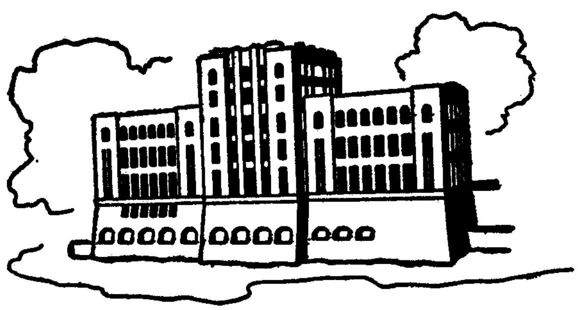
SEQUENTIAL OPERATIONS OF WATER RESOURCE SYSTEMS USING ADAPTIVE MULTI-OBJECTIVE TRADE-OFFS

by

Thomas E. Croley II

Sponsored by
Iowa Institute of Hydraulic Research
Water Resources Program
With Internal Funds



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ABSTRACT

With the advent of environmental, ecological, physical-chemical change, and human factor objectives, it is no longer possible to operate water resource systems for maximum economic gain only. Furthermore, the changing nature of objectives and of the system in real time indicates that adaptive decision-making techniques are imperatively needed. Existing trade-off techniques and sequential optimization techniques are reviewed and combined to provide policies for adaptive, multi-objective operation. Illustrative derivations indicate that such policies may have wide application to a large class of water management problems. These policies allow consideration of non-commensurate multi-objectives that are subjectively discerned and evaluated without fixing values and priorities prior to decision making. Also, they allow the adaptive inclusion of additional data and of relevant changes in objectives, priorities, and system models, in real time. Their application may be made without regards to a fixed operation horizon. They are heuristic, enabling clear understanding of their use and having reduced computation requirements. They are feasible for systems having reduced optimization horizon features described herein.

KEY WORDS

Adaptability; Computation; Management; Multi-objectives; Optimization; Real time; Reservoir operation; Sequential decisions; Simulation; Statistical analysis; Stochastic processes; Trade-offs; Water resources.

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INTRODUCTION

There is a recognized increase in water resource multi-objectives which are very difficult to consider in operations. They are very often subjectively discerned, evaluated, and compared; they are usually non-commensurate; and they are apt to change in importance with respect to each other in real time. There are sustained research efforts to develop multi-objective optimization techniques for the efficient consideration, design, and operation of water resource systems with respect to all objectives. Certain groups of "trade-off" techniques consider non-commensurate objectives and provide the best means for incorporating subjective judgments in the design at the proper stage. Although trade-off techniques surmount many problems, all multi-objective techniques may be deficient in practical applications. The techniques are unable to cope with real time problem changes at the design stage and lack adaptive features for continuous decision making in real time. Furthermore, the operation of a system in practice often extends beyond the original design period (instead of the system being scrapped) with altered operation objectives. However, to find optimum operation for a system, using present techniques, a fixed operation horizon must be considered. There is then a need for an adaptive technique in which a multi-objective optimization may proceed for indefinite operation.

Certain considerations have enabled the development of sequential single-objective techniques. It is demonstrated that these techniques have the advantages, over other methodologies, of: adaptability, ease of understanding (inherent in heuristic decision rules), and low computation requirements. Furthermore, they enable stage-by-stage optimizations in real time to proceed indefinitely, while operating near "optimum." Although the advantages associated with these sequential techniques are illustrated for a single-objective system formulation, there is considerable promise of developing similar techniques for a system formulation with changing, multiple, non-commensurate objectives, by using sequential objective trade-offs. The purpose of this paper is to present relevant thinking involved in the formulation of these methodologies for determining multi-objective water resource operations with sequential trade-offs in real time.

BACKGROUND

Classical Systems Analysis - As indicated in Figure 1, water resource objectives have increased and more than one objective is now being considered in the operation of water resource systems besides the objective of "economic efficiency." Figure 1 is designed especially to illustrate various non-commensurate objectives which are likely to change with real time; e.g., human factor objectives are changing as well as being subjectively discerned, evaluated and compared.

For a given water resource operation problem, several objectives are formulated and the merit of any operation sequence must be evaluated with respect to each of these objectives. This evaluation is performed by determining a "value " of the operation sequence with respect to each of the objectives. As pointed out by Shelly and Bryan (44), the assessment of values in a multi-objective operation problem is difficult. Values are, in general, multi-dimensional (each dimension corresponding to each objective). Evaluation of different operation sequences with respect to all objectives must be based on

CLASSICAL

FLOOD CONTROL

WATER SUPPLY
- IRRIGATION
- LOW FLOW AUGMENTATION
- NAVIGATION
- URBAN CONSUMPTION

POWER GENERATION

WILDLIFE HABITAT

SEDIMENTATION

"CONTEMPORARY ADDITIONS"

ECOLOGICAL ENHANCEMENT AND/OR PRESERVATION – extensive

ENVIRONMENTAL ENHANCEMENT AND/OR PRESERVATION – extensive

PHYSICAL – CHEMICAL CHANGE OBJECTIVES – extensive

HUMAN FACTOR OBJECTIVES:

- RECREATION OBJECTIVES (boating, fishing, camping, swimming, hunting, exploring, hiking, etc.)
- AESTHETIC OBJECTIVES (natural beauty and wonders, parks, seclusion, solitude, natural scenes and settings, etc.)
- EDUCATIONAL OBJECTIVES (archeological sites, geologic structures, hydrologic phenomenon, ecological refuges)
- HISTORICAL SIGNIFICANCE OBJECTIVES (historical structures, historical sites)
- CULTURAL OBJECTIVES (enhancement of Indian cultures, other ethnic cultures, and religious activities)
- OTHER SOCIAL/POLITICAL OBJECTIVES (employment opportunities, housing, social interactions, public acceptance, property market shifts, etc.)

FIG. 1 – PARTIAL LIST OF SOME RESERVOIR OPERATION OBJECTIVES (2)

all values.

After assignment of operation sequence values with respect to each objective, a "utility scale" may be used to transform the multi-dimensional value of an operation sequence into a scalar measurement of merit. This utility scale measures the relative contribution to the measure of merit (utility) of each value. Furthermore, relative weightings may be assigned to each utility, so that the measurement of merit reflects relative objective importance.

A partial list of difficulties, associated with the value assessment problem and consideration of non-commensurate multi-objectives that are subjectively evaluated, include: 1) objective discernment, 2) objective valuation, 3) value quantifying (utilities), 4) determination of a common-scale-of-utility, and 5) determination of the relative importance of objectives (weights). Several methodologies from the field of operations research have been proposed and used for the problems of objective setting, value assessment, and utility determination in water resources and are reviewed elsewhere (6, 29, 35, 37, 48, 49). Applications of these methodologies may be found in the references (2, 23, 25, 43, 45). Although objective discernment, valuation, and quantization are very important, this paper is concerned with avoiding the problems of determining a common-scale-of-utility and of determining the relative importance of objectives *a priori*.

<u>Applications</u> - Many multi-objective "optimization" formulations are reviewed in the literature (13, 30, 39, 47). However, only relevant applications to water resource <u>operations</u> are considered here. Subsequent discussion of optimization is made in terms of maximization without loss of generality The state-of-the-art in practical water resource operations can be categorized in the following manner. The first grouping is those techniques which measure several objective fulfillments with the same utility scale. Dollars are usually

used to measure the value of all objectives and their sum is used to measure the total utility. The optimization proceeds by finding that operation sequence with the maximum total utility. Examples in practical water resource operations and designs may be found in the references (10, 20, 26, 28, 33, 38).

A second grouping is actually a subset of the first and is those techniques which use relative weightings in the objective function to reflect objective priorities. The problem is solved many times parametrically for different values of the weighting coefficients. Examples are contained in the references (13, 14, 15, 28, 31, 39). In various reservoir operation studies, some of the objective weightings are defined in terms of contract levels for the project (19, 20, 46). After varying the contract levels as parameters over given ranges, the values for these parameters, which result in the overall maximum for all objectives (measured as the sum of the same utility measure for all objectives) are selected.

A third grouping is those techniques which reduce objectives to constraints in the optimization. Certain objectives are specified at being realized at fixed levels, regardless of the level of achievement for the major objective. The procedure is to find the operation with the maximum total utility for the major objective, with the other objective fulfillments fixed. Some studies where this approach has been followed are given in the references (1, 24, 27, 40).

A fourth category attempts to optimize system performance in a "hierarchical" approach (16, 18). The system component designs may be optimized with respect to relevant objectives at one level of the "hierarchy." Then, the combination of system components may be optimized with respect to a high priority objective.

