shows the accuracy of the new computer program. From Figure 2 one also observes that both overshoot and undershoot of Gibbs phenomenon are smaller for the 20 term representation. Additionally the points of the overshoot and the undershoot has moved to near the zero time which agrees with the characteristic of Gibbs phenomenon. According to Gibbs phenomenon the point of overshoot should approach the initial zero if the number of term of the series which represent the thermocouple response is increased to infinite.

## VII. VERIFICATION OF OSCILLATORY SOLUTION

As a measure of applicability of the present inversion solution, a test problem was solved for the case of a slab subjected to a preiodic surface temperature variation on one surface and held to the initial temperature on the other. The analytic solution for the problem is given in Appendix IV where a more suitable form of the solution than the one given by Carslaw and Jaeger [4] is derived and tabulated for the thermocouple response at one tenth of the slab thickness from the surface. The

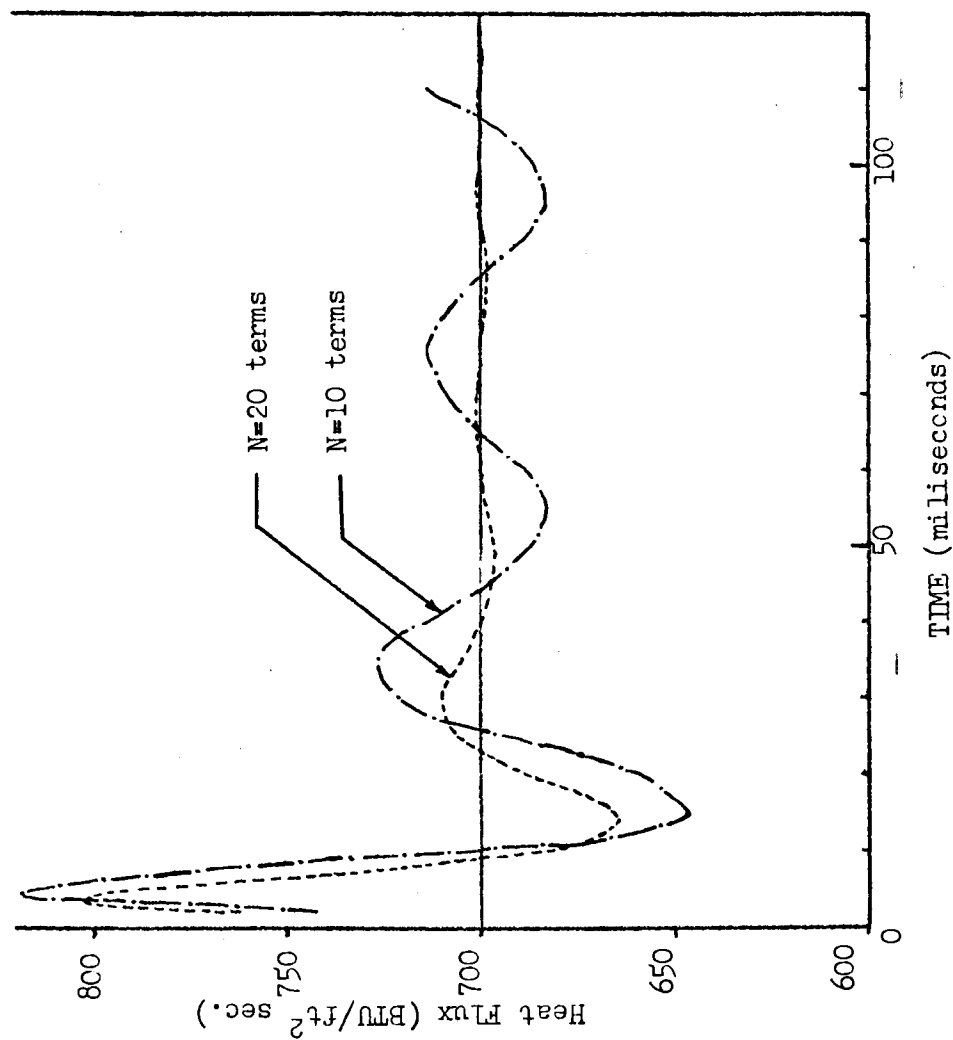


Figure 2 Comparison of Different Polynomial Representations



TABLE 1

Comparison of Inversion Prediction and Exact Solution

N	t	f(t)	$\theta(0,t)$	$\theta(0,t)$	ERROR*	$\frac{\partial \theta}{\partial x}(0,t)$	$\frac{\partial \theta}{\partial x}(0,t)$	ERROR*
		$\theta(1,t)$	EXACT	PREDICTED	%	EXACT	PREDICTED	%
20	0.1	0.0624	0.3758	0.3773	+0.399	1.0000	0.9873	-1.27
	0.2	0.1628	0.5315	0.5314	+0.019	1.0000	1.0014	+0.14
	0.4	0.3395	0.7516	0.7516	0.00	1.0000	1.0001	+0.01
	0.6	0.4882	0.9205	0.9205	0.00	1.0000	1.0000	0.00
	0.8	0.6183	1.0629	1.0628	-0.009	1.0000	0.9996	-0.04
	1.0	0.7353	1.1884	1.1878	-0.05	1.0000	1.9986	-0.14
10	0.1	0.0624	0.3758	0.4048	+7.71	1.0000	1.0796	+7.96
	0.2	0.1628	0.5315	0.5284	-0.58	1.0000	0.9391	-6.09
	0.4	0.3395	0.7516	0.7516	0.00	1.0000	1.0179	+1.79
	0.6	0.4882	0.9205	0.9205	0.00	1.0000	0.9923	-0.77
	0.8	0.6183	1.0629	1.0626	-0.028	1.0000	1.0043	+0.43
	1.0	0.7353	1.1884	1.1887	+0.025	1.0000	0.9963	-0.37

$$*ERROR \% = ((PREDICTED)-(EXACT))/(EXACT)$$

surface is subjected to a periodic temperature variation with a period of 8 milliseconds. Fifteen data points of the temperature response at the thermocouple location are then input to the inversion program for prediction of the surface temperature and heat flux. The result is shown in Figure 3 and Appendix III where the data symbol "2" denotes the thermocouple response and "1" the surface temperature. The accuracy of the inversion program is shown in Table 2. Except for the extremely short time period of 0.4 milliseconds the prediction by the inversion program with 15 term representation is within 2 percent of error.

The accuracy can be improved more if more data points are used. Figure 4 shows the predicted surface heat flux which we were unable to compute from the series solution (Eq. (5) of Appendix IV). This demonstrates the versatility of the inversion solution.

#### IV. APPLICATION OF THE INVERSION PROGRAM

Three sets of data (see Appendix III) provided by Rock Island Arsenal for the temperature response of a thermocouple embedded in a M60 gun barrel were utilized to evaluate the inversion solution. The inversion prediction for the surface heat flux from all three sets of data were extremely high when compared with other known data calculated by Chen and Chiou [15]. Since the program correctly predicted the surface heat flux for other sets of experimental data it was judged that the three sets of data may contain inaccurate initial time. For most experimentations the recording instrument is likely to experience some delay in responding to the extremely fast transient heat flux typical in gun bores. Therefore an advanced shift of time of 2 milliseconds in the data was tested. The

PREDICTED SURFACE AND THERMOCOUPLE TEMP. (ORD. DEG.F.) VS TIME (ABS. SEC)

