A COMPARISON OF THREE METHODS FOR COMPUTING THE ADDED MASS OF SHIP SECTIONS

by

Matilde Macagno

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Fundamental Hydromechanics Research Program
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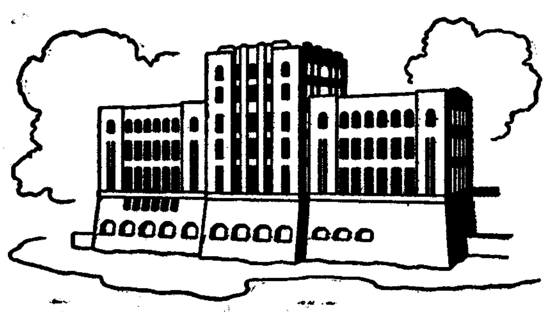

IIHR Report No. 104
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Erratum

A Comparison of Three Methods for Computing the
Added Mass of Ship Sections

by

Matilde Macagno

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The statement of sponsorship should be corrected to read as follows:

Sponsored by
Naval Ship Research and Development Center
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Abstract

Three methods that have been developed for computing added-mass coefficients of two-dimensional forms, oscillating horizontally or vertically at a free surface are compared by application to a set of four cargo-ship sections. Computer programs are listed for two of these methods. The method employing conformal mapping is recommended as the best of the three.

Introduction

In this work, a comparison of three methods previously developed by Landweber and Macagno, [1, 2, 3] for computing added-mass coefficients of two-dimensional forms oscillating at a free surface, will be presented.

In the first of these papers, general expressions for added mass for horizontal and vertical oscillations were derived, but were applied only to a two-parameter family, the so-called Lewis forms. In order to extend the family of Lewis forms, a three-parameter family was treated in reference [2]. In practice the two-parameter method has been found more convenient and has been more generally used. Since both of these methods are indirect and approximate, a procedure employing conformal mapping, which yields the actual added-mass coefficients of a given section, was developed and reported in reference [3].

In order to encourage the wider use of the three-parameter method, which is intrinsically more accurate than the two-parameter, and to clarify the application of the conformal mapping procedure, it seemed necessary to prepare complete computing programs for the latter two methods.

The purpose of this work is then two-fold; first to present these computer programs and second, by applying them to a family of ship sections, to determine how well the results by the three methods agree, and which to recommend for future use.

Methods of Determining Added Masses

(a) The two-parameter technique

In the first paper [1], the general theory was applied to obtain expressions for the added-mass coefficients of arbitrary two-dimensional forms, on the assumption that the conformal transformation of the exterior of the closed form into the exterior of the unit circle in the ζ-plane is known and given by the expression

$$z = \zeta + \frac{a_1}{\zeta} + \frac{a_3}{\zeta^3} + \ldots \qquad a_1, a_3, \ldots . \text{ real} \tag{1}$$

where

$$z = x + iy \quad , \quad \zeta = \xi + i\eta$$

A particular family of ship-like sections, the Lewis forms, were then derived from the unit circle by this transformation for the case where a_1 and a_3 are the only nonzero coefficients. In parametric form, the equations of these forms are

$$\left.\begin{aligned} x &= (1 + a_1) \cos\theta + a_3 \cos 3\theta \\ y &= (1 - a_1) \cos\theta - a_3 \sin 3\theta \end{aligned}\right\} \tag{2}$$

If b denotes the half-beam of the section at the water line, and H the draft at the keel line, the coefficients a_1 and a_3 are related to b and H by the expressions

$$b = 1 + a_1 + a_3 \qquad H = 1 - a_1 + a_3$$

or, solving for a_1 and a_3,

$$a_1 = b\,\frac{1-\lambda}{2} \qquad a_3 = b\,\frac{1+\lambda}{2} \tag{3}$$

where $\lambda = \frac{H}{b}$.