Disadvantages associated with these categories of techniques include:

1) prior to the optimization, some techniques assign objectives, values, utilities, a utility scale, and relative importance of (competing) objectives, which may not be competent; 2) constraint objectives fix the relative importance of those objectives with respect to the other objectives unless a parametric consideration is made; 3) the utility scale of dollars is not relevant for many objectives; and 4) the same utility scale for all objectives is not relevant for widely diverse (non-commensurate) objectives. In avoiding these disadvantages, a fifth category of techniques has developed which can be termed "trade-off" techniques; for examples, see (3, 8, 17, 32, 34). The two trade-off techniques which have the most application for operation decisions, are described.

The first trade-off technique (8) is a straightforward application of simple ideas to achieve a realistic value assessment-optimization approach for two-objective problems. The basic philosophy of trade-off techniques is illustrated in this method. Basically, the procedure is first to choose the objectives considered relevant for the design; determine the relevant objective values; and determine the corresponding utilities for quantitative measurement of the design fulfillment of each objective. In determining these utilities, it is not necessary to assign a common utility scale for these objectives. Each objective fulfillment is expressed in terms of its most relevant utility. Also, a weighting which reflects relative importance of the objectives is not made. The approach then involves optimization by selecting decisions which give the maximum utility with respect to one objective, with the other objective expressed as a maximum (or minimum) allowable limit constraint in the optimization. The optimization is then repeated in a parametric determination for other constraint limits of the second objective, until a relationship is established which expresses maximum utility (for objective 1) as a function of maximum (or minimum) constraint limits of the other objective. Thus, a relation like Figure 2 is generated. From this relation, the utility of the optimum designs (with objective constraints) is subtracted from that with no constraint. This results in the excess utility as marginal benefit of one objective with respect to the other, as in Figure 3. Now, the procedure is to select that design (or optimum operation sequence) with the desired "trade-off" between objectives, based on the trade-off function just derived. This can be applied to more than two objectives, but the trade-offs must then be made between more than two objectives and the situation becomes complex for subjective judgments as well as computationally complex.

The second trade-off technique (17) is referred to as the Surrogate Worth Trade-Off Method. Its underlying philosophy is similar to that described above, but its formulation is for the general n-objective case. Although it has been applied as a design technique, it will have much application for operation decisions, if few decisions are required. The optimum with respect to each objective (all other objectives ignored) is determined. The multi-objective problem is formulated by specifying the objectives in constraints and the maximum tolerable constraint levels (for each objective) are related to the optimums from the single-objective problems. This determines the necessary trade-off functions. The subjective judgments as to the relative worth of the objectives are embodied in "surrogate ratios" which are used to establish desired trade-offs between objectives. The concepts of duality and Lagrange multipliers are utilized to provide the basis for the construction of the trade-off matrix relations. The resulting trade-off determinations are then made by using the trade-off functions and the worth ratios. This approach has the most application for problems with more than two objectives; however, the computation may become large for complex problems. Also, the subjective

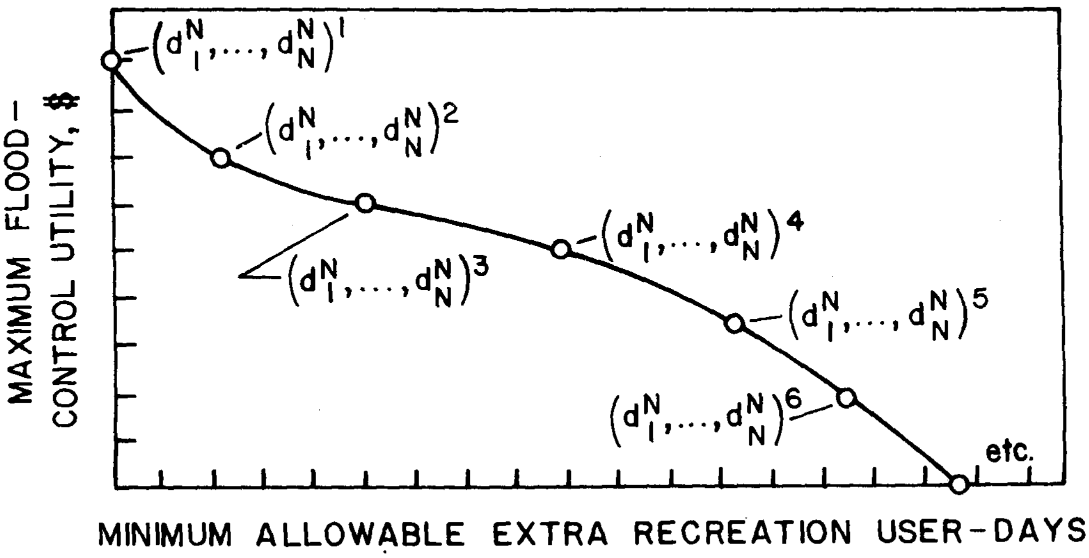

FIG. 2 – OPTIMUM FLOOD CONTROL UTILITY
VS RECREATION UTILITY

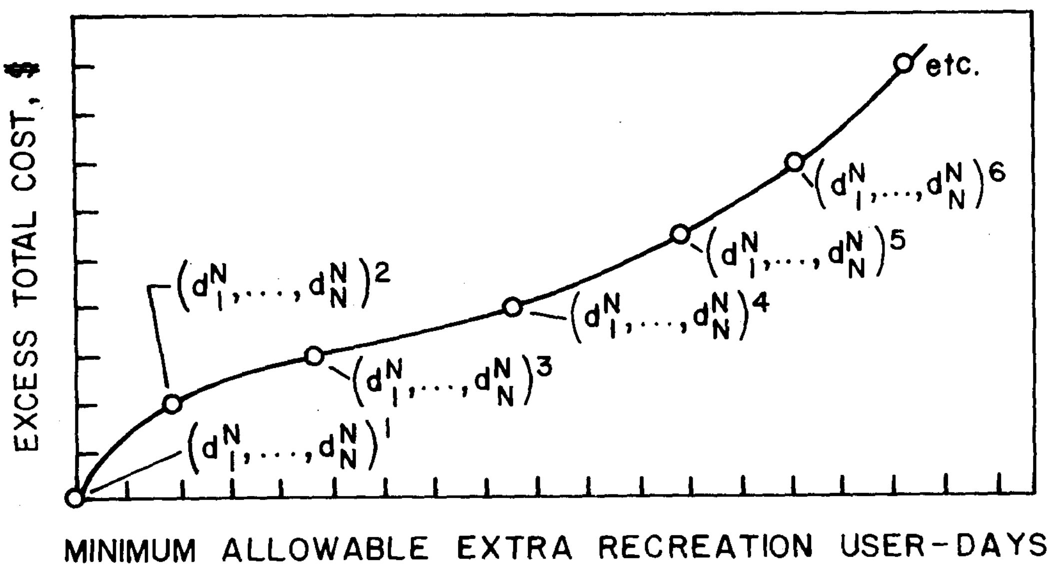

FIG. 3 – EXCESS TOTAL COST OF OPTIMUM DESIGN
VS RECREATION UTILITY

evaluations of objective worth are restricted to their representation as surrogate ratios.

There are several advantages, associated with objective trade-off techniques, over the other techniques. The design fulfillment for each objective is measured with the most relevant utility for each objective. The subjective determination of a common utility scale is avoided. The subjective determination of relative importance of objectives is not made *a priori*; thus the relative importance may be established after relative effects of objectives are established (trade-offs). The trade-offs involved in determining the relative importance of non-commensurate objectives can be identified quantitatively. The decision maker has alternatives identified in the best practicable manner: objective trade-offs; this is the most relevant expression as well as being more concise than consideration of all decisions.

As mentioned in the Introduction, there are general disadvantages associated with all of these categories of techniques (including trade-off techniques). In practical operations, objectives, their relative importance, their utilities, the system formulation, etc., change with time in a manner which is generally unknown at the time of design, making an adaptive decision making methodology important. The computation requirements associated with the reapplication of existing techniques at every stage of operation (attempting adaptive decision making) prohibit this application. Real operations often extend beyond the original design period and shouldn't be dependent upon often arbitrary operation horizons; thus, stage-by-stage decision determinations are necessary which may proceed indefinitely and without dependence on a fixed operation horizon.

Sequential Techniques - The disadvantages associated with the aforementioned optimum operation methodologies prompt the consideration of sequential optimization techniques. Many sequential optimization techniques have been formulated for the single-objective problem in water resource systems to achieve reduction in computation. The determination of operating decisions for reservoir operation with dynamic programming provide many examples (11, 12, 20 22, 50). The modification of these techniques for decomposable objective functions has been made to create sequential techniques which provide adaptive real time decision making techniques and not just "decision rules" or an "operation plan" (4,5,9, 21, 51). In particular, modifications were made for the cases of deterministic and stochastic hydrologies and are referred to respectively as the Modified Application of Deterministic Optimization Techniques (MADOT) and the Alternate Stochastic Optimization (ASO) technique. Both ASO and MADOT are described elsewhere (4, 5) respectively. Basically, they both replace the N stage decision problem with N problems, each with decisions to be determined over the remaining operation horizon. Each of the N problems is solved sequentially, but only over just the next few stages to determine the best single stage decisions at each present stage. It has been shown (4, 5, 7) that, as the number of stages considered in each reduced operation problem increases, the overall decision sequence approaches the true optimum. Also, for many problems only a few stages are necessary in the optimizations to achieve very near-to-optimum results.