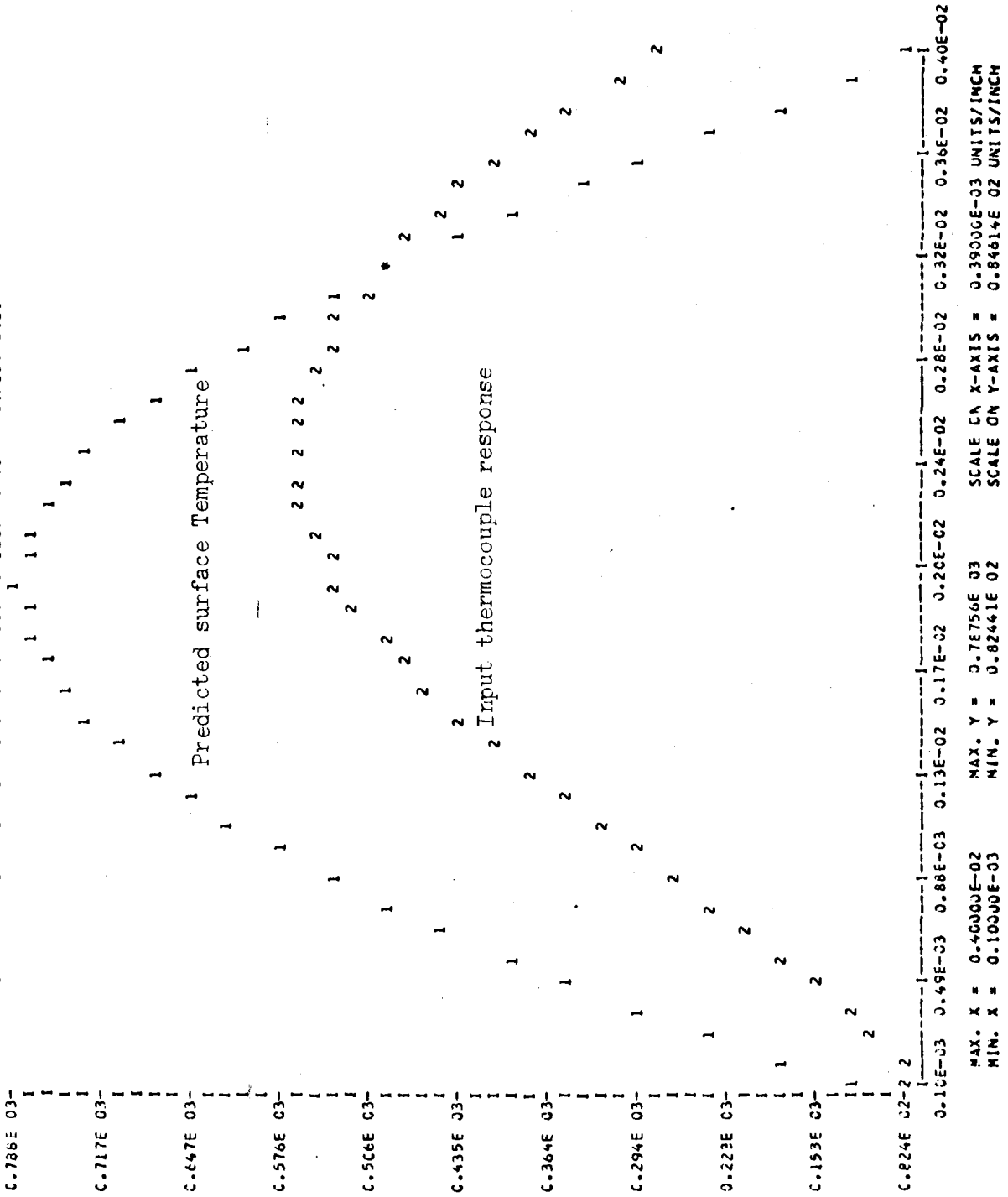


Figure 3 Predicted Surface Temperature

Table 2

## Comparison of Inversion Prediction and Exact Solution

Time (sec.)	Surface Temperature (theoretical)	Surface Temperature (predicted)	Error* %
0.0002	189.5041	193.4060	+3.563
0.0004	296.3119	301.4311	+2.367
0.0006	397.7934	403.8998	+1.921
0.0008	491.4497	498.3988	+1.689
0.0010	574.9747	582.5683	+1.534
0.0012	646.3119	654.3358	+1.417
0.0014	703.7046	711.9396	+1.321
0.0016	745.7396	753.9650	+1.236
0.0018	771.3818	779.3802	+1.157
0.0020	780.0000	787.5615	+1.08
0.0022	771.3818	778.3093	+1.00
0.0024	745.7396	751.8527	+0.918
0.0026	703.7046	408.8443	+0.874
0.0028	646.3119	650.3441	+0.712
0.0030	574.9747	577.7933	+0.569
0.0032	491.6697	492.9790	+0.372
0.0034	397.7934	397.9893	+0.0616
0.0036	296.3119	295.2761	-0.679
0.0038	189.5041	188.8522	-0.595

\*Error = ((Predicted - (Exact))/(Exact)) x 100

PREDICTED SURFACE HEATFLUX (ORD. BTU/FTSQ SEC) VS TIME (ABS. SEC)

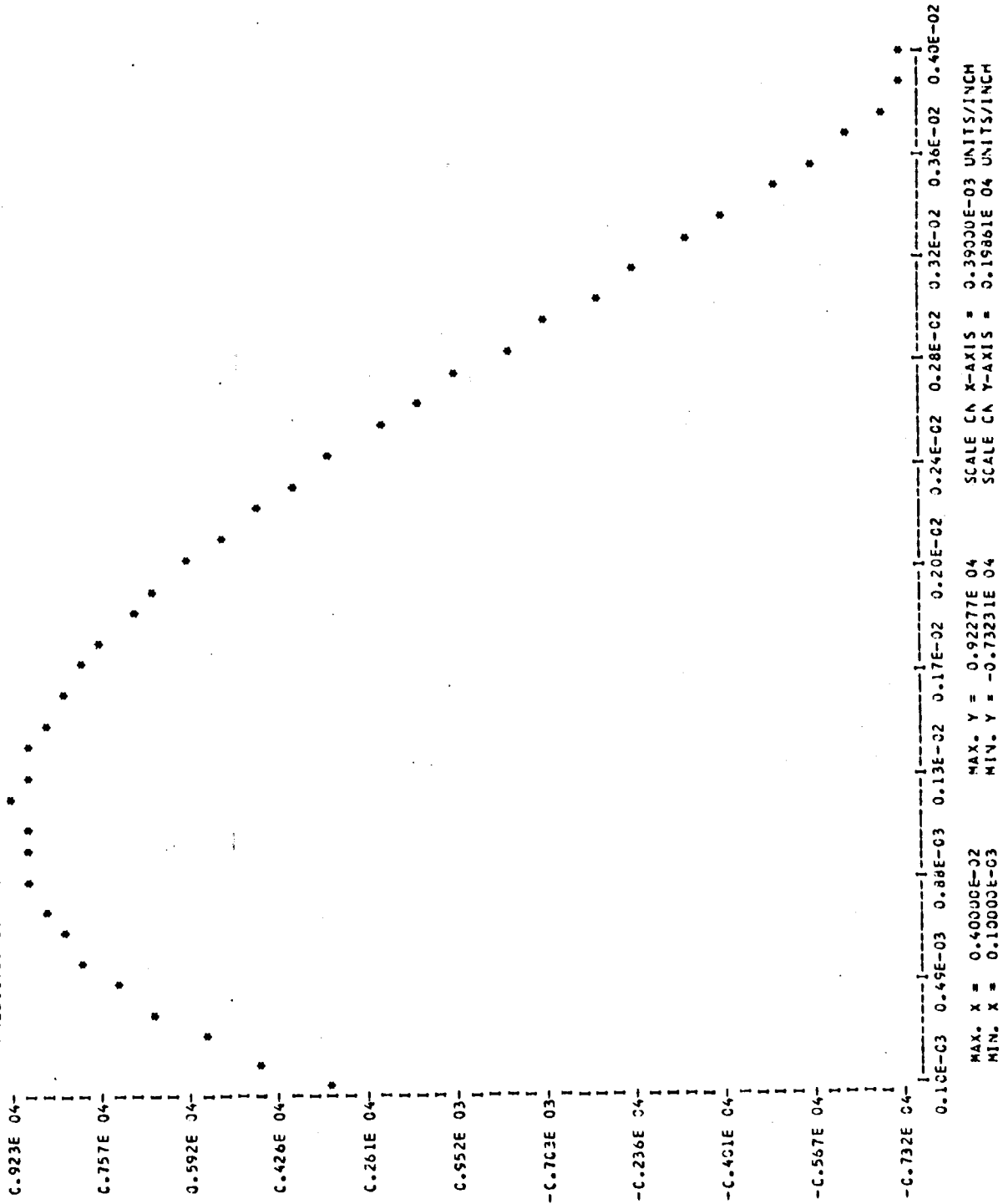
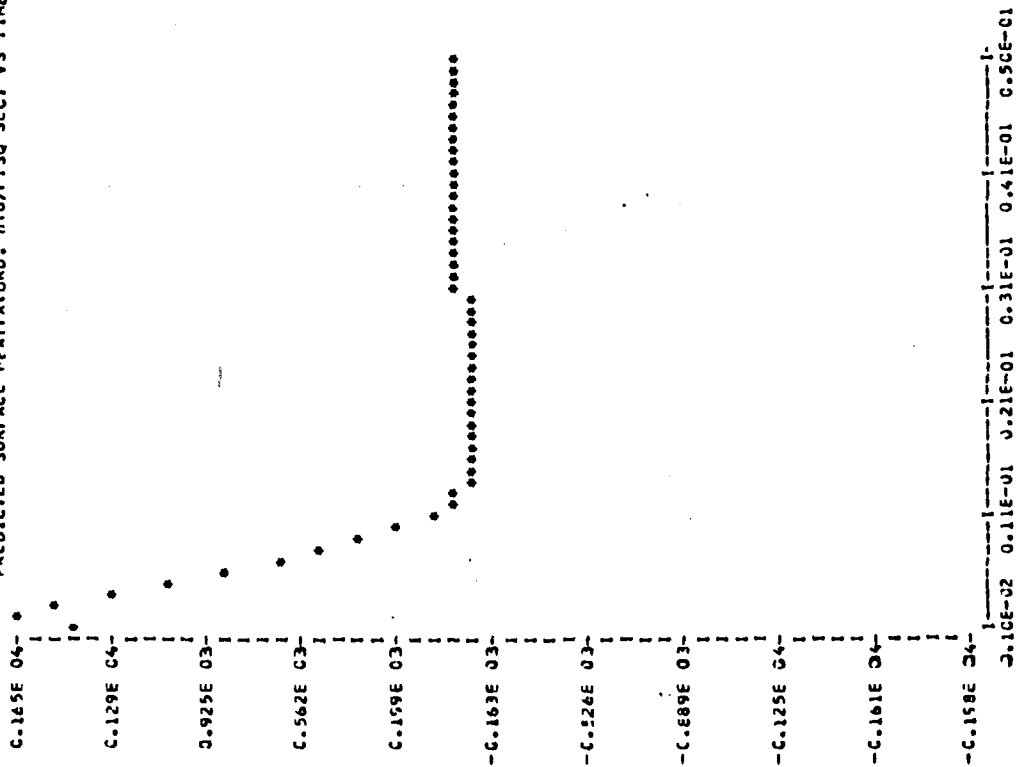


Figure 1 Predicted Surface Heat Flux

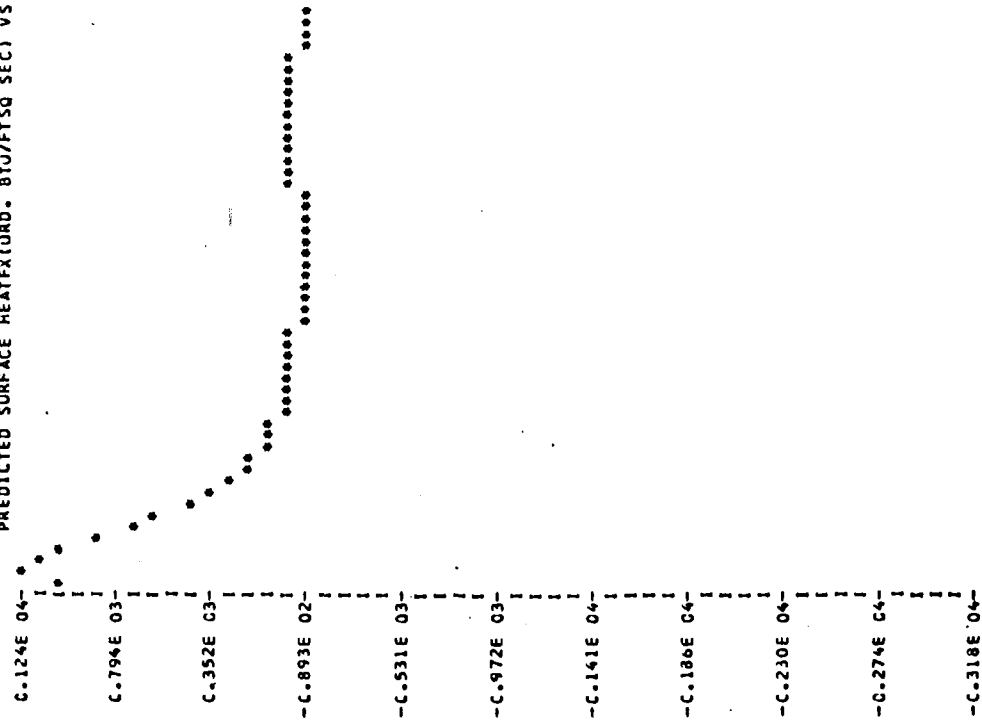
result is shown in Appendix III-3. Figures 5, 6 and 7 respectively show the predicted maximum heat flux of 1650, 1235 and 2695 Btu/ft<sup>2</sup> sec. approximately at 2 milliseconds after the firing, while Figures 8, 9 and 10 show that the surface temperature are 273, 234 and 396 degree F. approximately at 6 milliseconds after the firing.

