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**THE WAVEMAKING OF A SHIP:  
APPROXIMATE SOLUTIONS  
BASED ON A PARAMETRIC FORMULATION**

GEOLOGICAL SURV

FEB 24 1975

by

**F. Noblesse**

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Sponsored by  
Office of Naval Research  
Contract N00014-68-A-0196-0011



**IIHR Report No. 172**

**Iowa Institute of Hydraulic Research  
The University of Iowa  
Iowa City, Iowa**

**January 1975**

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## ABSTRACT

Two simple approximate solutions to the problem of the wavemaking of a ship in an inviscid fluid are presented. These approximate solutions are somewhat similar to those proposed previously by Guilloton, Wehausen, Dagan and Noblesse, but the present analysis leads to a new interpretation of the approximate solutions obtained in this paper. The analysis is based on a mapping of the actual flow region onto a parametric space bounded by the horizontal undisturbed free surface and a vertical cut on the ship center-plane. This parametric formulation is related to the approaches used by Guilloton, Wehausen, Dagan and Noblesse. The differences and similarities between these different approaches are discussed in detail, as well as the differences with the usual thin-ship theory.

## ACKNOWLEDGEMENT

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THE WAVEMAKING OF A SHIP: APPROXIMATE SOLUTIONS  
BASED ON A PARAMETRIC FORMULATION

I. INTRODUCTION

Guillotot (1964) proposed a method in which the flow around the hull of a ship is obtained from the classical solution of Michell (1898) applied to a fictitious, so-called linearized hull, rather than to the real hull as in the usual linearized theory. Guillotot's derivation of the transformation relating the linearized hull to the real hull largely relies upon his intuitive physical understanding. The method of Guillotot was found by Gadd (1973) to be remarkably successful in predicting the wave resistance of ship models. A transformation similar to that of Guillotot was also obtained by Wehausen (1969) as part of a second-order thin-ship theory based on the use of Lagrangian coordinates, as pointed out by Wehausen in the discussion of Gadd's paper.

A theoretical foundation for Guillotot's transformation was simultaneously proposed by Dagan (1974) and Noblesse (1974) who developed similar thin-ship perturbation analyses based on a mapping of the physical space onto a strained space by means of a slight straining of the coordinates in the spirit of Lighthill's method of strained coordinates. Dagan and Noblesse obtained similar incomplete (inconsistent) second-order solutions in which the boundary conditions are satisfied to second order and the field equations to first order; and these solutions are essentially similar to that of Guillotot.

In the present paper, a parametric formulation of the problem of the wavemaking of a ship in an inviscid fluid is presented, and two approximate solutions are then derived. This parametric formulation is related to the Lagrangian coordinates used by Wehausen, as well as to the strained coordinates of Dagan and Noblesse. Indeed, Wehausen's formulation appears as a particular case of the general parametrization presented here in Sections III(A) and (B), while the papers by Dagan and Noblesse essentially develop perturbation analyses of this parametric formulation. As a matter of fact, the second of the approximate solutions which are presented here

is fairly close to those obtained by Dagan and Noblesse, and is also similar to that of Guilloton; a direct comparison with the solution of Wehausen is somewhat more difficult but the first of the approximate solutions presented here is essentially contained in Wehausen's formulation.

However, whereas the boundary conditions in the approximate solutions of Dagan and Noblesse appear to be satisfied to second order in a perturbation expansion in terms of the beam/length ratio, the kinematic conditions both at the free surface and on the ship hull are satisfied exactly in the first of the present approximate solutions, and the Bernoulli equation is also satisfied exactly in the second approximation. Thus, the second approximation presented in this paper is one in which all the boundary conditions are satisfied exactly while the field equations are satisfied approximately. In the analyses of Guilloton and Wehausen, the boundary conditions appear to be satisfied approximately only.

If perturbation expansions are introduced, then the field equations satisfied by the present approximate solutions are seen to differ from the exact field equations by second-order terms. An important point, however, is that the perturbation parameter  $\epsilon$ , say, in these perturbation expansions could just as well be regarded as the beam/length ratio, a slenderness parameter or the draft/length ratio, or even a submergence parameter for the case of a submerged body. In other words, whereas the approximations of Dagan, Noblesse, and also Wehausen, are thin-ship approximations, there are no reasons for regarding the present approximate solutions as thin-ship approximations rather than slender-, or flat-, ship approximations.

Thus, although the approximate solutions derived in the present paper do not differ very significantly from those of Dagan, Noblesse and Guilloton, and are also somewhat similar to that of Wehausen, they are given here a new and strikingly different interpretation.

## II. FORMULATION OF THE PROBLEM IN THE PHYSICAL SPACE

A. Coordinate Systems and Definition of the Ship Hull: Let us consider a ship in steady forward motion at the free surface of a calm sea which is supposed to be of infinite depth and infinite lateral extent. The constant forward speed of the ship is denoted by  $U$ . It is assumed that the ship has been moving for a infinite time so that the flow and surface waves induced by its motion are time-independent with respect to a coordinate system moving with the ship. The equivalent problem of the steady flow of a uniform stream of velocity  $U$  past a ship fixed in space is then considered. In order to take into account the effects of sinkage and trim, two systems of coordinates must be introduced, see Figure 1.

The coordinate system  $O'X'Y'Z'$  is fixed with respect to the ship, and is determined by the equilibrium position of the ship when at rest. The  $X'$  axis is chosen along the intersection of the ship centerplane with the (undisturbed) free surface when the ship is at rest, and is pointing towards the stern of the ship. The origin  $O'$  is taken, say, amidships. The  $Y'$  axis is in the ship centerplane, perpendicular to the  $X'$  axis, and pointing upwards (the  $Y'$  axis is then vertical when the ship is at rest). The  $Z'$  axis is horizontal and its direction is chosen so that the  $X'$ ,  $Y'$  and  $Z'$  axes form a right-handed coordinate system.

The other coordinate system is denoted by  $OXYZ$ . The  $X$  axis is taken along the intersection of the ship centerplane with the horizontal undisturbed free surface, and pointing towards the stern. The  $Y$  axis is in the centerplane of the ship, vertical, pointing upwards, and passes through the origin  $O'$  of the coordinate system  $O'X'Y'Z'$ . The origin  $O$  of the coordinate system  $OXYZ$  is then taken at the point where the vertical line drawn from the origin  $O'$  pierces the undisturbed free surface. The  $Z$  axis is horizontal, and its direction is chosen so that the  $X$ ,  $Y$  and  $Z$  axes form a right-handed coordinate system. The two systems of coordinates coincide when the ship is at rest but differ when it is in motion (unless the ship is fixed in position, as may be the case for a model in a towing tank).

The hull of the ship is defined by the equations  $Z' = \pm F'(X', Y')$  and

$Z = \pm F(X,Y)$  in the coordinate systems  $O'X'Y'Z'$  and  $OXYZ$ , respectively. The unknown hull equation  $Z = \pm F(X,Y)$  is related to the given equation  $Z' = \pm F'(X',Y')$  by

$$\left. \begin{aligned} X' &= X \cos \alpha + (Y - h) \sin \alpha \\ Y' &= -X \sin \alpha + (Y - h) \cos \alpha \\ Z' &= \pm F'(X',Y') = \pm F(X,Y) = Z \end{aligned} \right\} \quad (1)$$

which follow easily from Figure 1.