There are several advantages associated with these techniques for the single-objective problem. Sequential decisions may be made in real time for operation of existing systems, where future changes are not known in advance. The application of the decision making techniques is adaptive since all relevant information (such as changes in objectives, priorities, system models, etc.) can be included. The computation requirements of these decision techniques are small when compared to equivalent applications of other methods. The techniques are straightforward, enabling clear understanding of their application in real time. Decisions may then be made sequentially without regards to a fixed total operation horizon, allowing near optimum operation of the system to continue indefinitely. There is generally high confidence that the resulting decisions are near optimum for a wide variety of water resource operation problems. Application of any relevant optimization technique (including objective trade-off determinations) may then be made in sequential reapplications over reduced operation horizons at each stage of operation.

The development of MADOT and ASO were made starting with decomposable objective functions (where Bellman's Principle of Optimality applies to make dynamic programming decompositions). However, the application of these techniques may be made for a more general class of problem formulations. The following theory presentation is designed to indicate this larger applicability. This presentation is illustrative only (not rigorous) and is made for the purpose of motivating succeeding heuristic technique combinations.

THEORY

If the objective function for the single-objective problem is:

$$B = B(d_1,\ldots,d_N) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which B = total utility over the N stages of the system operations (operation horizon) and d_i = decision vector at i^{th} stage (in real time) for the system. Assuming a unique optimum exists, the maximization of the total utility with respect to the operating decisions is:

$$\max_{d_1,\ldots,d_N} B = \max_{d_1,\ldots,d_N} B(d_1,\ldots,d_N) = B(d_1^N,\ldots,d_N^N) \quad \ldots\ldots\ldots\ldots \quad (2)$$

in which d_i^X = optimum decision vector at the i^{th} stage from an X-dimension optimization (X decision vector values to be determined) over stages N-X+1 through stage N using the following X-dimension objective function:

$$B^X = B(d_1^N,\ldots,d_{N-X}^N,d_{N-X+1},\ldots,d_N) \qquad (3)$$

Thus, d_i^X is only defined for $i \geq N-X+1$, $X \leq N$. For $X = N$, then $d_i^X = d_i^N$, $i = 1,\ldots,N$ and the d_i^N are the optimum decision vectors from an N-dimension optimization over all N stages of the problem using the original objective function (Eq. 1). Any and all relevant optimization constraints are implied in the maximization notation. Now, the following relation can be made in general:

$$\max_{d_1,\ldots,d_N} B = \max_{d_1} \{\max_{d_2,\ldots,d_N} B(d_1,\ldots,d_N)\},$$

$$= \max_{d_1,\ldots,d_{i-1}} \{\max_{d_i,\ldots,d_N} B(d_1,\ldots,d_N)\},$$

$$= \max_{d_i,\ldots,d_N} B(d_1^N,\ldots,d_{i-1}^N, d_i,\ldots d_N) \qquad (4)$$

$$\max_{d_1,\ldots,d_N} B = B(d_1^N,\ldots,d_{i-1}^N, d_i^{N-i+1},\ldots,d_N^{N-i+1}) \qquad (5)$$

in which the (N-i+1)-dimension optimization uses the objective function identified in Eq. 4 (see Eq. 3 with X = N-i+1). Equations 2 and 5 imply:

$$(d_i^{N-i+1},\ldots,d_N^{N-i+1}) \equiv (d_i^N,\ldots,d_N^N),$$

$$[\text{given } (d_1,\ldots d_{i-1}) \equiv (d_1^N,\ldots,d_{i-1}^N)] \qquad (6)$$

In particular, the first decision vector of each sequence are equivalent:

$$d_i^{N-i+1} \equiv d_i^N, \; [\text{given } (d_1,\ldots,d_{i-1}) \equiv (d_1^N,\ldots,d_{i-1}^N)] \qquad (7)$$

Equation 6 says that if the first i-1 optimum decision vectors from an N-dimension optimization are known, then an (N-i+1)-dimension optimization (using the objective function of Eq. 4) over the remaining N-i+1 stages yield the same last N-i+1 decision vector values that are obtained in the N-dimension optimization. Reapplying Eq. 7 for all i, i=1,...,N, the result is:

$$(d_1^N,\ d_2^{N-1},\ d_3^{N-2},\ldots,d_{N-2}^3,\ d_{N-1}^2,\ d_N^1) \equiv (d_1^N,\ d_2^N,\ldots,d_{N-1}^N,\ d_N^N). \quad . \quad . \quad . \quad . \qquad (8)$$

The "decomposition" of Eq. 8 is not a dynamic programming decomposition; it applies for all single-objective optimization problems but it is not particularly useful. Equation 8 says that instead of solving for the N optimum decision vector values in a single N-dimension optimization, the first decision is found from an N-dimension optimization (the other N-1 optimum decision vectors are ignored); the second decision vector is found from an (N-1)-dimension optimization (ignoring other decision vectors); etc. Thus, the same amount of computation is involved in finding the first stage optimum decision vector (on the left side of Eq. 8) as in finding the entire optimum decision vector sequence (on the right side of Eq. 8). Although Eq. 8 is true, it has not, of course, been used as an alternate method of finding optimum decision vectors.

Equation 8 can serve other purposes however. Researchers (4, 5, 7, 41, 42) have noted two factors which can make the length of analysis in an optimization problem shorter than the operation horizon for many water resource operation problems: 1) the larger the period of analysis, the higher is the likelihood of an event or combination of events that will cause previous operational policy to be irrelevant to the future state of the system; and 2) discounting the value of future production relative to current production by amortization in some objective functions makes later decisions have small influence on the present. These factors allow the following approximation to be made:

$$\bar{d}_i^k \cong d_i^{N-i+1}, \text{ given } (d_1,\ldots,d_{i-1}) \equiv (d_1^N,\ldots,d_{i-1}^N),$$

$k<<N$, $k = N-i+1$ for $i > N-k+1$. . . . . (9)

in which $\bar{d}_i^k$ = optimum decision vector at the i^{th} stage from a k-dimension optimization over the stages i through i+k-1 using the k-dimension objective function:

$$\bar{B}^k = B(d_1^N,\ldots,d_{i-1}^N,\ d_i,\ldots,d_{i+k-1},\ c_{i+k},\ldots,c_N). \quad (10)$$

in which $c_{\bullet}$ = selected constants.

The optimization problems are now restricted to those where the objective function satisfies the following relation:

$$d_j^N - \varepsilon_j \le d_j \le d_j^N + \varepsilon_j\ ,\ j=1,\ldots,N$$

$$\Longleftrightarrow B(d_1,\ldots,d_N) \ge B(d_1^N,\ldots,d_N^N) - \alpha \quad (11)$$

in which $\varepsilon_{\bullet}$ are selected vector deviations and α is an arbitrarily small deviation. This class of problems includes all of the objective functions contained in the practical applications of the references, and in general includes many water resource objective functions. It is interesting to note that this class of problems includes some, but is not limited to, problems amenable to dynamic programming and the Principle of Optimality (36). Thus, for problems of this type (see Appendix III):

$$\bar{\bar{d}}_i^k \cong d_i^{N-i+1} \text{ given } (d_1,\ldots,d_{i-1}) = (\bar{\bar{d}}_1^k,\ldots,\bar{\bar{d}}_{i-1}^k) \cong (d_1^N,\ldots,d_{i-1}^N),$$

$k<<N$, $k = N-i+1$ for $i > N-k+1$ . . . . (12)

in which $\bar{\bar{d}}_i^k$ = "near"-optimum decision vector at the i^{th} stage from a k-dimension optimization over the stages i through i+k-1 using the k-dimension objective function:

$$\bar{\bar{B}}^k = B(\bar{\bar{d}}_1^k,\ldots,\bar{\bar{d}}_{i-1}^k,\ d_i,\ldots,d_{i+k-1},\ c_{i+k},\ldots,c_N). \quad (13)$$