Figure 5 M60 GUN THERMOCOUPLE 10 21 INCHES FROM BREACH  
 PREDICTED SURFACE HEATFLUX(D. BTU/FTSQ SEC) VS TIME(ABS. SEC).



MAX. X = 0.10000E 00  
 MIN. X = 0.10000E-02  
 MAX. Y = 0.14504E 04  
 MIN. Y = -0.15771E 04  
 SCALE CN X-AXIS = 0.99000E-02 UNITS/INCH  
 SCALE CN Y-AXIS = 0.43500E 03 UNITS/INCH

Figure 6 M60 GUN THERMOCOUPLE 7 15.0 INCHES FROM BREACH  
 PREDICTED SURFACE HEATFLUX(D. BTU/FTSQ SEC) VS TIME(ABS. SEC)



MAX. X = 0.10000E 00  
 MIN. X = 0.10000E-02  
 MAX. Y = 0.12354E 04  
 MIN. Y = -0.31802E 04  
 SCALE CN X-AXIS = 0.9910E-02 UNITS/INCH  
 SCALE CN Y-AXIS = 0.527E 03 UNITS/INCH

Figure 7 M60 GUN THERMOCOUPLE 1 9.0 INCHES FROM BREICH  
 PREDICTED SURFACE HEATFX(ORD. BTU/FYSQ SEC) VS TIME(ABS. SEC)

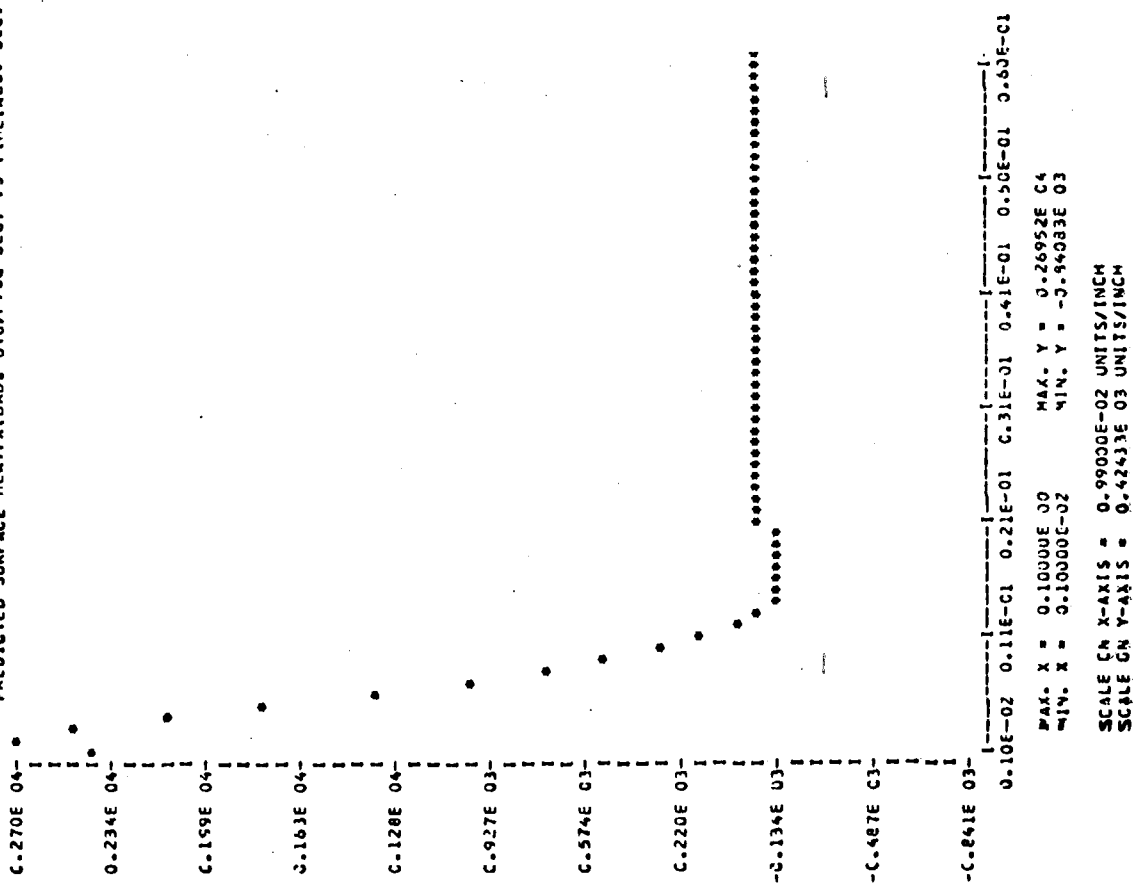


Figure 8 M60 GUN THERMOCOUPLE 10 21 INCHES FROM BREICH  
 PREDICTED SURFACE AND THERMOCOUPLE TEMP.(ORD. DEG.F.) VS TIME(ABS. SEC)

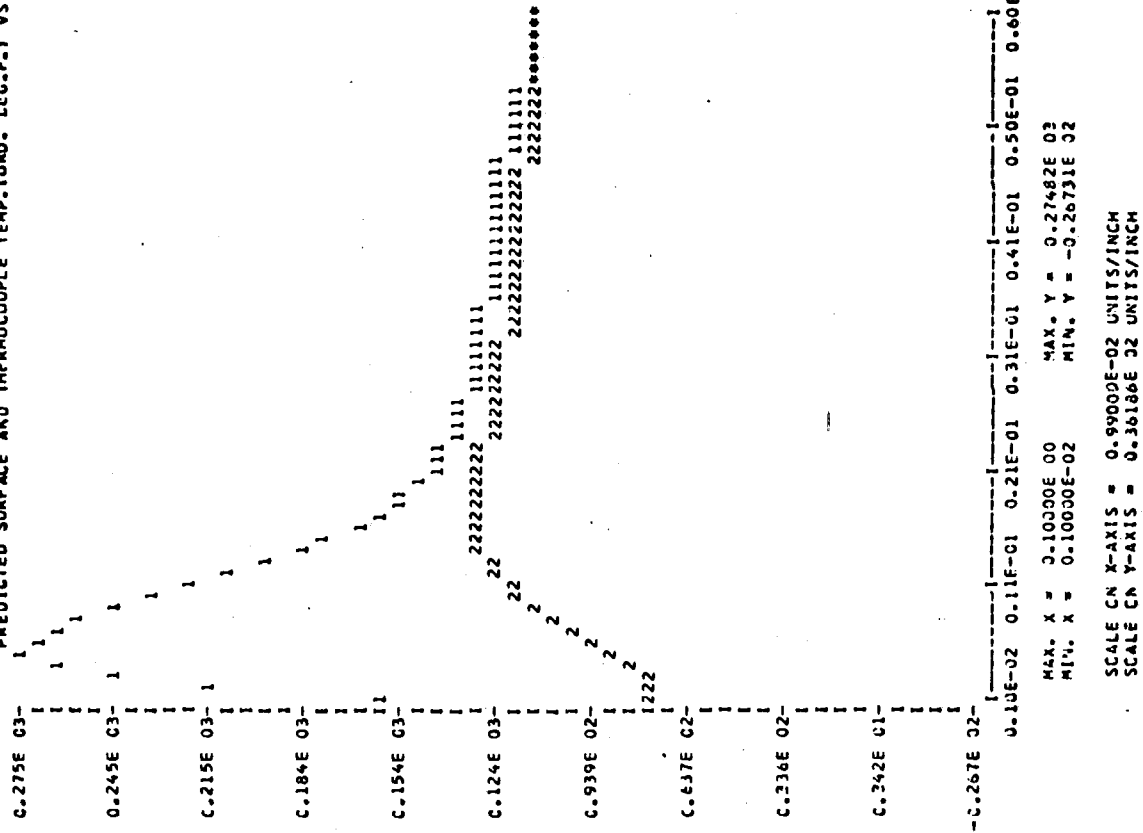




Figure 9 M40 GUN THERMOCOUPLE 7 15.0 INCHES FROM BREECH  
 PREDICTED SURFACE AND THERMOCOUPLE TEMP. (ORD. DEG.F.) VS TIME (ABS. SEC)