B. Sinkage and Trim: The sinkage  $h$  and trim angle  $\alpha$  are determined by the condition that the weight  $mg$  of the ship, the propelling thrust  $T$  and the hydrodynamic forces and moment exerted by the water upon the ship form a system of forces in equilibrium. The action of the water upon the ship results in a horizontal force  $R$ , called the wave resistance, a vertical lift  $L$  and a moment  $M$  about the  $Z$  axis at the origin  $O$  of the coordinate system  $OXYZ$ . The quantities  $R$ ,  $L$ ,  $M$ ,  $T$ ,  $h$  and  $\alpha$ , and their signs, are shown in Figure 1. The equations stating the equilibrium of the various forces acting upon the ship are

$$\begin{aligned} R &= T \cos \alpha & L &= mg + T \sin \alpha \\ M &= mg(\ell_G \cos \alpha + d_G \sin \alpha) + T(d_T - h \cos \alpha) \end{aligned}$$

where the quantities  $\ell_G$ ,  $d_G$  and  $d_T$ , and their signs, are defined in Figure 1. These equations immediately yield the trim angle  $\alpha$  and sinkage  $h$

$$\left. \begin{aligned} \tan \alpha &= (L - mg)/R \\ h &= \frac{mg}{T} \left( \ell_G + d_G \frac{L - mg}{R} \right) + (Td_T - M)/R \end{aligned} \right\} \quad (2)$$



where the thrust  $T$  is given by

$$T = \left[ R^2 + (L - mg)^2 \right]^{1/2} \quad (3)$$

When accounted for, sinkage and trim effects are usually explicitly introduced into the analysis in the form of perturbation expansions for  $h$  and  $\alpha$ . More precisely, these expansions, together with expansions for the velocity potential, the free-surface elevation and the propelling thrust  $T$ , are substituted into the exact equations of the problem, leading to a sequence of (linear) problems in which sinkage and trim appear explicitly. This approach was first employed by Peters and Stoker (1957) and is fully described in Wehausen (1973, p. 138). A different approach, which was used by Noblesse (1974), is adopted in the present analysis. In this approach, the solution of the hydrodynamic problem is pursued in the coordinate system OXYZ for a supposedly known hull equation  $Z = \pm F(X,Y)$ , leading to expressions for the wave resistance  $R$ , lift  $L$  and moment  $M$  in terms of the hull function  $F(X,Y)$ . Equations (1), (2) and (3) can then be used to derive the unknown hull function  $F(X,Y)$  from the given function  $F'(X',Y')$  by an iterative procedure. The necessity of this iterative procedure may appear as an inconvenience. However, it is a feature of the solution obtained in the present paper that, even if the effects of sinkage and trim were ignored, an iterative procedure would still be required.

C. Statement of the Problem in the Physical Space: Hereafter in the analysis, the following notations are used. The free-surface elevation, measured from the undisturbed level  $Y = 0$ , is denoted by  $E(X,Z)$ . The equation of the free surface is then given by  $Y = E(X,Z)$ . The coordinates of a point within the flow region are denoted by  $X, Y, Z$  where  $-\infty < X < +\infty$ ,  $-\infty < Y \leq E(X,Z)$  and  $F(X,Y) \leq Z < +\infty$ ,  $-\infty < Z \leq -F(X,Y)$  (the ship centerplane is a plane of symmetry for the flow). The  $X, Y, Z$  components of the fluid velocity vector at point  $(X, Y, Z)$  are denoted by  $U + U(X, Y, Z)$ ,  $V(X, Y, Z)$ ,  $W(X, Y, Z)$ . The disturbance velocity field induced

by the ship is then given by the vector field  $\vec{U}(X,Y,Z)$  with components U, V and W.

The boundary conditions and field equations to be satisfied by U, V, W and E, under the assumption that the fluid is inviscid, incompressible and without surface tension, are well known - see, for instance, Wehausen (1960, p. 447) - and are given below:

- the fact that there is no flow across the free surface  $Y = E(X,Z)$  is expressed by the kinematic boundary condition

$$V = UE_X + UE_X + WE_Z, \quad \text{on } Y = E(X,Z) \quad (4)$$

- the Bernoulli equation, with the condition that the pressure is constant at the free surface, yields the dynamic boundary condition

$$uU + \frac{1}{2} (U^2 + V^2 + W^2) + gE = 0, \quad \text{on } Y = E(X,Z) \quad (5)$$

- on the ship hull  $Z = \pm F(X,Y)$ , we have the kinematic boundary condition

$$\pm W = UF_X + UF_X + VF_Y, \quad \text{on } Z = \pm F(X,Y) \quad (6)$$

which expresses that there is no flow across the hull

- the condition that the fluid is incompressible is expressed by the continuity equation

$$U_X + V_Y + W_Z = 0 \quad (7)$$

- the assumption that the fluid is inviscid leads to the conclusion that the flow must be irrotational, which is expressed by the equations

$$W_Y - V_Z = U_Z - W_X = V_X - U_Y = 0 \quad (8)$$

- finally, it is necessary to impose a radiation condition specifying that waves are not propagated upstream from the ship but only downstream.

The fact that the pressure  $P(X,Y,Z)$  does not appear in the above formulation of the problem is, of course, a direct consequence of the assumption of irrotationality. The pressure  $P(X,Y,Z)$  is readily given in terms of the solution  $\vec{U}(X,Y,Z)$  of the problem stated above by the Bernoulli equation

$$\frac{P}{\rho} + UU + \frac{1}{2} (U^2 + V^2 + W^2) + gY = 0 \quad (9)$$

where  $\rho$  is the density of water.

D. Review and Discussion of the Thin-Ship Theory: The field equations (7) and (8) and the hull boundary condition (6) are linear and present no insurmountable difficulty. The real difficulties of the problem come from the free-surface boundary conditions (4) and (5), which are nonlinear and hold on a surface of unknown location. Due to the intractable nature of these nonlinear free-surface conditions, no exact solution of the problem has yet been obtained and, instead, various methods of approximation have been proposed. A recent review of these approximate theories is given by Wehausen (1973). A brief discussion of the thin-ship theory is included here for the purpose of comparison with the analysis presented in this paper.

The thin-ship theory is based on the approximation introduced by Michell in 1898. In this approximation, the free-surface conditions (4) and (5) are linearized and applied at the undisturbed free-surface position  $Y = 0$ ; the hull condition (6) is similarly simplified by applying it at the centerplane  $Z = \pm 0$  and neglecting the terms  $UF_X + VF_Y$ ; and the field equations (7) and (8) are satisfied exactly, although not in the actual flow domain. As shown by Peters and Stoker (1957), this approximation can be imbedded in a systematic perturbation scheme by assuming perturbation expansions in terms of the beam/length ratio  $\epsilon$ , as follows:

$$\begin{aligned} \vec{U}(X,Y,Z) &= \epsilon \vec{U}_1(X,Y,Z) + \epsilon^2 \vec{U}_2(X,Y,Z) + \dots \\ E(X,Z) &= \epsilon E_1(X,Z) + \epsilon^2 E_2(X,Z) + \dots \end{aligned} \quad \left. \vphantom{\begin{aligned} \vec{U}(X,Y,Z) &= \epsilon \vec{U}_1(X,Y,Z) + \epsilon^2 \vec{U}_2(X,Y,Z) + \dots \\ E(X,Z) &= \epsilon E_1(X,Z) + \epsilon^2 E_2(X,Z) + \dots \end{aligned}} \right\} \quad (10)$$

In the usual presentation of this theory - see, for instance, Wehausen (1973, p. 138) - perturbation expansions are also introduced for the sinkage  $h$ , trim  $\alpha$  and thrust  $T$ , as noted earlier. Furthermore, the analysis is usually carried out in terms of the velocity potential  $\phi$  rather than the velocity vector  $\vec{U} = \nabla\phi$ . These differences, however, need not concern us in the present brief discussion, which is merely aimed at pointing out the main features of the theory.