Using the approximation of Eq. 12, Equation 8 suggests that

$$(\bar{\bar{d}}_1^k, \bar{\bar{d}}_2^k, \bar{\bar{d}}_3^k, \ldots, \bar{\bar{d}}_{N-k}^k, \bar{\bar{d}}_{N-k+1}^k, \bar{\bar{d}}_{N-k+2}^{k-1}, \ldots, \bar{\bar{d}}_{N-2}^3, \bar{\bar{d}}_{N-1}^2, \bar{\bar{d}}_N^1)$$

$$\cong (d_1^N, \ldots, d_N^N) \quad \ldots \quad (14)$$

To apply Eq. 14 in real time (where the stage is identified with discrete time) the following conditions are necessary for each i:

$$\left.\begin{array}{l} 0 \geq g_{i,1}(d_i, \ldots, d_{i+k-1}) = f_{i,1}(I_{i+k-1}, \ldots, I_1, d_{i+k-1}, \ldots, d_1) \\ \vdots \\ 0 \geq g_{i,n}(d_i, \ldots, d_{i+k-1}) = f_{i,n}(I_{i+k-1}, \ldots, I_1, d_{i+k-1}, \ldots, d_1) \end{array}\right\} \ldots \quad (15)$$

Equations 15 simply state that the n constraint equations (which have so far been implied in the maximization notation) in each k-dimension optimization must be completely determined as known functions of parameters or variables (input vectors, I_i) whose values have already occurred (in real time) prior to stage i or will occur in the next k stages. Some or all of the inequalities in Eq. 15 and succeeding equations may be replaced with equalities, as appropriate. These constraint equations include the system models, resource and operation limitations, system constraints and boundary conditions, and any other constraints on the selection of the k-dimension optimization decisions (including maximum allowable constraints on utilities with respect to a second objective). Therefore, given all past conditions, at stage i (in real time):

$$\left.\begin{array}{l} 0 \geq g_{i,1}(d_i, \ldots, d_{i+k-1}) = f'_{i,1}(I_{i+k-1}, \ldots, I_i, d_{i+k-1}, \ldots, d_i) \\ \vdots \\ 0 \geq g_{i,n}(d_i, \ldots, d_{i+k-1}) = f'_{i,n}(I_{i+k-1}, \ldots, I_i, d_{i+k-1}, \ldots, d_i) \end{array}\right\} \ldots \quad (16)$$

Most practical water resource system operations satisfy Eqs. 15 even for small k. Thus, the procedure suggested by Eq. 14 is depicted in Figure 4 [see also reference (5)]. In practical problems, the future input vectors are not known,

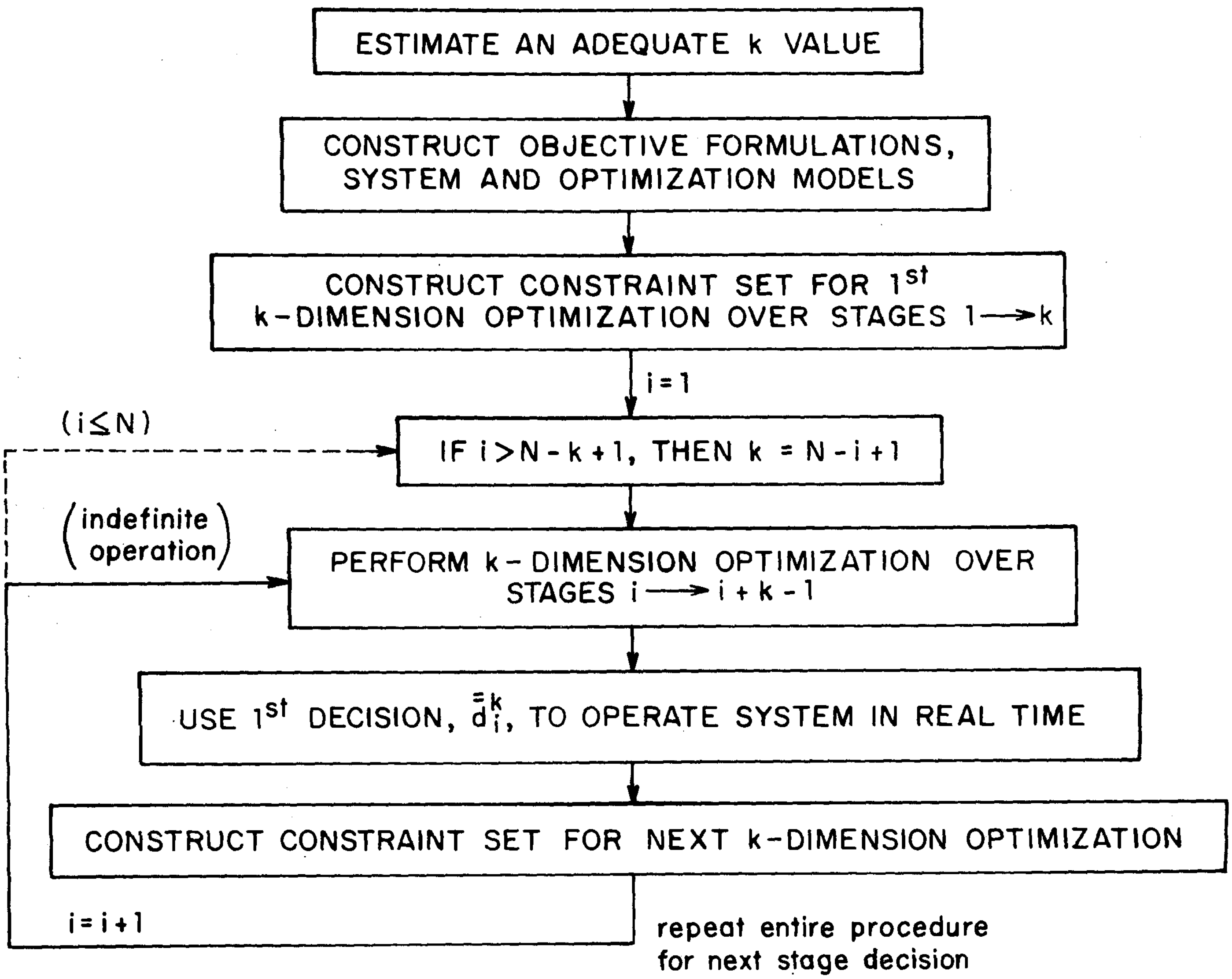

FIG. 4—MADOT APPLICATION WHEN FUTURE INPUTS ARE KNOWN

but may be estimated (for the next few stages in real time) by relevant forecasting methods. In this case, k should not be selected to be larger than the number of stages for which there is confidence in the forecasts. Furthermore, the greatest advantage of these procedures is that operating decisions are made in real time, allowing continuing reformulation of the problem, to adapt to real time changes. Assuming that the problem reformulations still satisfy the general requirements of Eqs. 11 and 15, and making use of the real-time adaptability feature, Eq. 14 suggests the procedure outlined in Figure 5.

It may be desired to forecast the optimum decision vector directly for use in operating the system at each stage in a stochastic optimization approach. There are several approaches, but the following is particularly suited to adaptations in real time. It is suggested by Eq. 8, using the modification of Eq. 12. In this procedure, all possible input sequences for the next k remaining stages and their probabilities are known (or estimated). Then the optimum decision sequence for each input sequence is calculated and only the first (present) stage decision vector is noted. All optimum present-stage decision vectors and their associated probabilities are used to formulate the probability distribution for the optimum present-stage decision vector. The choice of the actual decision is then made on the basis of this distribution. A data generation approach has been suggested to estimate this "present-stage optimum decision vector distribution" (4). Its essence is embodied in the outline of Figure 6.

Special Applications - In some applications, the reconstructions in the adaptive approach may not be necessary nor sufficiently worthwhile at every stage in real time. If this is the case, the reconstructions identified in Figure 5 (with a single asterisk) may then be made only at every m^{th} stage,