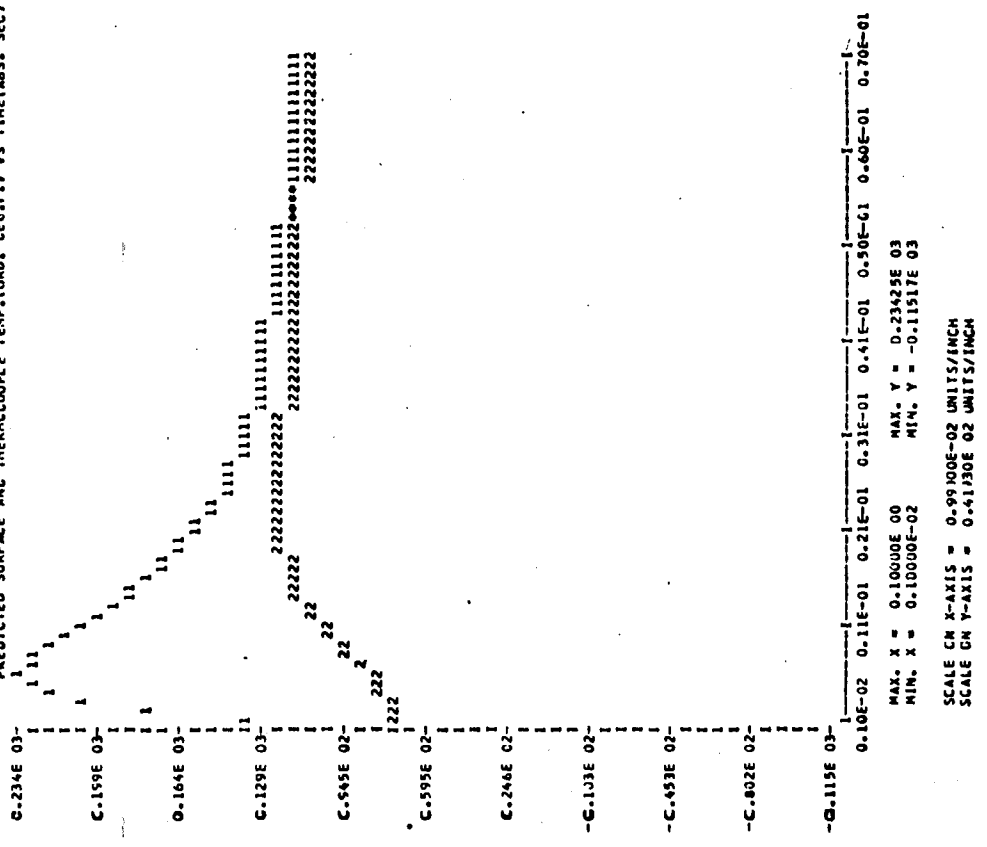
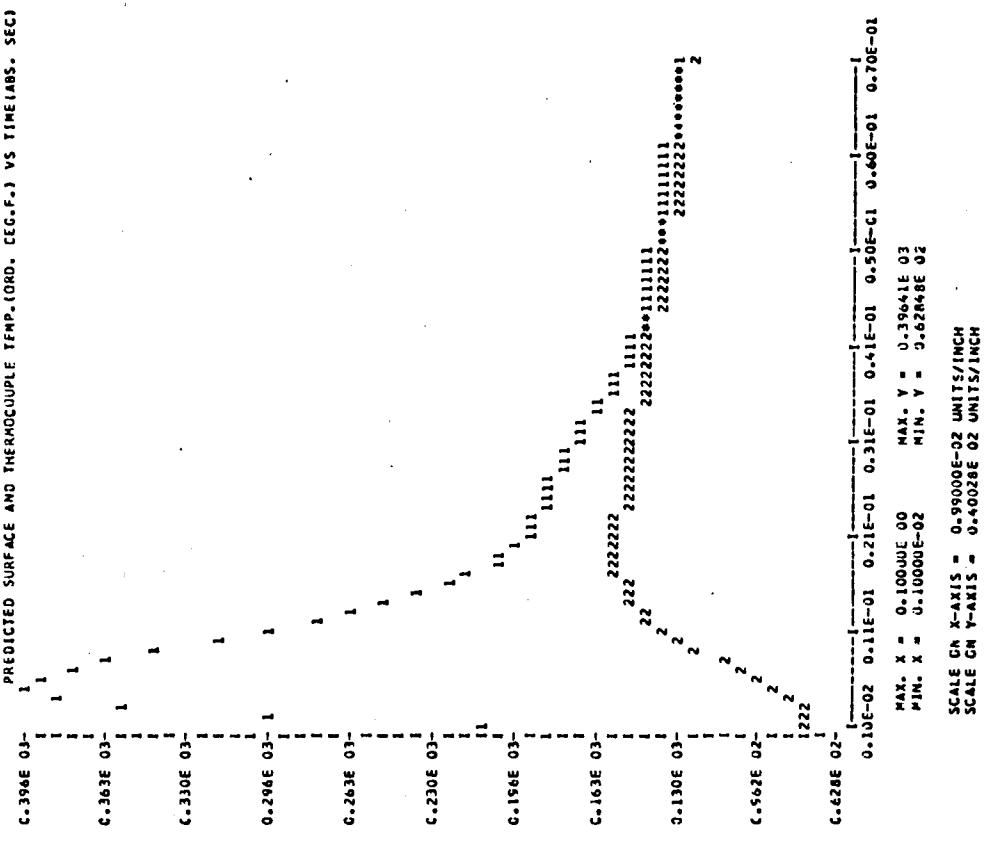


Figure 10 M60 GUN THERMOCOUPLE 4 9.0 INCHES FROM BREECH  
 PREDICTED SURFACE AND THERMOCOUPLE TEMP. (ORD. DEG.F.) VS TIME (ABS. SEC)



## V. CONCLUSION AND SUGGESTION

The new inversion computer program was thoroughly tested for its applicability and accuracy. The program can invert an intrinsic temperature response to predict the case of a constant surface heat flux or the case of a periodic surface temperature within 2% of deviation except at the extremely short time period. This was achieved with the maximum number of 20 input data points.

It is expected that the accuracy of the prediction will increase if the number of data input is increased.

Based on the experience gained in working with the program the following suggestions are thought relevant.

- (1) In conducting an experiment it is vital that both temperature and the time in the intrinsic measurement of surface heat flux and temperature must be much more accurate than the direct measurement. This is because the inversion problem always involves predicting a large heat flux or temperature variation from the data with small variation. Therefore a slight error in the time or temperature measurement a large error will result in the prediction of the surface heat flux or temperature. For example, in the prediction of gun barrel heat flux considered in Section III an error of two milliseconds in time will lead to 100 percent error in prediction of the surface heat flux.

(2) In selecting the data points for input to the computer program care must be exercised not to create a locally abrupt jump in the data. An abrupt change in data points will often introduce an abnormal fitting of a curve in its neighborhood and hence resulting in an incorrect prediction of the surface heat flux and temperature. If indeed the abrupt jump of the data must be used, then more data points in its neighborhood must also be chosen.

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## APPENDIX I. ANALYSIS OF THE INVERSION PROBLEM

Consider a slab, having a sufficient wall thickness,  $L$ , such that the outer surface temperature has a negligible response when the inner surface is exposed to a transient heat flux. A probe, for example a thermocouple, is located at  $X = X_1$  and it is normally desirable, as reported by Chen and Li [16], to be close to the heating surface since a better transient response and more accurate experimental measurements can be obtained to reduce error amplification in the mathematical inversion program. Under these circumstances we thus assume in the analysis  $L/X_1 \gg 1$ . The governing equation for the transient heat conduction may be written in a dimensionless form as

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} \quad (1)$$

with the initial and boundary conditions

$$\theta(x, 0) = 0 \quad (2)$$

$$\theta(\infty, t) = 0 \quad (3)$$

$$\theta(1, t) = f(t) \quad (4)$$

Where  $x = X/X_1$  is a dimensionless distance from the heating surface and  $t = \alpha\tau/X_1^2$  is a dimensionless time or Fourier number with  $\alpha$  being the thermal diffusivity and  $\tau$  the real time.  $\theta = (T - T_0)/T_0$  is the dimensionless temperature above the initially uniform temperature  $T_0$ .

$f(t) = (F(\tau) - T_0)/T_0$  is the dimensionless measured temperature response at  $x = 1$  with  $F(\tau)$  being the measured absolute temperature. The

inversion problem is then given the interior temperature  $f(t)$  to predict the surface temperature  $\theta(0,t)$  and the surface heat flux per unit area  $q(0,t)$  or  $\left. \frac{\partial \theta}{\partial x} \right|_0 = -q(0,t)X_1/T_0\kappa$ . Here  $\kappa$  is the thermal conductivity of the solid.

The above problem may be solved by Laplace Transformation. Let the transformation be:

$$\bar{\theta}(x,s) = \int_0^{\infty} \theta(x,t)e^{-ts} dt \quad (5)$$

where  $\theta$  is continuous otherwise satisfied the Dirichlet's condition. The temperature function  $\theta$  is recovered by inversion of the Laplace Transformation as:

$$\theta(x,t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \bar{\theta} e^{st} ds \quad (6)$$

where  $c$  is a suitable positive value. Equation (1) and the initial condition (2) under transformation (5) become:

$$\frac{d^2 \bar{\theta}}{dx^2} = s\bar{\theta} \quad (7)$$

which has a solution, with integration constants  $A$  and  $B$ ,

$$\bar{\theta}(x,s) = A e^{\sqrt{sx}} + B e^{-\sqrt{sx}} \quad (8)$$

The transformation of boundary conditions (3) and (4) into the Laplace

plane give  $\bar{\theta}(\infty, s) = 0$ ,  $\bar{\theta}(1, s) = \bar{f}(s)$ . Substitution of the boundary conditions into equation (8), we get  $\bar{\theta}$  and its derivative as:

$$\bar{\theta}(x, s) = \bar{f}(s) e^{\sqrt{s}(1-x)} \quad (9)$$

$$-\frac{\partial \bar{\theta}}{\partial x} = \sqrt{s} \bar{f}(s) e^{\sqrt{s}(1-x)} \quad (10)$$

According to Chen and Thomsen [14], we choose the temperature response as measured by a probe to be represented by a polynomial

$$f(t) = \sum_{n=1}^N b_n (4t)^n \Gamma(n+1) i^{2n} \operatorname{erfc} \left( \frac{1}{2\sqrt{t}} \right) \quad (11)$$

or in Laplace plane

$$\bar{f}(s) = e^{-\sqrt{s}} \sum_{n=1}^N \Gamma(n+1) \frac{b_n}{s^{1+n}} \quad (12)$$

The  $b_n$ 's are coefficients of the expansion to be determined such that the  $N$  term polynomial describes the temperature response  $f(t)$  measured at  $x = 1$ . With equation (12), equations (9) and (10) can be simplified to

$$\bar{\theta}(x, s) = e^{-x\sqrt{s}} \sum_{n=1}^N \Gamma(n+1) \frac{b_n}{s^{1+n}} \quad (13)$$

$$-\frac{\partial \bar{\theta}}{\partial x} = e^{-x\sqrt{s}} \sum_{n=1}^N \Gamma(n+1) \frac{b_n}{s^{1/2+n}} \quad (14)$$

The inversion of equations (13) and (14) at  $x = 0$  give:

$$\theta(0,t) = \sum_{n=1}^N b_n t^n \quad (15)$$

$$-\left. \frac{\partial \theta(x,t)}{\partial x} \right|_{x=0} = \sum_{n=1}^N b_n t^{n-1/2} \frac{\Gamma(n+1)}{\Gamma(n+1/2)} \quad (16)$$

where  $\theta(0,t)$  gives the surface temperature and  $-\frac{\partial \theta(0,t)}{\partial x}$  gives the surface heat flux  $\frac{qX_1}{\kappa T_0}$  as a function of time.