The thin-ship theory is based on Taylor expansions, used in order to transfer the boundary conditions (4) - (6) from the actual free surface  $Y = E(X,Z)$  and hull  $Z = \pm F(X,Y)$  to the undisturbed free surface  $Y = 0$  and ship centerplane  $Z = \pm 0$ , respectively, together with asymptotic expansions (10), the purpose of which being to generate a sequence of linear problems. Thus, the original nonlinear problem in the domain bounded by the hull of the ship and the (unknown) free surface is transformed into a sequence of linear problems in the (known) domain bounded by a cut on the ship centerplane and the undisturbed free surface.

The first-order problem, i.e., that for  $\vec{U}_1$  and  $E_1$  yields the classical solution of Michell. The general expression of the solution  $\vec{U}_2$  and  $E_2$  of the second-order problem has been derived in a complete form by Wehausen (1963) and Maruo (1966), and is given by Wehausen (1973, p. 223). The second-order velocity field  $\vec{U}_2$  consists of various terms representing the second-order corrections to Michell's first-order approximations to the free-surface conditions (4) and (5) and hull condition (6). In particular, the first-order approximation to the kinematic conditions (4) and (6) involves an inconsistency along the intersection of the undisturbed free surface with the ship centerplane - see Noblesse (1974) - which is corrected at second-order by means of a line integral of Kelvin sources. When the sinkage and trim effects are explicitly incorporated into the analysis, the expression for  $\vec{U}_2$  also involves

terms which are directly related to these effects, see again Wehausen (1973, p. 223).

The only attempts to carry through a complete second-order calculation are those of Eggers (1970) and Dagan (1973). The calculations are extremely difficult, even though both authors have limited themselves to simple mathematical hull shapes and have not incorporated the effects of sinkage and trim. It then seems that, in Dagan (1974)'s own words, "there is little hope that computation of nonlinear corrections via (4) - (8) will become useful in the near future." Furthermore, the higher-order thin-ship theory appears to be singular at low Froude numbers, as shown by Salvesen (1969) and Dagan (1972).

For the sake of comparison with the parametric formulation to be presented in the following section, it is perhaps instructive to examine in more detail the reasoning underlying the transfer of the boundary conditions from the actual free surface and the hull of the ship to the undisturbed free surface  $Y = 0$  and the ship centerplane  $Z = 0$ , respectively, by means of Taylor series. For the purpose of this discussion, let us consider a section  $X = \text{constant}$ ,  $Z \geq 0$  of the ship and the flow field, as shown in Figure 2. The lines AB and BC represent the wetted hull and the free surface, respectively, with the load waterline below and above the level of the undisturbed free surface in cases (a) and (b), respectively. As noted in the previous section, the problem to be solved is that of solving eqs. (7) and (8) in the region bounded by the wetted hull AB and the free surface BC, along which the hull condition (6) and the free surface conditions (4) and (5) hold. In the thin-ship theory, this exact problem is replaced by an "equivalent problem" in the region bounded by the ship centerplane OA and the plane Obc of the undisturbed free surface, with the exact field equations (7) and (8) and "equivalent boundary conditions" along OA and Obc. These equivalent boundary conditions are obtained from the exact boundary conditions (4) - (6) as follows. Let us first consider the free-surface conditions. By means of Taylor expansions about  $Y = 0$ , the exact free-surface conditions (4) and (5), which hold along the actual free surface BC, yield equivalent conditions along the projection bc of BC onto  $Y = 0$ . However, it should first

be noticed that this transfer of boundary conditions from BC to bc can be achieved in an approximate way only since an infinite number of terms in the Taylor expansions would be required for the transfer to be exact. More important is the fact that this equivalent free surface condition along bc is assumed to hold not only along bc but also along Ob. An equivalent hull boundary condition is similarly obtained from the exact hull condition (6), which holds along the actual wetted hull AB, and Taylor expansions about  $Z = 0$ . In case (a), the equivalent hull condition, which holds along BA, is assumed to apply along Ob also, while in case (b), the hull condition along IB is somehow not used. It then appears that the definition of the "equivalent problem", upon which the thin-ship theory is built, involves some difficulties. It will now be seen that these difficulties vanish when the actual flow region is mapped continuously onto the domain bounded by the ship centerplane and the undisturbed free surface, with the wetted hull AB and the actual free surface BC mapped onto the ship centerplane  $Z = 0, Y \leq 0$  and the undisturbed free surface  $Y = 0, Z \geq 0$ , respectively.

### III. PARAMETRIZATION OF THE PROBLEM

A. Definition of the Parametrization: We seek a solution to the problem of the wavemaking of a ship, stated in Section II(C), in the following parametric form

$$\left. \begin{aligned} U(X,Y,Z) &= Uu(x,y,z) \\ V(X,Y,Z) &= Uv(x,y,z) \\ W(X,Y,Z) &= Uw(x,y,z) \end{aligned} \right\} \quad (11)$$

$$\left. \begin{aligned} X &= x + \xi(x,y,z) \\ Y &= y + \eta(x,y,z) \\ Z &= z + \zeta(x,y,z) \end{aligned} \right\} \quad (12)$$

$$P(X,Y,Z) = \rho u^2 p(x,y,z) \quad (13)$$

The vector  $\vec{u}$ , with components  $u$ ,  $v$  and  $w$ , evaluated at point  $(x,y,z)$  in the parametric space then gives the disturbance velocity  $\vec{U}$ , divided by the speed  $U$  of the ship, at point  $(X,Y,Z)$  in the physical space. The correspondence between the points  $(X,Y,Z)$  and  $(x,y,z)$  is given by eqs. (12) which define a mapping of the physical space  $X,Y,Z$  onto the parametric space  $x,y,z$ . More precisely, it is assumed that the actual flow region  $-\infty < X < +\infty$ ,  $Y \leq E(X,Z)$ ,  $|Z| \geq F(X,Y)$  is mapped onto the domain  $-\infty < x < +\infty$ ,  $y \leq 0$ ,  $|z| \geq 0$  with the free surface  $Y = E(X,Z)$  mapped onto the plane  $y = 0$  and the ship hull  $Z = \pm F(X,Y)$  mapped onto  $z = \pm 0$ . The mapping functions  $\xi$ ,  $\eta$  and  $\zeta$  are called the longitudinal, vertical and lateral displacements, respectively, and are supposed to be continuously differentiable over the domain  $-\infty < x < +\infty$ ,  $y \leq 0$ ,  $|z| \geq 0$ , except possibly at isolated points or along a line in the case of blunt ships. The displacements  $\xi$ ,  $\eta$ ,  $\zeta$  are also assumed to vanish at infinity upstream.