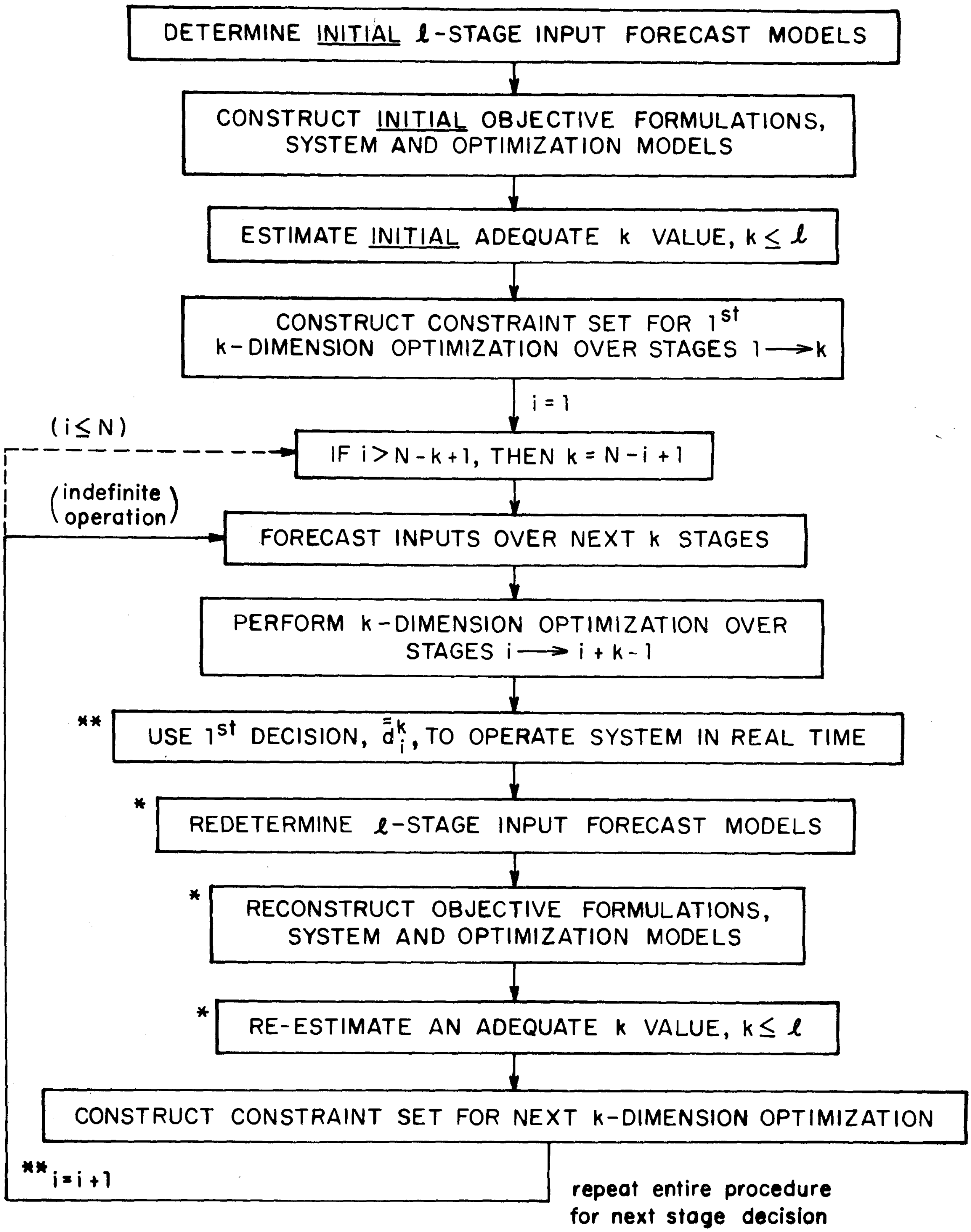

FIG. 5—MADOT APPLICATION FOR ADAPTIVE, PRACTICAL OPERATIONS

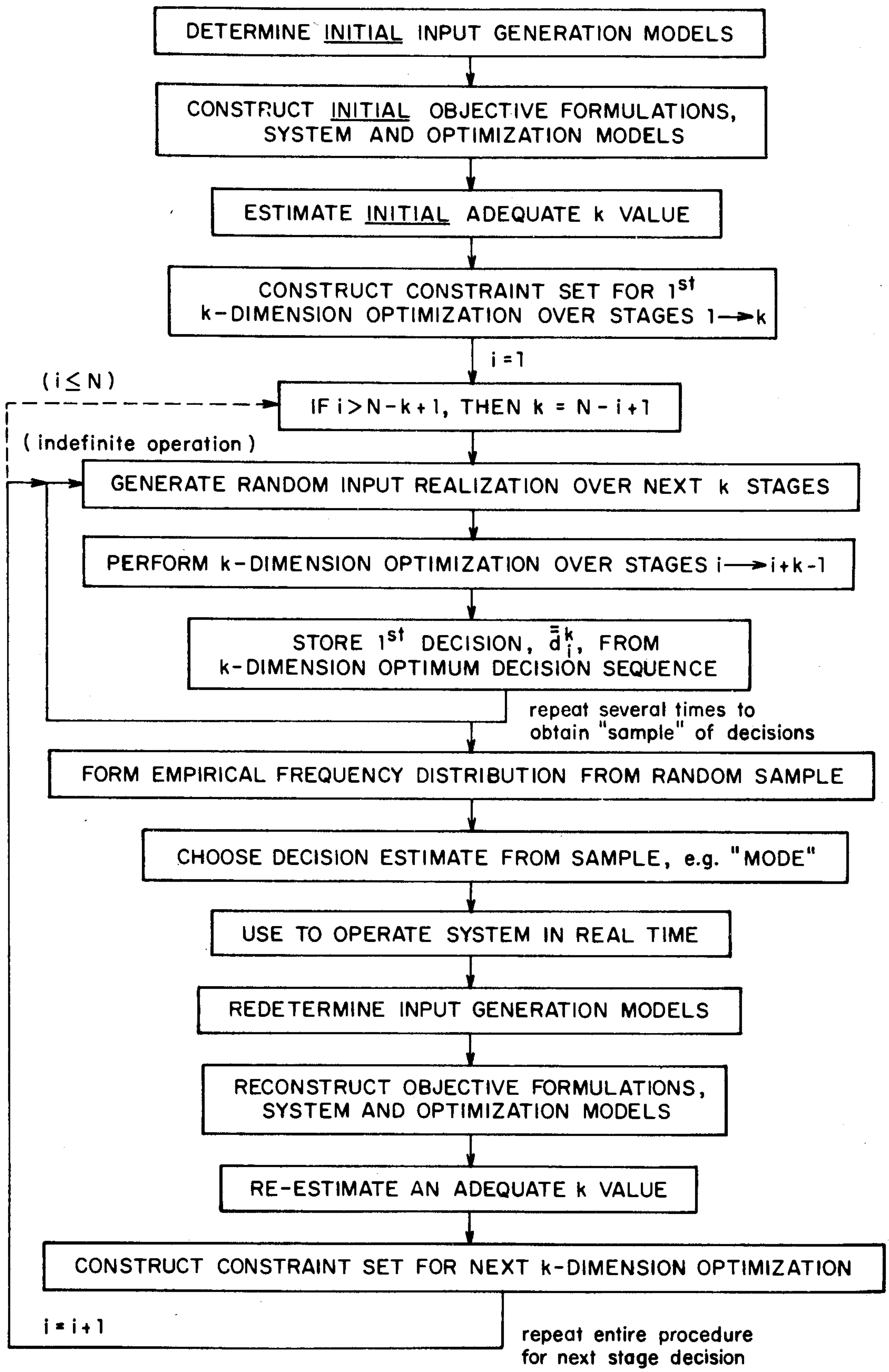

FIG. 6—ASO APPLICATION FOR ADAPTIVE, PRACTICAL APPLICATIONS

by-passing this step at other stages. Likewise, the optimizations also may not be necessary nor sufficiently worthwhile at every stage in real time for a sufficiently fine discretization . Then, the optimizations can be repeated only at every m^{th} stage, $m \leq k$. Equation 6 also implies:

$$(d_i^{N-i+1}, d_{i+1}^{N-i+1}, \ldots, d_{i+m-1}^{N-i+1}) \equiv (d_i^N, \ldots, d_{i+m-1}^N),$$

$$\text{given } (d_1, \ldots, d_{i-1}) \equiv (d_1^N, \ldots, d_{i-1}^N) \quad \ldots \quad (17)$$

which can then be reapplied to get a result similar to Eq. 8:

$$(d_1^N, \ldots, d_m^N, d_{m+1}^{N-m}, \ldots, d_{2m}^{N-m}, d_{2m+1}^{N-2m}, \ldots, d_{3m}^{N-2m}, \ldots, \ldots, d_{N-2m+1}^{2m}, \ldots$$

$$\ldots, d_{N-m}^{2m}, d_{N-m+1}^{m}, \ldots, d_N^m) \equiv (d_1^N, \ldots, d_N^N) \quad \ldots \quad (18)$$

In Eq. 18, it is assumed that N is evenly divisible by m, for convenience of notation. Using a similar analysis as before:

$$(d_1^{1,k}, \ldots, d_m^{1,k}, d_{m+1}^{m+1,k}, \ldots, d_{2m}^{m+1,k}, d_{2m+1}^{2m+1,k}, \ldots, d_{3m}^{2m+1,k}, \ldots, \ldots$$

$$\ldots, d_{N-m+1}^{N-m+1,m}, \ldots, d_N^{N-m+1,m}) \cong (d_1^N, \ldots, d_N^N) \quad \ldots \quad (19)$$

In Eq. 19, $d_i^{j,k}$ = "near"-optimum decision vector at the i^{th} stage ($j \leq i \leq j+k-1$) from a k-dimension optimization <u>over the stages j through j+k-1</u> using the k-dimension objective function:

$$B^{j,k} = B(d_1^{1,k}, \ldots, d_m^{1,k}, \ldots, \ldots, d_j, d_{j+1}, \ldots, d_{j+k-1}, c_{j+k}, \ldots, c_N) \quad \ldots \quad (20)$$

Equation 19 suggests a procedure similar to that outlined in Figure 5 but with the appropriate simple changes made in the steps identified with a double asterisk. This modified procedure repeats optimizations and reconstructions only at every m^{th} stage when system changes and optimizations are believed to be significant enough for a re-evaluation. Procedures for the stochastic optimization application can also be similarly derived.