The integral of the error function in (11) is defined as:

$$i^{2n} \operatorname{erfc}\left(\frac{1}{2\sqrt{t}}\right) = \int_{\left(\frac{1}{2\sqrt{t}}\right)}^{\infty} i^{2n-1} \operatorname{erfc}(y) dy \quad (17)$$

with  $n = 0$ ,  $\operatorname{erfc}(y) = \frac{2}{\sqrt{\pi}} \int_y^{\infty} e^{-x^2} dx$ .  $\Gamma(n)$  in equations (11) and (16) is the gamma function or Euler's integral function of the second kind.

$$\Gamma(n) = \int_0^{\infty} e^{-\omega} \omega^{n-1} d\omega \quad (18)$$

It should be remarked that the choice of the particular form (11) is to ensure the convergence of the solution on the Laplace plane and an analytic inversion back to the physical plane. With  $b_n$  coefficients determined from equation (11) and the experimental measurement of the temperature response  $f(t)$  at  $x = 1$ , the surface temperature,  $T_w(t)$ , is obtained from equation (15) as

$$T_w(t) = T_0 \left( 1 + \sum_{n=1}^N b_n t^n \right) \quad (19)$$



and the heat flux,  $q(t) = \frac{-T_o \kappa}{X_1} \frac{\partial \theta}{\partial x} \Big|_{x=0}$ , from equation (16) as:

$$q(t) = \frac{-T_o \kappa}{X_1} \sum_{n=1}^N b_n t^{n-1/2} \frac{\Gamma(n+1)}{\Gamma(n+1/2)} \quad (20)$$

The above solution is the exact solution for predicting the transient surface heat flux and temperature valid for both short and long time durations as long as the slab is thick enough such that the outer surface maintains its initial temperature. The feature of the present solution is the polynomial (11) which on the Laplace plane gives a term in equation (12)  $\exp(-\sqrt{s})$  to cancel the term  $\exp(\sqrt{s})$  in equations (9) and (10). This polynomial (11) as suggested by Chen and Thomsen [15] makes the present solution simpler than many inversion solutions derived in the past and valid in any time.

If the fluid temperature,  $T_g(t)$ , away from the surface of the slab is known, then the instantaneous heat transfer coefficients,  $h(t)$ , can be determined from Newton's cooling law as:

$$h(t) = \frac{q(t)}{T_g(t) - T_w(t)} \quad (21)$$

where heat flux,  $q(t)$ , and wall temperature  $T_w(t)$  are given by equations (19) and (20), and the average heat transfer coefficient up to time  $t$ ,  $\bar{h}(t)$ , can be defined as:

$$\bar{h}(t) = \frac{\int_0^t q(t') dt'}{\int_0^t [T_g(t') - T_w(t')] dt'} \quad (22)$$

APPENDIX II

IMPROVED COMPUTER PROGRAM

Cartesian Inversion Problem

PAGE 0001

15/33/00

DATE = 76157

MAIN

LEVEL 21

FORTRAN IV 6

```

C I      ** DATA INPUT **
C I      *****
C I      TITLE OF DATA BLANK FIRST 20 SPACE)
C I      9CRAD=BORE RADIUS OF CYLINDER (FT.)
C I      OTR= OUTER RADIUS (FT.)
C I      THICK=OUTER RADIUS-BORRED (FT.)
C I      DIS=BORE TO THERMOCOUPLE DISTANCE (FT.)
C I      TEMPO=INITIAL THERMOCOUPLE TEMPERATURE(F.)
C I      TG(1)= INITIAL GAS CORE TEMPERATURE AT NORM TIME 0.00001
C I      ALP=THERMAL DIFFUSIVITY OF MATERIAL(F75C/SEC)SEVENTH CARD
C I      THCON=THERMAL CONDUCTIVITY OF MATERIAL(8TU/FT.SEC.DEG.F)
C I      NP=NUMBER OF TIME-TEMPERATURE DATA PAIRS UP TO 25 PAIRS
C I      NB= NUMBER OF B COEFF. TO BE FITTED
C I      TIME(1)=TIME FROM THERMOCOUPLE RESPCASE CURVE(SEC.)
C I      TEMPI(1)= TEMPERATURE FROM THERMOCOUPLE RESPONSE (DEG. F)
C I      TSMFT = TIME SHIFTED FROM DATA READ IN(SEC) LAST CARD
C I
C C
C C
C I      DECLARATION STATEMENTS START COMPUTER PROGRAM
C I

```

```

0001      IMPLICIT REAL*(A-H,O-Z)
0002      DIMENSION TG(210),OM(210),DTWM(210),HM(210),TGM(210),
0003      ITWM(210),TOT(210),TMOI(210),HT(210),EQIP(210),DIEMR(210),
0004      DIMENSION B(26),BERFC(52),TEMP(26),TIME(26),T(26),X(26),GAM(28),
      1C(26,26),FN(26), G(26,210),M(26,210),ECUE(210),DTEMP(210),
      2,TC(26),TEMC(210),TREN(26,26),OT(26,26)
      DATA EPS/.1D-75/

```

```

C I      READ DATA CARDS INTO PROGRAM
C I

```

```

0005      CALL ERNSET(208,0,-1,1,1)
0006      199      1      FORPAT( 20X,50H
0007      200      FORMAT(F20.10)
0008      201      FORPAT(12)
0009      1      READ (5,159,END=999)
0010      READ (5,200) 9ORRAD
0011      READ (5,200) CVTR
0012      READ(5,200) DIS
0013      READ(5,200) TEMPO
0014      READ (5,200) TG(1)
0015      READ(5,200) ALP
0016      READ(5,200) THCON
0017      READ(5,201, END=959) NP
0018      2      READ (5,201) NB
0019      3      WRITE (6,198)
0020      158      FORMAT (1M1)
0021      WRITE (6,199)

```

FIRST  
SECOND  
THIRD  
FOURTH  
FIFTH  
SIXTH  
SEVENTH  
EIGHTH  
NINTH  
TENTH

```

0022 WRITE(6,100J)
0023 FORMAT ( 20X,' BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM')
0024 WRITE (6,109) N6
0025 DC 10 I=1,NP
0026 RE20(5,202) TIME(I),TEMP(I)
0027 CONTINUE
0028
0029 DO 16 I=1,NP
0030 WRITE(6,107)
0031 DO 16 I=1,NP
0032 WRITE(6,108) TIME(I),TEMP(I)
0033 FORMAT(2F20,10)
0034 REAC (5,200) TSHFT
0035 WRITE (6,130) TSHFT

```

```

C I NEW THAT THE DATA HAS BEEN INPUT START CALCULATIONS
C I FIRST SETUP DATA NEEDED FOR THE CALL TC PERFC
C I WHERE PERFC IS INTEGRATED ERROR FUNCTION
C I

```

```

0036 DO 5 I=1,NP
0037 TIME(I) = TIME(I) - TSHFT
0038 THICK=CUTR-BORRAD
0039 RO=0.500
0040 GI=1.000
0041 TRI=ALP/(DIS*2)

```

```

C I
C I CALCULATE DIMENSIONLESS TIME INCREMENT 'TEMIN'
C I THE CONSTANT CRTL VARIES WITH WHAT REAL TIME RANGE WE WANT
C I BUT NOT DEPENDS ON DIFFERENT MATERIAL OR THERMOCOUPLE
C I LOCATION
C I

```

```

0042 CRTL=0.1D-2
0043 TEMIN=TRI*CRLT
0044 XXI=0.500*GI
0045 WRITE (6,203) GI,TRI
0046 FORMAT ('-FOR CHECKING USE GI=',1F7.4,' TRI=',1F7.4 )
0047 DO 20 I=1,NP
0048 T(I)=TIME(I)*TRI
0049 X(I)=0.500/DSORT(T(I))
0050 CONTINUE

```

```

C I
C I USE THE SUBROUTINE PERFC TO CALCULATE INTEGRATED ERROR
C I FUNCTION AND SET UP C(I,J) MATRIX FOR THE SOLUTION OF B
C I

```

```

0051 NN1 =NB*1
0052 FORMAT ('-NN1 = ', 12)
0053 WRITE (6,204) AN1
0054 DC 30 I=1,NP
0055 C(I,NN1)=(TEMP(I)-TEMPO )/(460.000*TEMPO)
0056 DO 40 I=1,NP
0057 NB2=NB*2.000
0058 XI = X(I)
0059 CALL PERFC (NB2,BERFC,XI)