The assumption that the free surface  $Y = E(X,Z)$  is mapped onto the plane  $y = 0$  is expressed by eqs. (12) as follows

$$\left. \begin{aligned} X &= x + \xi(x,0,z) \\ Y &= E(X,Z) = \eta(x,0,z) \\ Z &= z + \zeta(x,0,z) \end{aligned} \right\} \quad (14)$$

Equations (14) are the parametric equations of the free surface. Similarly, the hull  $Z = \pm F(X,Y)$  is defined by the parametric equations

$$\left. \begin{aligned} X &= x + \xi(x,y,0) \\ Y &= y + \eta(x,y,0) \\ Z &= \pm F(X,Y) = \zeta(x,y,\pm 0) \end{aligned} \right\} \quad (15)$$

B. Statement of the Problem in the Parametric Space: Let us now introduce the parametrization defined by eqs. (11) - (13) into the set of equations and boundary conditions (4) - (9). The kinematic free-surface condition (4) is considered first. Equations (14) yield

$$E_X dX + E_Z dZ = \eta_x dx + \eta_z dz$$

$$dX = (1 + \xi_x) dx + \xi_z dz$$

$$dZ = \zeta_x dx + (1 + \zeta_z) dz$$

where all the quantities on the right-hand sides are evaluated at  $y = 0$ . Hence we have

$$(1 + \xi_x) E_X + \zeta_x E_Z = \eta_x$$

$$\xi_z E_X + (1 + \zeta_z) E_Z = \eta_z$$

from which it follows that

$$E_X = (\eta_x + \eta_x \zeta_z - \eta_z \zeta_x) / (1 + \xi_x + \zeta_z + \xi_x \zeta_z - \xi_z \zeta_x)$$

$$E_Z = (\eta_z + \eta_z \xi_x - \eta_x \xi_z) / (1 + \xi_x + \zeta_z + \xi_x \zeta_z - \xi_z \zeta_x)$$

The kinematic free-surface condition (4) then becomes

$$\begin{aligned} v(1 + \xi_x + \zeta_z + \xi_x \zeta_z - \xi_z \zeta_x) &= \eta_x + \eta_x \zeta_z - \eta_z \zeta_x \\ + u(\eta_x + \eta_x \zeta_z - \eta_z \zeta_x) + w(\eta_z + \eta_z \xi_x - \eta_x \xi_z) &\quad \text{on } y = 0 \quad (16) \end{aligned}$$

the kinematic condition (6) on the ship hull can similarly be shown to



become

$$\begin{aligned}
 w(1 + \xi_x + \eta_y + \xi_x \eta_y - \xi_y \eta_x) &= \zeta_x + \zeta_x \eta_y - \zeta_y \eta_x \\
 + u(\zeta_x + \zeta_x \eta_y - \zeta_y \eta_x) + v(\zeta_y + \zeta_y \xi_x - \zeta_x \xi_y) &\text{ on } z = \pm 0 \quad (17)
 \end{aligned}$$

The dynamic free-surface condition (5) is then readily seen to become

$$u + \frac{1}{2} (u^2 + v^2 + w^2) + k_o \eta = 0 \quad \text{on } y = 0 \quad (18)$$

where  $k_o = g/U^2$ .

The field equations (7) and (8) are now considered. In order to obtain compact expressions for the X, Y and Z first-order partial derivatives of the velocity components U, V and W, it is convenient to introduce the following notations:

$$\begin{aligned}
 (X_1, X_2, X_3) &\equiv (X, Y, Z) & (x_1, x_2, x_3) &\equiv (x, y, z) \\
 (U_1, U_2, U_3) &\equiv (U, V, W) & (u_1, u_2, u_3) &\equiv (u, v, w) \\
 U_{p,i} &\equiv \partial U_p / \partial X_i & u_{p,i} &\equiv \partial u_p / \partial x_i \quad (p, i = 1, 2, 3) \\
 (\xi_1, \xi_2, \xi_3) &\equiv (\xi, \eta, \zeta) & \xi_{i,j} &\equiv \partial \xi_i / \partial x_j \quad (i, j = 1, 2, 3)
 \end{aligned}$$

Equations (11) can then be written as

$$U_p(X_1, X_2, X_3) = u_{p,j}(x_1, x_2, x_3) \quad (p = 1, 2, 3)$$

from which it follows that

$$U_{p,i} dX_i = u_{p,j} dx_j$$

Equations (12) can similarly be written as

$$X_i = x_i + \xi_i(x_1, x_2, x_3) \quad (i = 1, 2, 3)$$

and this yields

$$dX_i = dx_i + \xi_{i,j} dx_j = (\delta_{ij} + \xi_{i,j}) dx_j$$

Hence we must have

$$U_{p,i} (\delta_{ij} + \xi_{i,j}) = U_{p,j} u_{p,j}$$

which can be written in the form

$$\begin{bmatrix} 1+\xi_{1,1} & \xi_{2,1} & \xi_{3,1} \\ \xi_{1,2} & 1+\xi_{2,2} & \xi_{3,2} \\ \xi_{1,3} & \xi_{2,3} & 1+\xi_{3,3} \end{bmatrix} \begin{bmatrix} U_{p,1} \\ U_{p,2} \\ U_{p,3} \end{bmatrix} = U \begin{bmatrix} u_{p,1} \\ u_{p,2} \\ u_{p,3} \end{bmatrix}$$

where  $p = 1, 2$  or  $3$ . The solution of this system of linear algebraic equations is given by

$$U_{p,i} = U D_{pi} / D \quad (p, i = 1, 2, 3) \quad (19)$$

where  $D$  denotes the determinant of the matrix  $(\delta_{ij} + \xi_{i,j})$ , and  $D_{pi}$  is given by

$$\begin{aligned} D_{pi} = & u_{p,i} + u_{p,i} \nabla \cdot \vec{\xi} - (u_{p,i} \xi_{i,i} + u_{p,j} \xi_{j,i} + u_{p,k} \xi_{k,i}) \\ & + u_{p,i} (\xi_{j,j} \xi_{k,k} - \xi_{j,k} \xi_{k,j}) + u_{p,j} (\xi_{j,k} \xi_{k,i} - \xi_{j,i} \xi_{k,k}) \\ & + u_{p,k} (\xi_{j,i} \xi_{k,j} - \xi_{j,j} \xi_{k,i}) \end{aligned} \quad (19a)$$

where  $p, i = 1, 2, 3$  and  $(i, j, k)$  is an even permutation of the integers  $(1, 2, 3)$ , that is,  $(i, j, k) = (1, 2, 3), (2, 3, 1)$  or  $(3, 1, 2)$ ; there is no summation convention implied in eq. (19a); the quantity  $\nabla \cdot \vec{\xi}$  denotes the divergence of the vector  $\vec{\xi}$ , the components of which are  $\xi_1, \xi_2, \xi_3$  or  $\xi, \eta, \zeta$ . Equation (19a) can be written in the somewhat more compact form

$$D_{pi} = u_{p,i} (1 + \nabla \cdot \vec{\xi}) - \nabla u_p \cdot (\vec{\xi}_{,i} - \nabla \xi_j \times \nabla \xi_k) \quad (19b)$$

It follows from eq. (19) that the equations of continuity (7) and irrotationality (8) becomes

$$\left. \begin{aligned} D_{11} + D_{22} + D_{33} &= 0 \\ D_{32} - D_{23} = D_{13} - D_{31} = D_{21} - D_{12} &= 0 \end{aligned} \right\} \quad (20)$$

with the quantities  $D_{pi}$  defined in terms of  $\vec{u}$  and  $\vec{\xi}$  by eqs. (19a or b). The (dimensionless) pressure  $p(x, y, z)$  is then obtained from the Bernoulli equation (9), which is readily seen to become

$$p + u + \frac{1}{2}(u^2 + v^2 + w^2) + k_0(y + \eta) = 0 \quad (21)$$

It is interesting to compare the formulations in the physical space  $X, Y, Z$  and in the parametric space  $x, y, z$ . Because of the assumption of irrotationality, both formulations are "kinematic", i.e., do not involve the pressure, which is given in terms of the kinematic quantities by the Bernoulli equation written in the form (9) or (21). In the formulation in the physical space, the unknowns are the disturbance velocity field  $\vec{U}(X, Y, Z)$  and the free-surface elevation  $E(X, Z)$ ; the governing field equations and boundary conditions are eqs. (4) - (8); and the domain over which the solution is to be determined is not known in advance. In the parametric formulation, the unknowns are the velocity field  $\vec{u}(x, y, z)$  and the displacement field  $\vec{\xi}(x, y, z)$ , which are defined over a domain known beforehand and must satisfy eqs. (20) and the conditions (16) - (18).