PROPOSAL

As suggested in the Introduction, a combination of the trade-off techniques and the sequential techniques can be made to construct adaptive techniques for making trade-off decisions in real time for water resource operations with changing, non-commensurate multi-objectives. These techniques would have the joint features associated with objective trade-off techniques and with the sequential optimization techniques, already described.

The two sequential techniques and the two trade-off techniques can be combined to yield the following techniques (and their variations):

1. Trade-Off Decisions in Real Time with Two Non-Commensurate Objectives using Hydrological Forecasts (or MADOT-1)
2. Trade-Off Decisions in Real Time with Two Non-Commensurate Objectives using Estimated Stochastic Hydrologies (or ASO-1),
3. Surrogate Worth Trade-Off Decisions in Real Time using Hydrological Forecasts (or MADOT-2), and
4. Surrogate Worth Trade-Off Decisions in Real Time using Estimated Stochastic Hydrologies (or ASO-2).

Although these possibilities exist, attention is devoted, herein, to construction of the first two heuristic techniques.

Combination of Techniques - The following discussion is illustrative only and pertains to the two-objective trade-off technique and MADOT and ASO. The other possible combinations are more involved, but the combination philosophy would be similar to the illustrations given herein.

When applying the trade-off technique to selection of an operation plan (all operation decisions), the derived trade-off relationship has entire "optimum" decision sequences corresponding to each point; see Figures 2 and 3. The points in Figure 3 represent an optimization (as in Eq. 2) for different values of the minimum allowable constraint limit b, of the second objective utility B_2, to get optimum decision sequences corresponding to each trade-off level $b^{\bullet}$:

$$(d_1^N,\ldots,d_N^N)^1 = (d_1^N,\ldots,d_N^N | B_2 \geq b = b^1),$$

$$(d_1^N,\ldots,d_N^N)^2 = (d_1^N,\ldots,d_N^N | B_2 \geq b = b^2),$$

$$\vdots$$

$$(d_1^N,\ldots,d_N^N)^q = (d_1^N,\ldots,d_N^N | B_2 \geq b = b^q) \quad \ldots\ldots\ldots \quad (21)$$

The trade-off decisions associated with a desired level of trade-off $b = b^*$, can thus be identified:

$$(t_1,\ldots,t_N) = (d_1^N,\ldots,d_N^N)^* = (d_1^N,\ldots,d_N^N | B_2 \geq b = b^*) \quad \ldots\ldots \quad (22)$$

in which t_i = trade-off decision vector at i^{th} stage (in real time) for the system. Equation 8 applies for any and all optimizations, including that of Eq. 2 for all minimum tolerable constraint limits (b) of objective B_2; therefore:

$$(t_1,\ldots,t_N) = [d_1^N(b^*),\ d_2^{N-1}(b^*),\ldots,d_{N-1}^2(b^*),\ d_N^1(b^*)]. \quad \ldots\ldots \quad (23)$$

in which $d_i^X(b)$ = optimum decision vector at the i^{th} stage from an X-dimension optimization <u>over stages N-X+1 through stage N</u> using the following X-dimension objective maximization:

$$\max B[d_1^N(b),\ldots,d_{N-X}^N(b),\ d_{N-X+1},\ldots,d_N] \text{ subject to (s.t.) } B_2 \geq b \quad \ldots \quad (24)$$

Assuming Eq. 12 also applies for all optimizations (all b), then:

$$(t_1,\ldots,t_N) \cong [\bar{\bar{d}}_1^k(b^*),\ \bar{\bar{d}}_2^k(b^*),\ldots,\bar{\bar{d}}_{N-k+1}^k(b^*),\ \bar{\bar{d}}_{N-k+2}^{k-1}(b^*),\ldots$$

$$\ldots,\bar{\bar{d}}_{N-1}^2(b^*),\ \bar{\bar{d}}_N^1(b^*)] \quad \ldots\ldots\ldots\ldots \quad (25)$$

where $\bar{\bar{d}}_i^k(b)$ = "near"-optimum decision vector at the i^{th} stage from a k-dimension optimization <u>over the stages i through i+k-1</u> using the k-dimension objective maximization:

$$\max B[\bar{\bar{d}}_1^k(b),\ldots,\bar{\bar{d}}_{i-1}^k(b),\ d_i,\ldots,d_{i+k-1},\ c_{i+k},\ldots,c_N] \quad \text{s.t. } B_2 \geq b \qquad (26)$$

In the deterministic realm (MADOT-1), the heuristic decision making procedure suggested by Eq. 25 can be identified as outlined in Figure 7; advantage is taken of the real time adaptability feature of the sequential technique and the problem reformulations are assumed to still satisfy Eqs. 11 and 15. Likewise, the heuristic procedure for the stochastic realm (ASO-1) can be identified as outlined in Figure 8.

The reformulation of the Surrogate Worth Trade-Off method could also be made, similar to that described herein, to construct the MADOT-2 and ASO-2 methodologies. Again, this reformulation would be made which gives the surrogate ratios and the objective trade-off relations for operations over just the next few stages of operation. Then the decision determinations will be limited to selection of the single-stage decisions at each "present" stage of operation.

ANALYSIS

The suitability of these combination techniques can only be determined in specific applications. While they are expected to have the advantages already identified with the component methodologies, there are several aspects of the application which are ill-defined.

The assumption of a unique optimum was made only for convenience in the theoretical discussion. Actually, the existence of more than one optimum decision sequence would improve the "closeness-to-optimum" of the reduced horizon optimization decisions. Therefore, the class of operation problems to which these management policies apply is quite broad.

The "selected constants," $c_{\bullet}$ identified in Eqs. 10, 13, 20, and 26, are not really defined herein. Actually their choice depends upon the nature of