```

```

0058 DO 40 J=1,NB
0059 J2=2*J
0060 C(I,J)=(4.000*T(I))**8*ERFC((J2)*DGAMMA(J+1.000)
0061 CONTINUE
40 C
C
C I NOW B COEFFICIENT FOR THE TEMPERATURE RESPONSE AT
C I THE THERMOCOUPLE ARE SOLVED EITHER BY EXACT OR LEAST SQUARE
C I
C
0062 NT=NP+1
0063 IF (NP - NB) 42, 43, 42
0064 42 CALL LYSQER (NP,NB,C,B,EPS,TIME,NT,TRAN,QTQ,TEMPO)
0065 GO TO 44
0066 43 DET = SIMUL (NP, C, B, EPS, 1,11)
0067 44 CONTINUE
C
C
C I NOW CALCULATE HEAT FLUX FUNCTION AND
C I CALCULATE TEMPERATURE FUNCTION
C I
C
0068 TEM = TEMIN
0069 DO 55 I=1,200
0070 DO 50 J=1,NB
0071 G(J,I)= TEM**J
0072 H(J,I)=-TEM**(J-0.500)*DGAMMA(J+1.000)/DGAMMA(J+0.500)
0073 CONTINUE
0074 TEM = TEM + TEMIN
0075 55 CONTINUE
C
C
C I CALCULATE TEMPERATURE AND HEAT FLUX
C I
C
0076 DO 75 I=1,200
0077 DELT=0.000
0078 QUE=0.000
0079 DO 70 J=1,NB
0080 DELT=DELT+9(J)*G(J,I)
0081 QUE=QUE+8(J)*H(J,I)
0082 DTEMP(I)=DELT
0083 DTEMP(I)=DTEMP(I)*(460.000+TEMPO)+TEMPO
0084 EQU(I)=-QUE
0085 EQU(I)= EQU(I)*(460.000+TEMPO)+THCON/DIS
C
C
C I CALCULATE INST. AND MEAN QUANTITIES FOR TEMP. AND HEAT T.COFF.
C I
C
0086 PI(I)=0.000
0087 TCM(I)=0.000
0088 QM(I)=0.000
0089 DTM(I)=0.000
0090 TMA(I)=0.000
0091 MA(I)=0.000
0092 SUNTW=0.000

```

FORTRAN IV G LEVEL 21 MAIN DATE = 76157 15/33/00

```

0140 FORMAT(' NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=',I2)
0141 WRITE (6,105) N9
0142 WRITE(6,107)
0143 DO 15 I=1,N9
0144 WRITE(6,108) TIME(I),TEMP(I)
0145 WRITE (6,130) TSMET
0146 FORMAT (' NUMBER OF B(I) COFF. TO BE FITTED = ',I2)
0147 FORMAT(' TIME OF DATA SHIFTED BY(SEC)',F10.6)
0148 FORMAT(' ',7X,F20.10,6X,F20.10)
0149 FORMAT(' ',13X,'TIME',5X,'TEMPERATURE')
0150 DC 41 I=1,N8
0151 FORMAT (' COEFFICIENTS OF B(I)=' , D16.8, ' I =',I2)
0152 WRITE (6,229) B(I),I
0153 WRITE (6,198)
0154 WRITE(6,113)
0155 FORMAT (30X,' TEMPERATURE AND HEAT FLUX %/30X,' PREDICTED BY',
1 ' JENQ-SHING CHIOU ',///,2X,
2'(NORTH) (REALTM) (NOR.FLUX) (REAL.FLUX) (NOR.T) (REAL.SUF.T) (
3TH.CO.T)')
0157 WRITE (6,115)
0158 FORMAT(' ',(DMLESS) (SEC.) (DMLESS) (BTU/FTSQ SEC) (DMLESS)
1(DEG.F.) (DEG.F.),/)
0159 TEM = TEMIN
0160 TINC=TEM
0161 TRINC= TEMIN/TRI
0162 TREP=TRINC
0163 DO 90 I=1,200
0164 WRITE (6,114) TEM,TREM,EQUE(I),EQU(I),DTEPP(I),DTEMR(I),TEMC
1(I)
0165 FORMAT (F9.6,F11.7,2(F10.4),2(F12.4),F15.8)
0166 TEM=TEP+TINC
0167 TREM=TREM+TRINC
0168 CCNT=I/N9
0169 CCNT=I/N9
0170 WRITE(6,113)
0171 WRITE (6,115)
0172 WRITE (6,199)
GO TO 1
C
C
C I THIS IS THE END OF THE MAIN PROGRAM
C I
C
999 WRITE(6,1000)
1000 FORMAT('---ALL DATA PROCESSED--EXECUTION OF EXECUTION')
0173
0174
0175
0176
END

```

```

0153 SUMTG=0.000
0154 SUMQ =0.000
0155 SUMDT=0.000
0156 TG(I)=(TG(I)-TEMPO)/(460.000+TEMPO)
0157 DO 80 I=1,200
0158 F=1
0159 SUQO = SUMO +EQUE(I)
0160 CR(I) = SUMQ/F
0161 SUMTG=SUMTG + TG(I)
0162 TGM(I)=SUMTG/F
0163 SUMDT = SUMDT + TG(I)- DTEMP(I)
0164 SUQTM = SUQTM + DTEMP(I)
0165 TMM(I)=SUMTM/F
0166 MM(I) = SUMQ /SUMDT
0167 HT(I)=EQUE(I)/( TG(I) - DTEMP(I))
0168 TG(I+1)=(DTEMP(I)+DTEMP(I+1))/2.000*(EQUE(I)+EQUE(I+1))/(2.000*
1 HT(I))
80 CONTINUE
C
C
C I CALCULATE THERMOCOUPLE TEMPERATURE FROM FITTED EQ. I
C I
C

```

```

0110 TEM = TEMIN
0111 DO 78 I=1,200
0112 TEMC(I) = 0.000
0113 XX=0.50/DSQRT(TEP)
0114 CALL PERFC (NB2,BERFC,XX)
0115 DO 76 J=1,NB
0116 J2=J*2.000
0117 TCT(J)=(TEM**4.000)**J*BERFC(J2)*DGAMMA(J+1.000)
0118 TEMC(I) = TEMC(I) + P(J)*TCT(J)
0119 CONTINUE
0120 TEM = TEM + TEMIN
0121 TEMC(I)=TEMC(I)*(TEMPO+460.000)+TEMPO
0122 CONTINUE
C
C
C I NOW OUTPUT THE DATA I
C I
C

```

```

0123 WRITE (6,198)
0124 WRITE (6,199)
0125 WRITE(6,101) BORRAD
0126 FOPMAT('=-BCRE RADIUS (FT.) =',F10.5)
0127 WRITE (6,115) CUTR
0128 FORMAT (' OUTER RADIUS (FT.)=',F10.5)
0129 WRITE(6,102) DIS
0130 FORMAT(' BOPE TO THERMOCOUPLE DISTANCE (FT)=',F10.6)
0131 WRITE(6,103) TEMPO
0132 FOPMAT(' INITIAL THERMOCOUPLE TEMPERATURE (F.)=',F10.4)
0133 WRITE (6,301) TG(1)
0134 FOPMAT (' INITIAL GAS TEMPERATURE (F.)=',F10.4)
0135 WRITE(6,104) ALP
0136 FOPMAT (' THERMAL DIFFUSIVITY (FTSQ/SEC) =',F15.8)
0137 WRITE(6,105) TMCEN
0138 FOPMAT(' THERMAL CONDUCTIVITY(8TU/FT.SEC.F.)=',F15.8)
0139 WRITE(6,106) NP

```

```

0001 SUBROUTINE PERFC(NP,BERFC,X)
C
C
C I THIS SUBROUTINE CALCULATES THE REPEATED INTEGRALS
C I OF THE COMPLEMENTARY ERROR FUNCTION
C I
C I NP= NUMBER OF REPEATS INTERALS TO BE CALCULATED
C I CERFC= REAL*8 ARRAY FOR ERROR F.
C I X= THE INITIAL VALUE FOR ERROR FUNCTION
C I
C I
C

```

```

0002 IMPLICIT REAL*8(A-H,O-Z)
0003 DIMENSION BERFC(52)
0004 DATA SRIPI/ 0.56418558355/
C

```

```

C I INITIALIZE & CALCULATE I(1)ERFC & I(2)ERFC DEPENDING
C I CN NP
C I
C

```

```

0005 CALL ERASET(208,0,-1,1,1)
0006 DO 11 I =1,50
0007 BERFC(I) =0.000
0008 11 CONTINUE
0009 X2=X*X
0010 BERFC1=SQ(PI)*DEXP(-X2)-X*DERFC(X)
0011 BERFC(1)=BERFC1
0012 IF(NP.EQ.1) RETURN
0013 BERFC2=0.2500*(1.000+2.000*X2)*DERFC(X)-2.000*SRIPI*X*DEXP(-X2)
0014 BERFC(2)=BERFC2
0015 IF(NP.EQ.2) RETURN
C

```

```

C I NOW GO INTO DO LOOP & CALCULATE I(NP)ERFC
C I
C

```

```

0016 DO 10 I=3,NP
0017 BERFC(I)=0.500/I*(BERFC1-2.000*X*BERFC2)
0018 BERFC1=BERFC2
0019 BERFC2=BERFC(I)
0020 10 DEBUG SUBCHK
0021 RETURN
0022 END
C

```

```

0001 SUBROUTINE LTSOER (M,JJ,PHI,8,EPS,TIME,MT,TRAN,QTQ,TEMPO)
C
C I THIS PROGRAM CALCULATES THE LEAST SQUARES COEFFICIENTS FOR
C I MODIFIED POLYNOMIAL WHICH IS POLYNOMIAL TIMES THE INTEGRATED
C I ERROR FUNCTION
C I
C I M IS THE NUMBER OF DATA PAIRS TO BE USED
C I JJ IS THE NUMBER OF DEGREE TO BE FITTED
C
C
C

```

```

0002 REAL*8 PHI(26,26),FX(26),TRAN(26,26),QTQ(26,26),DET,FPX,
0003 IR(26),TIME(26)
0004 CALL SRRSET(208,0,-1,1,1)
0005 LOIP=50
0006 NI=JJ+1
0007 DO 1001 I=1,M
0008 X(I)=TIME(I)
0009 FX(I)=PHI(I,NI)
0010 1001 CONTINUE
0011 ERROR =0.000

```

```

C I FIND THE TRANSPOSE OF THE PHI MATRIX FOR LISTO F TYING AND
C I PERFORM THE MULTIPLICATION AND GET JJ BY JJ MATRIX
C I
C