C. Discussion of Previous Parametric Formulations: Parametric formulations of the problem of the wavemaking of a ship have been used previously by Wehausen (1969), Dagan (1974) and Noblesse (1974). In the present notations, Wehausen's choice of parameters is as follows:  $x = Ut$ ,  $y = Y_\infty$ ,  $z = Z_\infty$  where  $t$  denotes the time, and  $Y_\infty$  and  $Z_\infty$  are the  $Y$  and  $Z$  coordinates of any streamline at infinity upstream. With this choice of parameters, eqs. (12) become

$$X = Ut + \xi(Ut, Y_\infty, Z_\infty)$$

$$Y = Y_\infty + \eta(Ut, Y_\infty, Z_\infty)$$

$$Z = Z_\infty + \zeta(Ut, Y_\infty, Z_\infty)$$

which are the same as eqs. (11) in Wehausen's paper. The above equations clearly show that the velocity is given by

$$u + U(X, Y, Z) = \partial X / \partial t = u + U \partial \xi / \partial x$$

$$v(X, Y, Z) = \partial Y / \partial t = U \partial \eta / \partial x$$

$$w(X, Y, Z) = \partial Z / \partial t = U \partial \zeta / \partial x$$

It then follows from eqs. (11) that Wehausen's particular choice of parameters implies the following relation between the disturbance velocity field  $\vec{u}(x, y, z)$  and the displacement field  $\vec{\xi}(x, y, z)$ :

$$(u, v, w) = \partial(\xi, \eta, \zeta) / \partial x \quad \text{or} \quad \vec{u} = \vec{\xi}_x \quad (22)$$

The parametrization defined by eqs. (11) - (13) differs from the one of Wehausen in that the parameters  $x$ ,  $y$  and  $z$  are left unspecified, except for the fact that the free surface and the ship hull are mapped onto  $y = 0$  and  $z = \pm 0$ , respectively. The advantage of not particularizing

the parametrization at the start of the problem formulation resides in the latitude this introduces into the analysis.

However, Wehausen's Lagrangian coordinates are a particularly convenient choice of parameters with respect to the kinematic conditions (4) and (6). Indeed, it is readily verified that the free-surface kinematic condition (16) is automatically satisfied if

$$(u,v,w) = (\xi_x, \eta_x, \zeta_x) \quad \text{on } y = 0 \quad (22a)$$

The hull boundary condition (17) is similarly automatically satisfied if

$$(u,v,w) = (\xi_x, \eta_x, \zeta_x) \quad \text{on } z = \pm 0 \quad (22b)$$

This ensures that the kinematic boundary conditions at the free surface and on the hull can be satisfied exactly via an iterative procedure, as shown in the following section.

In an attempt to establish a sound theoretical foundation for Guilloton's method, Dagan and Noblesse have developed similar thin-ship perturbation analyses in which the actual flow region is mapped onto the "undisturbed flow region" (bounded by the undisturbed free surface  $Y = 0$  and a cut on the ship centerplane to which the ship is supposed to reduce in the limit of vanishing beam/length ratio) by means of a slight straining of the coordinates. The analyses of Dagan and Noblesse are essentially based on the following perturbation expansions:

$$\vec{U}(X,Y,Z)/U = \epsilon \vec{u}_1(x,y,z) + \epsilon^2 \vec{u}_2(x,y,z) + \dots \quad (23a)$$

$$\vec{X} = \vec{x} + \epsilon \vec{\xi}_1(\vec{x}) + \epsilon^2 \vec{\xi}_2(\vec{x}) + \dots \quad (23b)$$

where  $\vec{X}$  and  $\vec{x}$  denote the position vectors of the points  $(X,Y,Z)$  and  $(x,y,z)$ , respectively, and  $\epsilon$  is the beam/length ratio. Equations (23a,b) should be compared with eqs. (10) and also with eqs. (11) and (12).

The approach adopted by Dagan and Noblesse is essentially that of

Lighthill's method of strained coordinates, and, in the spirit of that method, the parameters  $x, y, z$  in expansions (23a,b) are simply regarded as "strained coordinates" with no particular physical significance attached to them. Dagan, however, gave a kinematic interpretation of the coordinates straining and noted the similarity between the strained coordinates and Wehausen's Lagrangian coordinates. As a matter of fact, eqs. (23b) requires that, as  $\epsilon \rightarrow 0$ , the point  $(X, Y, Z)$  tend to the point  $(x, y, z)$ . This clearly implies that every point  $(X, Y, Z)$  on any given streamline corresponds to the same  $y$  and  $z$ , and  $y \equiv Y_\infty$ ,  $z \equiv Z_\infty$  where  $Y_\infty$  and  $Z_\infty$  are the  $Y$  and  $Z$  coordinates of the streamline at infinity upstream. Thus, it is implied in eqs. (23b) that the strained coordinates  $y$  and  $z$  coincide with Wehausen's Lagrangian coordinates  $Y_\infty$  and  $Z_\infty$ . Incidentally, eqs. (23b) also imply that the point  $(x, y, z)$  must lie within the undisturbed flow region  $-\infty < x < +\infty$ ,  $-\infty < y \leq 0$ ,  $|z| \geq 0$ . We also have  $y \equiv Y_\infty$  and  $z \equiv Z_\infty$  in the present study, as clearly implied by the assumption that the displacements  $\xi, \eta, \zeta$  tend to zero as  $x \rightarrow -\infty$ .

Wehausen's assumption  $x = Ut$  does not appear to be a necessary one, however. The relation  $x = Ut$  may be interpreted in the following physical terms: Consider a point  $A_0$ , with coordinates  $X_0, Y_0$  and  $Z_0$ , far upstream from the ship in the undisturbed incoming uniform stream and a point  $A$ , with coordinates  $X, Y$  and  $Z$ , on the streamline issued from  $A_0$  and in the region where the flow is disturbed by the ship. As  $\epsilon \rightarrow 0$ , the streamline passing through  $A_0$  tends to the horizontal line drawn from  $A_0$ , and the point  $A$  tends to the point  $a$ , with coordinates  $x, y$  and  $z$ , on the horizontal line. In the case  $\epsilon = 0$ , the time it would take for a fluid particle to travel from point  $A_0$  to point  $a$  is given by  $\tau = (x - X_0)/U$ . The relation  $x = Ut$  amounts to assuming that the time  $\tau$  is equal to the time  $t$  required for a fluid particle to travel from point  $A_0$  to point  $A$ . Incidentally, this is exactly the assumption made by Guilloton whose analysis then appears to be based, essentially, on the use of Lagrangian coordinates, like Wehausen's analysis.

The purpose of the above discussion was to indicate the similarities and differences between the present approach and that used by Wehausen, Dagan and Noblesse. In brief, Wehausen's formulation may be regarded as a

particular case of the general parametrization presented in the previous section, while the papers by Dagan and Noblesse essentially develop perturbation analyses of this general parametric formulation. Nevertheless, in spite of the great care that has been taken in the previous section to present as general a formulation as possible (the parameters  $x, y, z$  are left unspecified; the disturbance velocity field  $u, v, w$  and the displacement field  $\xi, \eta, \zeta$  are not assumed to be small), we shall fall back on some particular choice of parameters (the Lagrangian coordinates of Wehausen!) and neglect some of the nonlinear effects in the following section, where two approximate solutions are derived. As noted in the introduction, the first of these approximate solutions is essentially contained in Wehausen's formulation, while the second is fairly close to those of Dagan and Noblesse and is also similar to that of Guilloton.