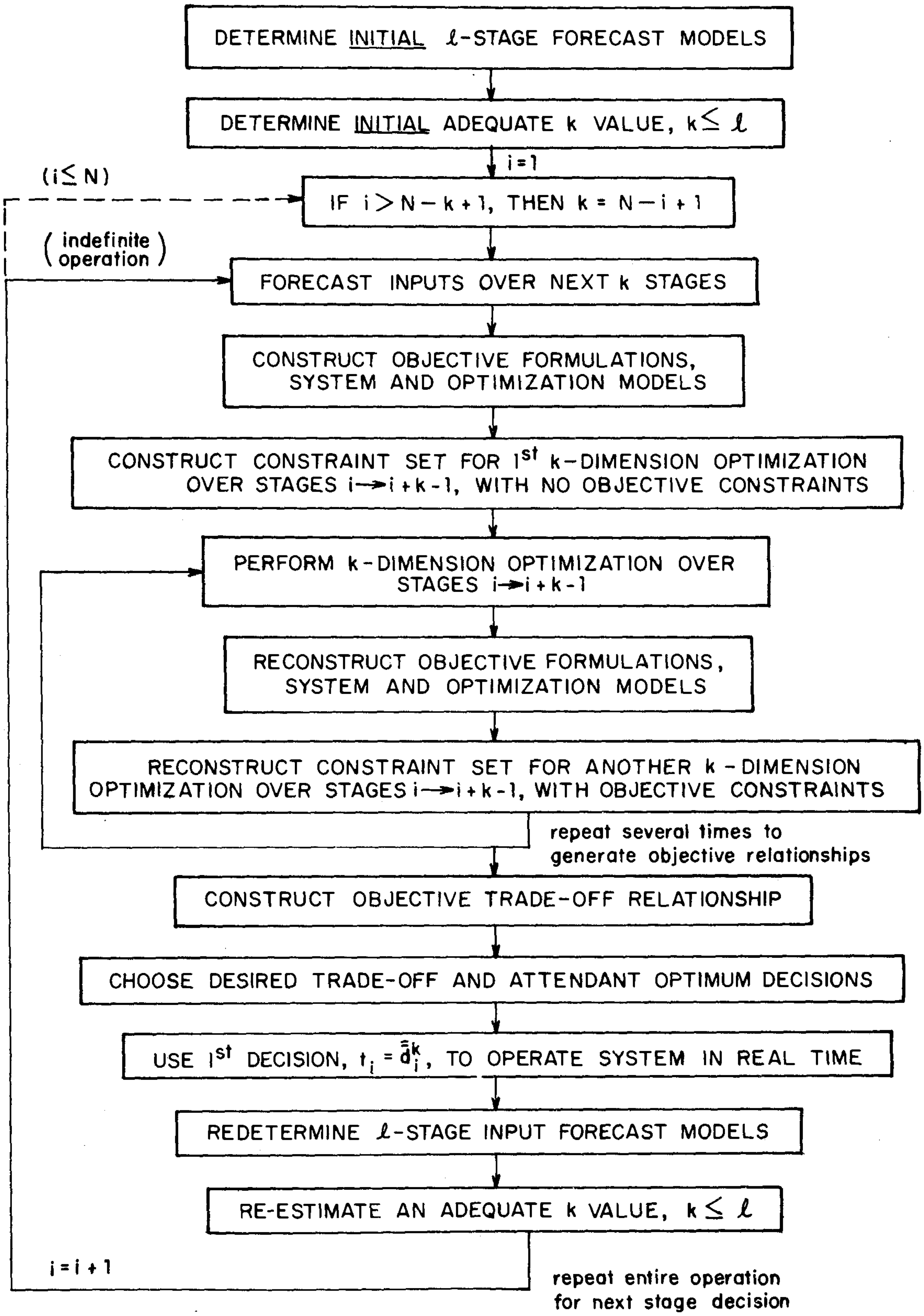

FIG. 7—MADOT APPLICATION FOR ADAPTIVE, PRACTICAL TRADE-OFF OPERATIONS

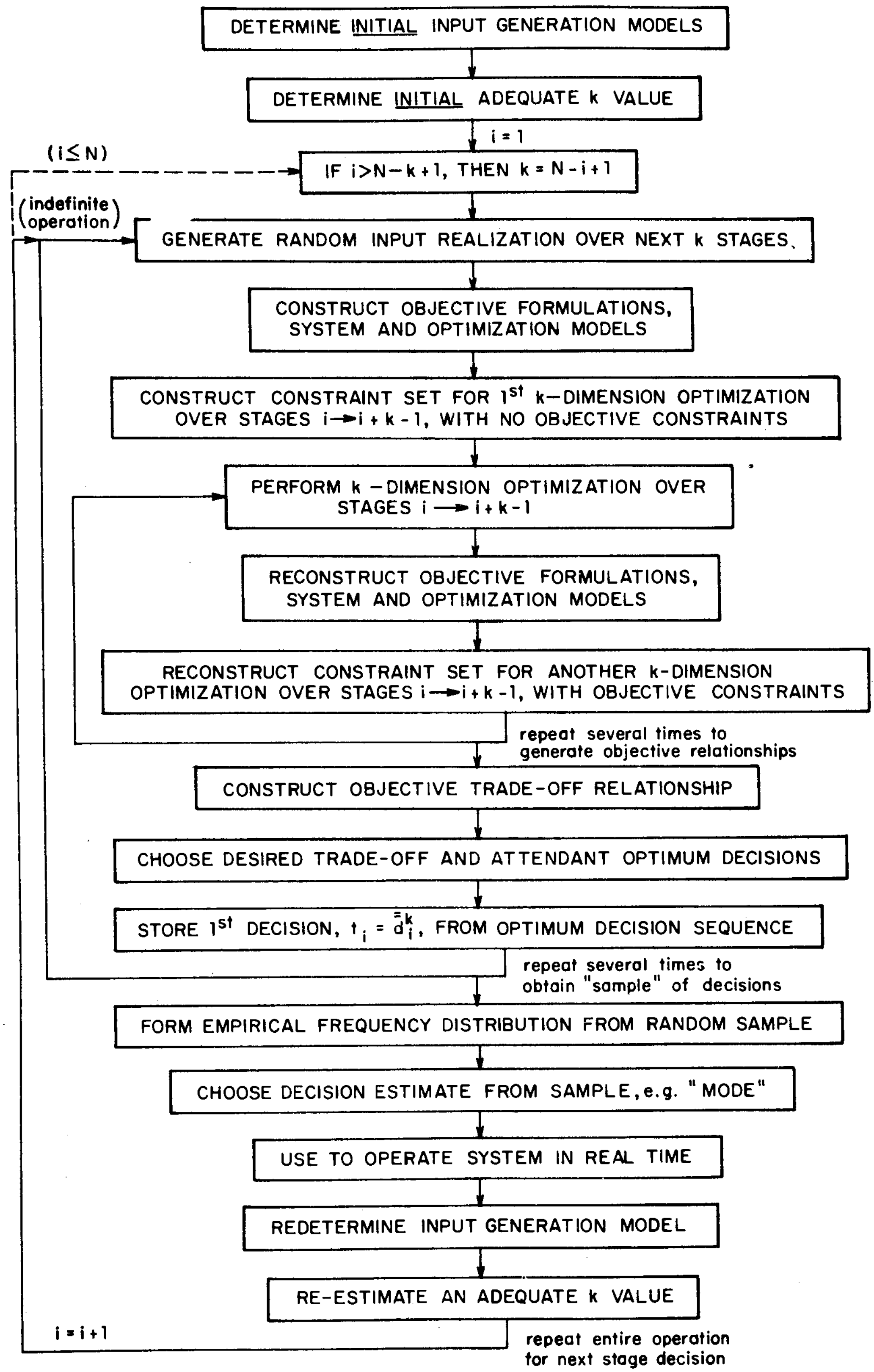

FIG. 8– ASO APPLICATION FOR ADAPTIVE, PRACTICAL TRADE-OFF OPERATIONS

the objective function. They can be selected so as to minimize deviations of total utility from the true maximum as a function of the k decision vectors. While this is not obvious, these constants may simply be set equal to zero in objective formulations which are a sum of benefits (or costs) over the stages of operation. This has resulted in satisfactory results elsewhere (4, 5, 7). Alternatively, special studies can investigate optimization results for alternative constants in specific applications to provide some preliminary guidelines. In some applications, the choice of the constants has no effect on the maximization results.

There is some question as to what length of the reduced operation horizon at each stage (k) is satisfactory. As k increases, so do the computation requirements but also, so does the degree to which results approach optimum (for each single-objective optimization). To determine an acceptable value for k, a preliminary statistical study on a similar hypothetical problem may be made. A value of k can be chosen where the additional approach to the optimum is offset by increases in computation costs. Alternatively, the value of k can be set equal to the number of stages for which there is a satisfactory level of confidence in the forecast models (in MADOT) or input generation models (in ASO).

The sample size at each stage in ASO and the selection of the mode as the optimum decision estimate are arbitrary (see Figures 6 and 8). The suitable size and estimator can be determined in preliminary studies also. In fact, the existence of optimization qualities such as those identified in Eq. 11 must be ascertained prior to the technique applications. While this is difficult to do in general, specific applications can involve this statistical determination in preliminary studies (4,5).

These heuristic operation determinations are only relevant for the real time operation of <u>existing</u> systems and would have little or no value for preliminary design. This is because many computations are involved for each stage decision and because the changing nature of the objectives, system, priorities, etc., are unknown for the future.

The worth of these techniques can be indicated in special application studies. The relative ease of understanding and application must be determined. The closeness of the results to "optimum trade-off" must be evaluated. The degree of computational difficulty must be established. The adaptability of the techniques to various changes must be determined. Special studies can indicate how the operations compare with straight applications of trade-off techniques. Of course, these studies must not involve changing objectives, priorities, or system models, so that straight applications of the trade-off techniques can also be made for comparison purposes.

Furthermore, the areas of applicability of these techniques can be indicated in special studies. The operation problems which are relevant must be delineated. The effect of the forecast or input generation models and of the system formulations on the required value of k must be determined (at least in an approximate manner). The formulations of objective functions and constraint spaces which effect the applicability and/or results and how they effect the results must be established. The types of changes in objectives, priorities, and system models which effect the results and how they effect them must also be determined.

Finally, the techniques can be applied to real operation problems. Thus, the methodology can be illustrated for a practical problem and the worth of the method(s) in a practical situation may be demonstrated. Also, operations for a multi-objective problem can be determined with real time trade-offs, allowing the best consideration of changing conditions.

CONCLUSIONS

Multi-objectives in water resource operations can be considered using modified applications of existing techniques. The inclusion of non-commensurate objectives, values, and priorities that are subjectively discerned, evaluated, compared and that are changing in real time is possible. The operation of water resource systems with respect to these changing, non-commensurate objectives can be determined in real time without regards to a fixed operation horizon. Thus, indefinite operation of dynamic (changing characteristics) systems is possible.

The methodologies are heuristic, enabling their understanding and application to existing operation problems. They are feasible for systems having reduced optimization horizon features described herein. Their evaluation, comparison with existing techniques, and demonstration wait on their application to both hypothetical and real world problems.