```

```

0011 CALL TRANS (PHI, TRAN,M,JJ,NT)
0012 CALL MPLY (PHI,TRAN,JJ,M,QTQ,MT)
0013 DO 4 J=1,JJ
0014 QTQ(J,NI)=0.000
0015 DO 4 K=1,M
0016 QTQ(J,NI)=QTQ(J,NI) + TRAN(J,K)*FX(K)
0017 WRITE (6,5) (I,QTQ(I,NI),I=1,JJ)
0018 5 FORMAT (' I=',I2,' QTQ(I,NI)=',D16.7)
C
C I FIND THE SOLUTION TO THE NORMAL EQUATIONS
C I
C

```

```

0019 DET = SIMUL (JJ,QTQ,8,EPS,1,20)
0020 WRITE (6,100)
0021 100 FORMAT (' LEAST SQUARE COEFFICIENTS ')
0022 WRITE (6,6) (I,B(I)) ,I=1,JJ)
0023 6 FORMAT (' I= ',I2,' B(I)= ',D16.7)
0024 WRITE (6,9)
0025 9 FORMAT ('// LEAST SQ COEF PUT '// KTH TIME',I15,'TEMP. GIVEN ',
0026 I30,'FITTED',I42,'ERROR',I55,'PER CENTAGE')
0027 DO 15 K=1,M
0028 FPX=TEMPJ
0029 DO 8 I=1,JJ
0030 FPX=FPX + QTQ(I,NI)* PHI(K,I)
0031 FX(K) = FX(K) + TEMPO
0032 EPR = PHI(K,NI) - FPX + TEMPO
0033 IF (PHI(K,NI).NE. 0.) PER=ERR/(PHI(K,NI) + TEMPO)*100.0
0034 IF (ABS(ERR)).LE.EPRCR) GO TO 15
0035 ERROR = ABS(ERR)

```



```

0035 WRITE (6,17) K,X(K),FX(K),FPX,ERR,PER
0036 FORMAT (113, 3(F10.5,5X),F10.6,3X,F6.2)
0037 WRITE (6,110) ERRCR
0038 FORMAT (' MAXIMUM ERROR =',D16.7)
0039 DEBUG SUBCHK
0040 RETURN
0041 END
    
```

```

0001 SUBROUTINE TRANS (PH1,TRAN,M,N,N1)
    
```

```

C
C I THIS SUBROUTINE GIVE THE TRANSPOSE OF THE MATRIX
C I
REAL*8 PH1(26,26),TRAN(26,26)
CALL ERRSET(208,0,-1,1,1)
DO 10 J=1,N
DO 10 I=1,M
TRAN(J,I)=PH1(I,J)
RETURN
END
    
```

```

0001 SUBROUTINE MULT (PH1,TRAN,M,M,QTQ,N1)
    
```

```

C
C I THIS SUBROUTINE GIVES MULTIPLICATION OF TWO MATRICES
C I
REAL*8 PH1(26,26),TRAN(26,26),QTQ(26,26)
CALL ERRSET(208,0,-1,1,1)
DO 1 I=1,N
DO 1 J=1,N
QTQ(I,J)=0.000
DO 1 K=1,M
QTQ(I,J)=QTQ(I,J)+TRAN(I,K)*PH1(K,J)
RETURN
END
    
```

FUNCTION SIMUL(N,A,X,EPS,INDIC,NRC)

.....GAUSS JORDAN TECHNIQUE MAXIMUM PIVOT  
.....PAGE 290 'APPLIED NUMERICAL METHODS' CARNAHAN

```

0002 IMPLICIT REAL*(A-H,O-Z)
0003 DIMENSION IROW(60),JCOL(60),JORD(60),Y(60),A(26,26 ),X(26)
0004 CALL ERASET(208,0,-1,1,1)
0005 MAX=N
0006 IF(INDIC.EQ.0) MAX=N+1
0007 .....IS N LARGER THAN 50
0008 IF(N.LE.50) GO TO 5
0009 WRITE (6,200)
0010 SIMUL=0.000
0011 RETURN
0012
0013 BEGIN ELIMINATION PROCEDURE
0014 DETER=1.000
0015 DO 18 K=1,N
0016 K=M1=K-1
0017 .....SEARCH FOR THE PIVOT ELEMENT
0018 PIVCT=0.000
0019 DO 11 I=1,N
0020 DO 11 J=1,N
0021 .....SCAN IROW AND JCOL ARRAYS FOR INVALID PIVCT SUBSCRIPTS
0022 IF(K.EQ.1) GO TO 9
0023 DO 8 I=SCAN=1,KM1
0024 DO 8 J=JSCAN=1,KM1
0025 IF(I.EQ.IROW(I,SCAN)) GO TO 11
0026 IF(J.EQ.JCOL(J,SCAN)) GO TO 11
0027 CONTINUE
0028 IF(CABS(A(I,J)).LE.CABS(PIVCT)) GO TO 11
0029 PIVCT=A(I,J)
0030 IROW(K)=I
0031 JCOL(K)=J
0032 CONTINUE
0033 .....INSURE THAT SELECTED PIVOT IS LARGER THAN EPS
0034 IF(CABS(PIVCT).GT.EPS) GO TO 13
0035 SIMUL=0.000
0036 RETURN
0037
0038 UPDATE THE DETERMINANT VALUE...
0039 IPRW=IROW(K)
0040 JCOL=JCOL(K)
0041 DETER=DETER*PIVCT
0042 .....NORMALIZE PIVOT ROW ELEMENTS
0043 DO 14 J=1,MAX
0044 A(IROW,K,J)=A(IROW,K,J)/PIVCT
0045 .....CARRY OUT ELIMINATION AND DEVELOPE INVERSE
0046 A(IROW,K,JCOL)=1./PIVCT
0047 DO 18 I=1,N
0048 AIJCK=A(I,JCOL)
0049 IF(I.EQ.IROW(K)) GO TO 18
0050 A(I,JCOL)=AIJCK/PIVCT
0051 DO 17 J=1,MAX
0052 IF(J.NE.JCOL) A(I,J)=A(I,J)-AIJCK*A(IROW,K,J)
0053 CCNTINJE
0054 .....ORDER SOLUTION VALUES AND CREATE JORD ARRAY

```

```

FCRTRAN IV G LEVEL 21          SIMIL          DATE = 76157          15/33/00          PAGE 0002

0044      DO 20 I=1,N
0045      IROWI=IROW(I)
0046      JCOLI=JCOL(I)
0047      JORD(IROWI)=JCOLI
0048      IF(INDIC-GE.0) X(JCOLI)=A(IROWI,MAX)
20      C      ADJUST SIGN OF DETERMINANT
           INTCH=0
           NPI=N-1
           DO 22 I=1,NMI
           IP1=I+1
           DO 22 J=IP1,N
           IF(JORD(J).GE.JORD(I)) GO TO 22
           JTEMP=JORD(J)
           JORD(J)=JORD(I)
           JORD(I)=JTEMP
           INTCH=INTCH+1
           CCNTINUE
22      C      IF(INTCH/2*.NE.-INTCH) DETER=-DETER
           ...IF INDIC IS POSITIVE RETURN WITH RESULTX
           IF(INDIC.LE.0) GO TO 26
           SIMUL=DETER
           RETURN
           ...IF INDIC IS NEGATIVE OR ZERO, UNSCRAMBLE THE INVERSE
           ...FIRST BY ROWS
26      C      DO 28 J=1,N
           DO 27 I=1,N
           IROWI=IROW(I)
           JCOLI=JCOL(I)
           Y(JCOLI)=A(IROWI,J)
27      C      DO 28 I=1,N
           A(I,J)=Y(I)
           ...THEN BY COLUMNS
           DO 30 I=1,N
           DO 29 J=1,N
           IROWJ=IROW(J)
           JCOLJ=JCOL(J)
           Y(IROWJ)=A(I,JCOLJ)
29      C      DO 30 J=1,N
           A(I,J)=Y(I)
           ...RETURN FOR INDIC NEGATIVE OR ZERO
           SIMUL=-DETER
           RETURN
           ...FORMAT OFR OUTPUT STATEMENT
           ...FORMAT110HON TOO BIG)
200      C      DEBUB SUBCHK
           END
0060
0061
0062
0063
0064
0065
0066
0067
0068
0069
0070
0071
0072
0073
0074
0075
0076
0077
0078
0079
0080
0081
0082

```

## APPENDIX III M60 DATA

M60 GUN THERMOCOUPLE 7 15.0 INCHES FROM BREECH  
BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM  
NUMBER OF B(I) COFF. TO BE FITTED = 10

TIME	TEMPERATURE
0.010000000	110.600000000
0.020000000	124.500000000
0.030000000	123.100000000
0.040000000	119.600000000
0.050000000	116.900000000
0.060000000	114.100000000
0.070000000	112.500000000
0.080000000	110.300000000
0.090000000	107.600000000
0.100000000	106.800000000

TIME OF DATA SHIFTED BY(SEC) = -0.002000  
BORE RADIUS (FT.) = 0.01250  
CUTER RADIUS (FT.) = 0.04417  
BORE TO THERMOCOUPLE DISTANCE (FT) = 0.001830  
INITIAL THERMOCOUPLE TEMPERATURE (F.) = 78.8000  
INITIAL GAS TEMPERATURE (F.) = 4.4937  
THERMAL DIFFUSIVITY (FTSQ/SEC) = 0.00010307  
THERMAL CONDUCTIVITY(BTU/FT.SEC.F.) = 0.00555555  
NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.) = 10  
NUMBER OF B(I) COFF. TO BE FITTED = 10

M60 GUN THERMOCOUPLE 10 21 INCHES FROM BREECH  
BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM  
NUMBER OF B(I) COFF. TO BE FITTED = 10

TIME	TEMPERATURE
0.010000000	126.200000000
0.020000000	131.600000000
0.030000000	124.200000000
0.040000000	119.600000000
0.050000000	116.800000000
0.060000000	113.300000000
0.070000000	109.800000000
0.080000000	107.100000000
0.090000000	105.700000000
0.100000000	104.200000000

TIME OF DATA SHIFTED BY(SEC) = -0.002000  
BORE RADIUS (FT.) = 0.01250  
CUTER RADIUS (FT.) = 0.03562  
BORE TO THERMOCOUPLE DISTANCE (FT) = 0.001670  
INITIAL THERMOCOUPLE TEMPERATURE (F.) = 78.8000  
INITIAL GAS TEMPERATURE (F.) = 4.4937  
THERMAL DIFFUSIVITY (FTSQ/SEC) = 0.00010307  
THERMAL CONDUCTIVITY(BTU/FT.SEC.F.) = 0.00555555  
NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.) = 10  
NUMBER OF B(I) COFF. TO BE FITTED = 10