A parametric formulation was also used by Landweber (1973), but for a different purpose. The aim of Landweber's analysis was to investigate the nature of the singularity distributions which generate the irrotational part of the actual flow field induced by the motion of a ship. Similar ideas, of mapping the actual flow region onto a reference domain known beforehand, likewise underlie the report by Yim (1968). See also Joseph (1973).

#### IV. TWO SIMPLE APPROXIMATE SOLUTIONS

A. An Approximate Solution Satisfying the Kinematic Boundary Conditions: If the disturbance velocity field,  $u, v, w$  and the displacement field  $\xi, \eta, \zeta$  are assumed to be small, then eqs. (19b) yield, to a first approximation,  $D_{pi} = u_{p,i}$ , and the equations of continuity and irrotationality (20) simply become

$$\nabla \cdot \vec{u} = \nabla \times \vec{u} = 0 \quad (24)$$

Clearly, the above argument can be rationalized by introducing perturbation expansions for the velocity and displacement fields, as follows:

$$\vec{u} = \epsilon \vec{u}_1 + \epsilon^2 \vec{u}_2 + \dots, \quad \vec{\xi} = \epsilon \vec{\xi}_1 + \epsilon^2 \vec{\xi}_2 + \dots \quad (25)$$

It then readily follows that eqs. (24) give a first-order approximation to the exact field equations (20), i.e., eqs. (24) differ from eqs. (20) by terms of order  $\epsilon^2$ . An interesting point which was already noted in the introduction is that the perturbation parameter  $\epsilon$  in the perturbation expansions (25) could clearly just as well be regarded as the beam/length ratio, a slenderness parameter, the draft/length ratio, or even, for the case of a submerged body, a submergence parameter. In fact, the perturbation parameter  $\epsilon$  should perhaps more significantly be directly related to the maximum curvature of the ship hull, as one would intuitively expect eqs. (24) to be a better approximation for fine bows and sterns than for blunt ones.

The dynamic free-surface condition (18) can similarly be simplified by neglecting the nonlinear terms. It yields

$$u + k_o \eta = 0 \quad \text{on } y = 0 \quad (26)$$

As noted in the previous section, the kinematic conditions (16) and (17) can be automatically satisfied by specifying that the velocity and displacement fields are related by means of eqs. (22a,b). In fact, eq. (22) may be assumed. The field eqs. (24) and the dynamic condition (26) then become

$$\nabla \cdot \vec{\xi} = \nabla \times \vec{\xi} = 0 \quad (27)$$

$$\xi_x + k_o \eta = 0 \quad \text{on } y = 0 \quad (28)$$

The problem defined by eqs. (27), the free-surface condition (28) and the condition

$$\zeta(x,y,\pm 0) = \pm f(x,y) \quad (29)$$



where  $f(x,y)$  is some given function, is recognized as the classical linearized ship-wave problem first solved by Michell; see for instance Wehausen (1960, p. 579). The solution is given by

$$\vec{\xi}(x,y,z) = \nabla\psi(x,y,z) \quad (30)$$

$$\psi(x,y,z) = \frac{1}{2\pi} \iint_{\sigma} G(x,y,z;x',y',0) f(x',y') dx' dy' \quad (31)$$

where  $\sigma$  denotes the area in the half plane  $z = 0, y \leq 0$  where the function  $f(x,y)$  is nonzero, and the function  $G$  is the well-known Havelock source function, the expression of which may also be found in Wehausen (1960, p. 484). The function  $\psi(x,y,z)$  will be referred to as the "displacement potential". The function  $f(x,y)$  is defined in terms of the (supposedly known) hull function  $F(X,Y)$  by means of the following implicit equations

$$\left. \begin{aligned} X &= x + \xi(x,y,0) \\ Y &= y + \eta(x,y,0) \\ F(X,Y) &= f(x,y) \end{aligned} \right\} \quad (32)$$

which readily follow from eqs. (15) and (29). The function  $f(x,y)$  plays a key role in the analysis, and will be referred to as the "equivalent hull function", which is perhaps more appropriate than "linearized hull function" used by Guilloton and Noblesse. We shall refer to the fictitious hull  $z = \pm f(x,y)$  as the "equivalent hull".

In summary, the solution given by eqs. (11), (12), (22), (30), (31) and (32) satisfies the kinematic conditions - both at the free surface and on the hull of the ship - exactly, but satisfies the dynamic free-surface condition and the field equations approximately only (to the first order, if the language of perturbation methods is used). It is readily seen that this approximate solution is just the classical solution of Michell

for the "equivalent hull"  $z = \pm f(x,y)$ . Indeed, eqs. (22), (30) and (31) yield

$$\vec{u}(x,y,z) = \nabla\phi(x,y,z)$$

where the velocity potential  $\phi(x,y,z)$  is given by

$$\phi(x,y,z) = \psi_x(x,y,z) = \frac{1}{2\pi} \iint_{\sigma} G_x(x,y,z;x',y',0) f(x',y') dx' dy'$$

The relation  $G_x(x,y,z;x',y',0) = -G_x(x,y,z;x',y',0)$  allows an integration by parts, from which it follows that the velocity potential  $\phi(x,y,z)$  is given by

$$\phi(x,y,z) = \frac{1}{2\pi} \iint_{\sigma} G(x,y,z;x',y',0) f_x(x',y') dx' dy'$$

and this is exactly the Michell-Havelock solution for the hull  $z = \pm f(x,y)$ , see for instance Wehausen (1960, p. 579).

Thus, it is seen that the classical solution of Michell yields two different approximations to the problem of the wavemaking of a ship. In the original version of Michell, the solution yields explicitly the disturbance velocity potential  $u(X,Y,Z)$  in terms of the real hull  $Z = \pm F(X,Y)$ , and this solution is approximate in the sense that the field equations are satisfied exactly but the boundary conditions are satisfied approximately (to first order). In the above version, which is essentially that of Guilloton, the solution of Michell is evaluated in a parametric space  $x,y,z$  for an "equivalent hull"  $z = \pm f(x,y)$  and yields an implicit solution (requiring an iterative procedure) in the physical space  $X,Y,Z$ ; this solution is approximate in that the kinematic boundary conditions at the free surface and on the ship hull are satisfied exactly but the dynamic free-surface condition and the field equations are satisfied approximately (to first order). Clearly, these two versions of the Michell solution are only first-order approximations to the exact solution, and there is no

evident reason for believing that one version should be better than the other. The numerical calculations of Emerson (1967), Gadd (1973) and Standing (1974) have shown, however, that there are significant numerical differences between the two solutions, and that the version of Guilloton seems to be in better agreement with measurements.

Finally, let us write down the expression of the pressure  $p(x,y,z)$  corresponding to the above solution. In accordance with the approximations made in this solution, the pressure  $p$  is obtained from the Bernoulli equation (21) by neglecting the nonlinear terms. It yields

$$p = -k_o(y + \psi_y) - \psi_{xx} \quad (33)$$

where eqs. (22) and (30) have been used.