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APPENDIX II - NOTATION

B = total utility obtained from system operation over N stages;

B_2 = total utility with respect to a second operation objective;

$B^{j,k}$ = objective utility for determining near-optimum decision, $j \le i \le j+k-1$, in k-dimension optimization over stages j through j+k-1, with near-optimum previous decisions;

B^X = objective utility for determining optimum decision, $N-X+1 \le i \le N$, in X-dimension optimization over stages N-X+1 through N, with optimum previous decisions;

$\bar{B}^k$ = objective utility for determining near-optimum decision in k-dimension optimization over stages i through i+k-1, with optimum previous decisions;

$\bar{\bar{B}}^k$ = objective utility for determining near-optimum decision in k-dimension optimization over stages i through i+k-1, with near optimum previous decisions

b = maximum allowable constraint limit for B_2;

$b^\bullet$ = trade-off levels;

b^* = desired trade-off level, subjectively selected as "best";

$c_\bullet$ = selected constants to reduce objective function dimensionality;

d_i = decision vector for stage i of systems operation;

$d_i^{j,k}$ = i^{th} stage decision vector from k-dimension optimization using $B^{j,k}$ over stages j through j+k-1 for $j \le i \le j+k-1$;

d_i^X = i^{th} stage decision vector from X-dimension optimization using B^X over stages N-X+1 through N for $N-X+1 \le i \le N$;

$d_i^X(b)$ = i^{th} stage decision vector from X-dimension optimization using the constrained objective optimization ($B_2 \ge b$) of Eq. 24;

$\bar{d}_i^k$ = i^{th} stage decision vector from k-dimension optimization using $\bar{B}^k$ over stages i through i+k-1;

$\bar{\bar{d}}_i^k$ = i^{th} stage decision vector from k-dimension optimization using $\bar{\bar{B}}^k$ over stages i through i+k-1;

$\bar{\bar{d}}_i^k(b)$ = i^{th} stage decision vector from k-dimension optimization using the constrained objective optimization ($B_2 \ge b$) of Eq. 26;

$g_{i,n}(\cdot)$ = n^{th} constraint equation for operation decision vectors at stage i;

I_i = input vector at stage i, representing hydrology and other system inputs;

N = number of stages in operation horizon;

t_i = trade-off decision for stage i of systems operation;

α = an arbitrarily small interval from the maximum total utility for near-optimum total utilities; and

ε_j = small vector deviation from true optimum decision at stage j for near-optimum decision d_j.

APPENDIX III - DERIVATION OF EQS. 12 AND 14

Note that in general:

$$\max_{d_i,\ldots,d_N} B(d_1^*,\ldots,d_{i-1}^*, d_i,\ldots,d_N) \ge B(d_1^*,\ldots,d_N^*) \;\forall\; (d_1^*,\ldots,d_N^*) \quad \ldots \quad \text{A}$$

Equation A suggests that, for the problems identified with Eq. 11:

$$d_j^N - \varepsilon_j \le d_j^* \le d_j^N + \varepsilon_j \;,\; j=1,\ldots,i-1$$

$$\Longrightarrow \max_{d_i,\ldots,d_N} B(d_1^*,\ldots,d_{i-1}^*, d_i,\ldots,d_N) \ge B(d_1^N,\ldots,d_N^N) - \alpha \quad \ldots\ldots\ldots \quad \text{B}$$

Let $d_i^X(d_1^*,\ldots,d_{N-X}^*)$ = optimum decision vector at the i^{th} stage from an X-dimension optimization over the stages N-X+1 through N using the X-dimension objective function:

$$B(d_1^*,\ldots,d_{N-X}^*,\ d_{N-X+1},\ldots,d_N) \qquad \text{C}$$

Then, from Eqs. 11, B, and C with X=N-i+1:

$$d_j^N - \varepsilon_j \leq d_j^* \leq d_j^N + \varepsilon_j \ , \ j=1,\ldots,i-1$$

$$\Longrightarrow B[d_1^*,\ldots,d_{i-1}^*,\ d_i^{N-i+1}(d_1^*,\ldots,d_{i-1}^*),\ldots,d_N^{N-i+1}(d_1^*,\ldots,d_{i-1}^*)]$$

$$\geq B(d_1^N,\ldots,d_N^N) - \alpha$$

$$\Longrightarrow d_j^N - \varepsilon_j \leq d_j^{N-i+1}(d_1^*,\ldots,d_{i-1}^*) \leq d_j^N + \varepsilon_j \ , \ j=i,\ldots,N \qquad \text{D}$$

In particular:

$$d_j^N - \varepsilon_j \leq d_j^* \leq d_j^N + \varepsilon_j \ , \ j=1,\ldots,i-1$$

$$\Longrightarrow d_i^N - \varepsilon_i \leq d_i^{N-i+1}(d_1^*,\ldots,d_{i-1}^*) \leq d_i^N + \varepsilon_i \qquad \text{E}$$

or,

$$d_j^* \cong d_j^N,\ j=1,\ldots,i-1 \Longrightarrow d_i^{N-i+1}(d_1^*,\ldots,d_{i-1}^*) \cong d_i^N. \qquad \text{F}$$

Applying the approximation of Eq. 9 to the optimization of Eq. C with X=N-i+1:

$$\bar{d}_i^k \cong d_i^{N-i+1}(d_1^*,\ldots,d_{i-1}^*) \text{ given } (d_1,\ldots,d_{i-1}) \equiv (d_1^*,\ldots,d_{i-1}^*) \qquad \text{G}$$

in which $\bar{d}_i^k$ = optimum decision vector at the i^{th} stage using the k-dimension objective maximization:

$$\max_{d_i,\ldots,d_{i+k-1}} B(d_1^*,\ldots,d_{i-1}^*,\ d_i,\ldots,d_{i+k-1},\ c_{i+k},\ldots,c_N) \qquad \text{H}$$

Combining Eqs. F, G, and 7:

$$d_j^* \cong d_j^N,\ j=1,\ldots,i-1 \Longrightarrow \bar{d}_i^k \cong d_i^{N-i+1}(d_1^*,\ldots,d_{i-1}^*) \cong d_i^N = d_i^{N-i+1} \quad . \ . \ . \qquad \text{I}$$

Thus, if $d_1^*,\ldots,d_{i-1}^*$ are approximately optimum, then so is $\bar{d}_i^k$, if the approximation of Eq. G is sufficiently close. It is demonstrated (4, 5, 7) that it gets closer as k increases. Using the $\bar{\bar{d}}_i^k$ definition of Eqs. 12 and 13 for the $\bar{d}_i^k$ of Eqs. G and H and reapplying Eq. I for i=1,...,N:

$$\bar{\bar{d}}_i^k \cong d_i^{N-i+1}(\bar{\bar{d}}_1^k,\ldots,\bar{\bar{d}}_{i-1}^k) \cong d_i^N = d_i^{N-i+1} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \qquad \text{J}$$

Equation J is Eq. 12 in the text. This equation and Eq. 8 may be combined to give Eq. 14.

Although the derivations in this appendix strictly apply for an unconstrained maximization (for which Eq. 11 is more often true), they give insight into the more realistic problems of constrained maximization. Indeed, if Eq. 11 applies for the constrained problem (as it often does), then Eqs. 12 and 14 also apply for the constrained problem. If Eq. 11 does not strictly apply for the constrained problem but does apply for the corresponding unconstrained problem, then Eqs. 12 and 14 may still have some application, depending upon the nature of the constraints. It may often be observed, for some constrained objective function maximizations, that earlier decisions which are <u>not</u> approximately optimum may still allow determination of succeeding decisions which give high total utility. In other words, the following relation may apply:

$$d_j \in D_j,\ j = 1,\ldots,N \Longleftrightarrow B(d_1,\ldots,d_N) \geq B(d_1^N,\ldots,d_N^N) - \alpha \quad . \ . \ . \ . \ . \ . \qquad \text{K}$$

in which the D_j represent classes of decision vectors at each stage j that still allow total utility B to be arbitrarily near optimum. Now, however, D_j may include decision vectors which are <u>not</u> close to the optimum decision vectors as in Eq. 11. Following a similar presentation as outlined in this appendix, the following equation would result:

$$B(\bar{\bar{d}}_1^k, \bar{\bar{d}}_2^k, \ldots, \bar{\bar{d}}_{N-k+1}^k, \bar{\bar{d}}_{N-k+2}^{k-1}, \ldots, \bar{\bar{d}}_{N-1}^2, \bar{\bar{d}}_N^1) \geq B(d_1^N, \ldots, d_N^N) - \alpha \quad . \quad . \quad . \quad . \qquad L$$

Thus, the near-optimum operation of some existing systems is possible, even for constrained maximizations and non-optimum previous results.