M60 GUN THERMOCOUPLE 4 9.0 INCHES FROM BREECH  
 BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM  
 NUMBER OF B(I) COFF. TO BE FITTED = 10

TIME	TEMPERATURE
------	-------------

0.0100000000	143.2000000000
0.0200000000	157.3000000000
0.0300000000	151.0000000000
0.0400000000	144.4000000000
0.0500000000	137.5000000000
0.0600000000	132.8000000000
0.0700000000	129.0000000000
0.0800000000	125.9000000000
0.0900000000	122.8000000000
0.1000000000	119.6000000000

TIME OF DATA SHIFTED BY(SEC) = -0.002000

BORE RADIUS (FT.) = 0.01250

CLTER RADIUS (FT.) = 0.05000

BORE TO THERMOCOUPLE DISTANCE (FT) = 0.001830

INITIAL THERMOCOUPLE TEMPERATURE (F.) = 78.8000

INITIAL GAS TEMPERATURE (F.) = 4.4937

THERMAL DIFFUSIVITY (FTSQ/SEC) = 0.00010307

THERMAL CONDUCTIVITY(BTU/FT.SEC.F.) = 0.00555555

NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.) = 10

NUMBER OF B(I) COFF. TO BE FITTED = 10

## APPENDIX IV. THE CASE OF OSCILATORY SURFACE TEMPERATURE

Consider a slab with a sufficient thickness,  $l$ , such that when a surface is subjected to a periodic surface temperature variation with a frequency  $w$  the other surface is held at the initial temperature  $T_0$ . If properties are assumed constant the governing equation for the problem can be written as

$$\frac{\partial v}{\partial t} = \alpha \frac{\partial^2 v}{\partial x^2} \quad (1)$$

with initial and boundary conditions as

$$v(x, 0) = 0 \quad (2)$$

$$v(l, t) = \sin wt \quad (3)$$

$$v(0, t) = 0 \quad (4)$$

where  $v = (T - T_0)/(T_{\max} - T_0)$  and  $\alpha$  is the thermal diffusivity.

The solution of the problem according to Carslaw and Jaeger [4] can be written as

$$v = 2\alpha\pi \sum_{n=1}^{\infty} (-1)(-1)^n \left[ (\alpha n^2 \pi^2 \sin wt - w l^2 \cos wt) + w l^2 e^{-\alpha n^2 \pi^2 t / l^2} \right] \cdot \sin \left( \frac{n\pi x}{l} \right) / [\alpha n^4 \pi^4 + w^2 l^2] \quad (5)$$

Part III PREDICTION OF TRANSIENT SURFACE HEAT FLUX AND  
TEMPERATURE ON A HOLLOW CYLINDER

I. INTRODUCTION

In the study of transient heat transfer many efforts have been made on the so-called "inverse problem" [1,2] where a surface heat flux and temperature is to be predicted by the measured data at some location interior to a body.

In the previous works [1-6] the solution is represented in either an integral form after some manipulation of the contour integral from the inverse transform, or in a series form after the expansion of the solution for small and large times. Using Laplace transformation Chen and Thomsen [6] introduced a polynomial in terms of the error function to represent the response of thermocouple measurement and the inversion is accomplished for any transient surface heat flux at the inner surface of a cylindrical tube. However, their inversion solution is valid only for a short duration due to the asymptotic expansion of the modified Bessel function in the inverse Laplace transform. In this study an exact solution obtained from the inverse Laplace transform by the convolution method is given for the case of hollow cylinder. The solution is valid for both constant and variable heat flux and for both short and long time duration.

II. ANALYSIS

Consider a long hollow cylinder with sufficient wall thickness such that the outer surface temperature has a negligible response when the inner surface is exposed to a thermal pulse of a transient process. This condition considerably simplifies the theoretical analysis as the outer boundary may be assumed to be infinite, and only one interior probe of the cylinder

is required in the experimental measurement. The material of the cylinder is considered to be homogeneous and isotropic with constant thermal diffusivity,  $\alpha$ . Let  $R_i$  and  $R_o$  be, respectively, the inner and outer surface radii.  $R_1$  the radius of the probe location and  $t$  the dimensionless time. If the temperature of the cylinder is initially uniform at  $T_o$ , the mathematical problem governing the temperature  $T$ , may be written as

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \quad 1 < r < r_o = \infty \quad (1)$$

$$\theta(r, 0) = 0 \quad (2)$$

$$\theta(\infty, t) = 0 \quad (3)$$

$$\theta(r_1, t) = f(t) \quad 1 < r_1 < \infty \quad (4)$$

where  $\theta = T - T_o$ ,  $r = R/R_i$ ,  $t = \alpha\tau/R_i^2$ , and  $f(t)$  is the interior temperature response of the thermocouple measured at  $r = r_1$  at the dimensionless time  $t$ . The problem is to predict the surface temperature  $\theta(1, t)$  and heat flux per unit area

$$q = - (K/R_i) (\partial\theta/\partial r) \Big|_{r=1} \quad (5)$$

where  $K$  is the thermal conductivity.

The problem can be solved by Laplace transformation. Let the transformation be

$$\bar{\theta}(r, s) = \int_0^{\infty} \theta e^{-ts} dt \quad (6)$$



when  $\theta$  satisfies the Dirichlet's condition the temperature function  $\theta$  is recovered by inversion of the Laplace transformation as

$$\theta(r, t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \theta e^{st} ds \quad (7)$$

where  $c$  is a suitable positive value. Equation (1) and (2) under transformation (6) becomes

$$\frac{d^2 \bar{\theta}}{dr^2} + \frac{1}{r} \frac{d\bar{\theta}}{dr} = s\bar{\theta}$$

which has a solution of the form

$$\bar{\theta} = A I_0(pr) + B K_0(pr) \quad (8)$$

where  $I_0$  and  $K_0$  are modified Bessel functions of the first and second kind with  $p = (s)^{1/2}$ . With the boundary conditions (3) and (4), (8) becomes

$$\bar{\theta} = \bar{f}(s) [K_0(pr)/K_0(pr_1)] \quad (9)$$

where  $\bar{f}(s)$  is the Laplace transform of the boundary condition (4).

The temperature response measured at  $r = r_1$  can be expressed by a polynomial or numerous other suitable functions. In the present analysis, for reasons to be explained later,  $f(t)$  will be represented as

$$f(t) = \sum_{n=1}^N b_n \int_0^t F_1(\tau) F_n(t - \tau) d\tau \quad (10)$$

If we choose  $F_1(t) = \frac{1}{2\tau} e^{-\frac{r_1^2}{4\tau}}$  and  $F_n(t - \tau)$  being any arbitrary function

depending on  $n$ , for example  $(t - \tau)^n$  etc. then the Laplace transform of Eq. (10) gives

$$\bar{f}(s) = \sum_{n=1}^N b_n K_o(pr_1) \bar{F}_n(s) \quad (11)$$

where  $\bar{F}_n(s)$  is the Laplace transform of  $F_n(t)$ . Substituting Eq. (11) into Eq. (19) we have

$$\bar{\theta}(s, r) = \sum_{n=1}^N b_n K_o(pr) \bar{F}_n(s) \quad (12)$$

It is noted that the  $K_o(pr_1)$  in Eq. (9) has been cancelled by this substitution which explains the choice of  $F(t) = \frac{1}{4K} e^{-\frac{r_1^2}{4K} t}$  in Eq. (10).

The inversion of Eq. (12) gives

$$\theta(t, r) = \sum_{n=1}^N b_n \int_0^t \frac{1}{2\tau} e^{-\frac{r^2}{4\tau}} F_n(t - \tau) d\tau \quad (13)$$

At surface  $r = 1$

$$\theta(t, 1) = \sum_{n=1}^N b_n \int_0^t \frac{1}{2\tau} e^{-\frac{1}{4\tau}} F_n(t - \tau) d\tau \quad (14)$$

The temperature gradient and hence heat flux at surface is

$$\left. \frac{\partial \theta}{\partial r} \right|_{r=1} = - \sum_{n=1}^N b_n \int_0^t \frac{1}{4\tau^2} e^{-\frac{1}{4\tau}} F_n(t - \tau) d\tau \quad (15)$$

some examples of  $F_n(t - \tau)$  function and the representation of the thermocouple response are:

Case 1      If  $\bar{F}_n(s) = \frac{1}{s^2 + n^2}$

then  $f(t) = \sum_{n=1}^N b_n \int_0^t \frac{1}{2\tau} e^{-\frac{r_1^2}{4\tau}} \frac{1}{n} \sin n(t - \tau) d\tau$       (16)

Case 2      If  $\bar{F}_n(s) = \frac{s}{s^2 + n^2}$

then  $f(t) = \sum_{n=1}^N b_n \int_0^t \frac{1}{2\tau} e^{-\frac{r_1^2}{4\tau}} \cos n(t - \tau) d\tau$       (17)

Case 3      If  $\bar{F}_n(s) = \frac{1}{(s + a)^n}$

then  $f(t) = \sum_{n=1}^N b_n \int_0^t \frac{e^{-\frac{r_1^2}{4\tau}} (t - \tau)^{n-1} e^{-a(t-\tau)}}{2\tau(n-1)} d\tau$       (18)

Case 4      If  $\bar{F}_n(s) = s^{-(n+1/2)}$

then  $f(t) = \sum_{n=1}^N b_n \int_0^t \frac{e^{-\frac{r_1^2}{4\tau}} 2^n (t - \tau)^{n-1/2}}{2\tau \cdot 1.3.5 \dots (2n-1)\sqrt{\pi}} d\tau$       (19)

Case 5      If  $\bar{F}_n(s) = \frac{1}{s^n}$

then  $f(t) = \sum_{n=1}^N b_n \int_0^t e^{-\frac{r_1^2}{4\tau}} \frac{(t - \tau)^{n-1}}{2\tau(n-1)!} d\tau$       (20)

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