#### B. An Approximate Solution Satisfying All the Boundary

Conditions Exactly: The solution derived in the previous section will now be modified so that the dynamic free-surface boundary condition (18) is also satisfied exactly. The Bernoulli equation (21) can be written as

$$(1 + u)^2 = 1 - 2p - 2k_o(y + \eta) - v^2 - w^2$$

this yields

$$u = [1 - 2p - 2k_o(y + \eta) - v^2 - w^2]^{1/2} - 1 \quad (34)$$

Let us substitute  $p$ ,  $\eta$ ,  $v$  and  $w$  on the right-hand side of the above equation by the corresponding expressions derived in the previous section, i.e., eqs. (30) and (22) yield  $\eta = \psi_y$ ,  $v = \psi_{xy}$ ,  $w = \psi_{xz}$  and  $p$  is given by eq. (33). Equation (34) then gives

$$u = (1 + 2\psi_{xx} - \psi_{xy}^2 - \psi_{xz}^2)^{1/2} - 1 = \psi_{xx} + u_2$$

where the displacement potential  $\psi$  is given by eq. (31), and the second-order quantity  $u_2$  is defined by

$$u_2 = (1 + 2\psi_{xx} - \psi_{xy}^2 - \psi_{xz}^2)^{1/2} - 1 - \psi_{xx} \quad (35)$$

It is readily verified that the solution defined by

$$\left. \begin{aligned} u &= \psi_{xx} + u_2 = \xi_x; & v &= \psi_{xy} = \eta_x; & w &= \psi_{xz} = \zeta_x \\ \xi(x,y,z) &= \psi_x(x,y,z) + \int_{-\infty}^x u_2(x',y,z) dx'; & \eta &= \psi_y; & \zeta &= \psi_z \end{aligned} \right\} \quad (36)$$

satisfies exactly the kinematic conditions (16) and (17) [since eq. (22) is verified] and the dynamic free-surface condition (18) [in fact, the Bernoulli equation (21) is satisfied everywhere in the flow field]. Of course, this solution satisfies the field eqs. (20) only approximately. In fact, the field equations satisfied by the solution (31), (35) and (36) differ from both the approximate equations (24) and the exact ones (20) by second-order terms.

In summary, eqs. (11), (12), (36), (35), (31) and (32) define an approximate solution in which the boundary conditions are satisfied exactly but the field equations are satisfied approximately. This solution differs from that derived in the previous section only by a second-order correction  $u_2$  to the longitudinal velocity component  $u$  and a corresponding correction to the longitudinal displacement  $\xi$ . Again, only comparison with measurements can tell whether these corrections actually give better agreement. Finally, it should be noted that in both solutions, the pressure  $p$  is given by eq. (33).

By expanding eqs. (36) and grouping the terms of the same order (assuming  $\psi$  to be a small quantity of order  $\epsilon$ ), the longitudinal velocity component  $u$  becomes

$$u = \psi_{xx} - \frac{1}{2} \left( \psi_{xx}^2 + \psi_{xy}^2 + \psi_{xz}^2 \right) + \frac{1}{2} \psi_{xx} \left( \psi_{xx}^2 + \psi_{xy}^2 + \psi_{xz}^2 \right) + \dots$$

in agreement with the results of Noblesse and, for the first two terms, with those of Dagan (1974).

V. HYDRODYNAMIC FORCES AND MOMENT

The action of the water upon the ship results in a horizontal force R called the wave resistance, a vertical lift L and a moment M about the Z axis at the origin O of the coordinate system OXYZ. The wave resistance R, lift L and moment M at the origin are given by

$$\left. \begin{aligned} R &= 2 \iint_{\Sigma} P F_X dXdY, & L &= 2 \iint_{\Sigma} P F_Y dXdY \\ M &= 2 \iint_{\Sigma} P (X F_Y - Y F_X) dXdY \end{aligned} \right\} \quad (37)$$

where  $\Sigma$  denotes the projection of the actual wetted hull on the ship centerplane and P is the pressure acting on the surface of the hull. Equations (32) yield

$$\left. \begin{aligned} F_X &= (f_x + f_x \eta_y - f_y \eta_x) / \Delta \\ F_Y &= (f_y + f_y \xi_x - f_x \xi_y) / \Delta \\ dXdY &= |X_{x_y} Y_y - X_y Y_x| dx dy = \Delta dx dy \end{aligned} \right\} \quad (38)$$

where  $\Delta = 1 + \xi_x + \eta_y + \xi_x \eta_y - \xi_y \eta_x$  and  $\xi_x, \xi_y, \eta_x$  and  $\eta_y$  are evaluated at  $z = 0$ . Equations (37) then become

$$R = 2\rho u^2 \iint_{\sigma} p(f_x + f_x \eta_y - f_y \eta_x) dx dy \quad (39)$$

$$L = 2\rho u^2 \iint_{\sigma} p(f_y + f_y \xi_x - f_x \xi_y) dx dy \quad (40)$$

$$M = 2\rho u^2 \iint_{\sigma} p[(x + \xi)(f_y + f_y \xi_x - f_x \xi_y) - (y + \eta)(f_x + f_x \eta_y - f_y \eta_x)] dx dy \quad (41)$$

where eq. (13) has been used. Equations (39) - (41) are exact expressions for the wave resistance, lift and moment.

Approximate expressions for R, L and M, corresponding to the approximate solutions derived above, can readily be obtained from eqs. (39) - (41) by substituting the pressure p by the approximate expression (33). The following equations

$$(k_o y + k_o \psi_y + \psi_{xx}) (f_x + f_x \eta_y - f_y \eta_x) = k_o y f_x + \psi_{xx} f_x + (\psi_{xx} + k_o \psi_y) (f_x \eta_y - f_y \eta_x) + k_o \{ [(y\eta)_y f]_x - [(y\eta)_x f]_y \} \quad (42)$$

$$(k_o y + k_o \psi_y + \psi_{xx}) (f_y + f_y \xi_x - f_x \xi_y) = k_o y f_y + k_o (\xi f_x + \eta f_y) + \psi_{xx} f_y + (\psi_{xx} + k_o \psi_y) (f_y \xi_x - f_x \xi_y) + k_o \{ [(y\xi)_x f]_y - [(y\xi)_y f]_x \} \quad (43)$$

$$\begin{aligned}
 & (k_o y + k_o \psi_y + \psi_{xx}) [(x + \xi)(f_y + f_y \xi_x - f_x \xi_y) - (y + \eta)(f_x + f_x \eta_y - f_y \eta_x)] = \\
 & k_o y (x f_y - y f_x) + k_o x (\xi f_x + \eta f_y) - k_o (\xi \xi_x + \eta \eta_x) f + \psi_{xx} (x f_y - y f_x) \\
 & + (\psi_{xx} + k_o \psi_y) [\xi f_y - \eta f_x + (x + \xi)(f_y \xi_x - f_x \xi_y) - (y + \eta)(f_x \eta_y - f_y \eta_x)] \\
 & + k_o \{ [(y x \xi)_x f]_y - [(y x \xi)_y f]_x \} + k_o \{ [(y^2 \eta)_x f]_y - [(y^2 \eta)_y f]_x \} \\
 & + \frac{1}{2} k_o \{ [(\xi^2 + \eta^2)_x y f]_y - [(\xi^2 + \eta^2)_y y f]_x \} \tag{44}
 \end{aligned}$$

where the relation  $\eta = \psi_y$  has been used, are easily verified. The first and last terms in eq. (42), the last term in eq. (43), and the last three terms in eq. (44) yield zero when integrated over  $\sigma$ .

After a few simple manipulations, the approximate expressions of R, L and M can be written as

$$R = -2\rho u^2 \iint_{\sigma} \psi_{xx} f_x dx dy - 2\rho u^2 \iint_{\sigma} (\psi_{xx} + k_o \psi_y) (\eta_y f_x - \eta_x f_y) dx dy \tag{45}$$

$$\begin{aligned}
 L = & 2\rho g \iint_{\sigma} (1 + \xi_x + \eta_y) f dx dy - 2\rho g \int_{x_b}^{x_s} x \eta_o f_o dx \\
 & - 2\rho u^2 \iint_{\sigma} \psi_{xx} f_y dx dy - 2\rho u^2 \iint_{\sigma} (\psi_{xx} + k_o \psi_y) (\xi_x f_y - \xi_y f_x) dx dy \tag{46}
 \end{aligned}$$

$$\begin{aligned}
 M = & 2\rho g \iint_{\sigma} x(1 + \xi_x + \eta_y) f dx dy - 2\rho g \int_{x_b}^{x_s} x \eta_o f_o dx \\
 & + 2\rho g \iint_{\sigma} (\xi + \xi \xi_x + \eta \eta_x) f dx dy - 2\rho u^2 \iint_{\sigma} \psi_{xx} (x f_y - y f_x) dx dy \\
 & - 2\rho u^2 \iint_{\sigma} (\psi_{xx} + k_o \psi_y) [\xi f_y - \eta f_x + (x + \xi) (\xi_x f_y - \xi_y f_x) \\
 & \quad - (y + \eta) (\eta_y f_x - \eta_x f_y)] dx dy \tag{47}
 \end{aligned}$$

where  $f_o \equiv f(x,0)$  and  $\eta_o \equiv \eta(x,0,0)$ , and  $x_b$  and  $x_s$  denote the abscissae of the bow and stern of the "equivalent hull". The physical meaning of each term in the above expressions should be self evident. In particular, the first two terms in the expression for  $L$ , and the first three terms in the expression for  $M$  correspond to hydrostatic effects while the remaining terms correspond to hydrodynamics effects.

#### VI. EFFECTS OF SINKAGE AND TRIM

In the previous sections, the flow around the supposedly known hull  $Z = \pm F(X,Y)$  has been studied, and approximate expressions for the wave resistance  $R$ , lift  $L$  and moment  $M$  exerted by the water upon the ship have been derived. The solution and, in particular, the required expression for  $R$ ,  $L$  and  $M$ , are expressed in terms of the equivalent hull function  $f(x,y)$ , which is related to the real hull function  $F(X,Y)$  by means of eqs. (32). These equations, together with eqs. (1), show that the equivalent hull function  $f(x,y)$  is related to the given equation  $Z' = \pm F'(X',Y')$  of the hull in the coordinate system  $O'X'Y'Z'$  fixed with respect to the ship by means of the equations



$$\left. \begin{aligned} X' &= [x + \xi_c(x,y)] \cos \alpha + [y + \eta_c(x,y) - h] \sin \alpha \\ Y' &= -[x + \xi_c(x,y)] \sin \alpha + [y + \eta_c(x,y) - h] \cos \alpha \end{aligned} \right\} \quad (48)$$

$$F'(X',Y') = f(x,y)$$

where  $\xi_c(x,y)$  and  $\eta_c(x,y)$  denote the longitudinal and vertical displacements  $\xi(x,y,0)$  and  $\eta(x,y,0)$  at the ship centerplane  $z = 0$ . Thus, the effects of the sinkage  $h$  and trim  $\alpha$  are now incorporated into the solution.

#### VII. CONCLUDING REMARKS

In conclusion, a few remarks concerning the field equations, which are satisfied approximately only in the above solutions, and the difficulties associated with blunt ship forms are appropriate.

Clearly, the field equations could be satisfied by distributing sources and vorticity in the parametric space  $-\infty < x < +\infty$ ,  $y \leq 0$ ,  $|z| \geq 0$ . Whereas, in principle, this approach would lead to an "exact theory", in practice it presents considerable numerical difficulties. A simpler solution is then desirable. Such a solution might be obtained by using the fact that the most significant error in the field equations seems likely to originate from the neighborhood of the free surface. This would suggest seeking an approximate solution of the field equations by means of Taylor expansions about  $y = 0$ .

The iterative procedure underlying the definition of the equivalent hull was implicitly assumed to be convergent in the above analysis. It may not be so, however, in the case of ship forms that are too blunt. This suggests a generalization of the above analysis in which the actual flow region is mapped onto a reference domain bounded by the undisturbed free surface and the wetted hull of the ship in position of rest. In this theory, the displacement field would be truly small, which has obvious advantages. However, such a theory would also be considerably more complicated.

Developments along the lines outlined above are now in progress and will be reported elsewhere.

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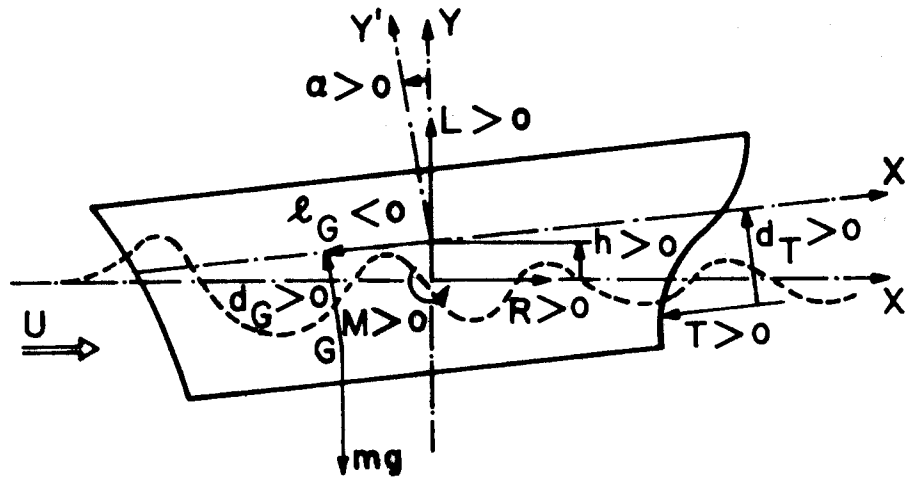


FIG. 1 - DEFINITION SKETCH

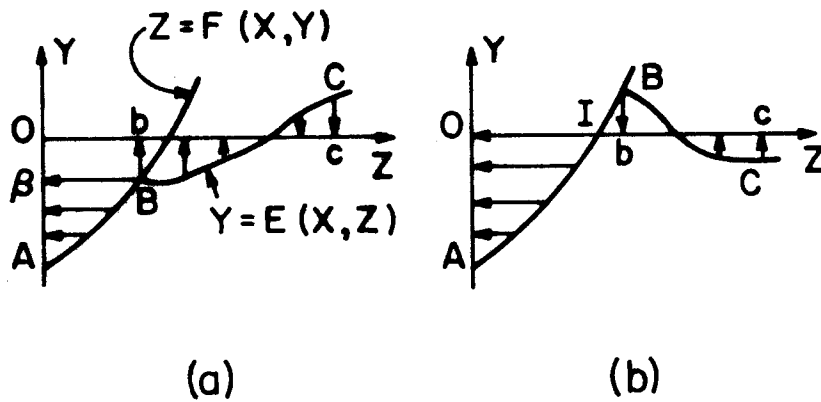


FIG. 2 - SECTION  $X = \text{CONSTANT}, Z \geq 0$   
OF THE SHIP AND THE FLOW FIELD

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<p>Two simple approximate solutions to the problem of the wavemaking of a ship in an inviscid fluid are presented. These approximate solutions are somewhat similar to those proposed previously by Guilloton, Wehausen, Dagan and Noblesse, but the present analysis leads to a new interpretation of the approximate solutions obtained in this paper. The analysis is based on a mapping of the actual flow region onto a parametric space bounded by the horizontal undisturbed free surface and a vertical cut on the ship centerplane. This parametric formulation is related to the approaches used by Guilloton, Wehausen, Dagan and Noblesse. The differences and similarities between these different approaches are discussed in detail, as well as the differences with the usual thin-ship theory.</p>			

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Ship Wave Resistance Mapping Parametrization Lagrangian Coordinates Strained Coordinates						