If H and b are given, the sectional area of the form is obtained from

$$S = \frac{\pi}{2}[-2 + 3b(1 + \lambda) - b^2(1 + \lambda + \lambda^2)]$$

and the section-area coefficient, defined by

$$\sigma = \frac{S}{2bH} = \frac{S}{2\lambda b^2}$$

is given in terms of a parameter $\alpha = 2/b$, by

$$\sigma = \frac{\pi}{8\lambda}[-\alpha^2 + 3\alpha(1 + \lambda) - 2(1 + \lambda + \lambda^2)]$$

If the shape characteristics λ and σ are given, the corresponding values of b, H, a_1, a_3 and α can be obtained, but not all values of α give possible useful forms; i.e., α must lie within a certain range. Once α is found, the coefficients for vertical vibrations, C_V, and for horizontal vibrations C_H are given by

$$\left.\begin{aligned} C_V &= 1 + (1 + \lambda - \alpha)(\lambda - \alpha) \\ C_H &= \frac{4}{\pi^2}[1 + \frac{4}{3\lambda^2}(1 + \lambda - \alpha)^2] \end{aligned}\right\} \quad (4)$$

A series of Lewis forms and corresponding curves for the added-mass coefficients are given in this paper. Therefore, given λ and σ for any ship-section that resembles a Lewis form, the values of C_V and C_H can be computed from the formulas (4) to an accuracy that depends on how closely the Lewis form represents the given ship section.

(b) The three-parameter technique

As was shown by Prohaska [4], the two parameters draft-beam ratio λ and section-area coefficient σ are insufficient to define an added mass and since Lewis forms cannot represent ship sections with area coefficients close to unity, a new method was developed [2]. In this paper, a more general three-parameter family of forms was derived, for which the added-mass coefficients were determined and presented as a series of curves. This new family of forms is a natural extension of the Lewis forms; to the two parameters that were previously used, a third parameter η was added, the ratio of the radius of gyration about the transverse axis in the free surface to the draft.

In this case, the family of forms was defined from the unit circle in the ζ-plane by the transformation

$$z = \zeta + \frac{a_1}{\zeta} + \frac{a_3}{\zeta^3} + \frac{a_5}{\zeta^5} \qquad a_1, a_3, a_5 \quad \text{real}$$

The equations of the form are now

$$\left.\begin{aligned} x &= (1 + a_1) \cos \theta + a_3 \cos 3\theta + a_5 \cos 5\theta \\ y &= (1 - a_1) \sin \theta - a_3 \sin 3\theta - a_5 \sin 5\theta \end{aligned}\right\} \tag{5}$$

If, as before, b denotes the half beam of the form and H the draft at the keel line, from equations (5) we obtain

$$\left.\begin{aligned} b &= 1 + a_1 + a_3 + a_5 \\ H &= 1 - a_1 + a_3 + a_5 \end{aligned}\right\} \tag{6}$$

The parameters, in terms of b, H and S, are given by

$$\alpha = \frac{2}{b} \quad , \quad \lambda = \frac{H}{b} \quad , \quad \sigma = \frac{S}{2bH} \quad , \quad \eta = \frac{I}{bH^3} \tag{7}$$

where I is the moment of inertia about the x-axis, given by

$$I = 2 \int_0^H xy^2 dy = - \frac{2}{3} \int_0^b y^3 \, dx \tag{8}$$

in which the second form is obtained from the first by integration by parts.

A quadratic expression relates the parameters α, λ, σ and a_5,

$$\sigma = \frac{\pi}{8\lambda} \{- \alpha^2(1 + 3a_5^2) + \alpha[3(1 + \lambda) + a_5(1 - \lambda)] - 2(1 + \lambda + \lambda^2)\} \tag{9}$$

Solving for a_1 and a_3 from equations (6), we obtain

$$a_1 = \frac{1-\lambda}{\alpha} - a_5$$
$$a_3 = \frac{1+\lambda}{\alpha} - 1 \tag{10}$$

and from (7), (8) and (10) we obtain for η

$$\eta = \frac{\pi\alpha^4}{128\lambda^3}\{2[\beta^3 + \beta^2 a_3 + 2\beta(a_3^2 - a_3a_5 + a_5^2) - a_3^2a_5] - $$
$$- \beta^4 - 2\beta^3a_3 - 2\beta^2(4a_3^2 - 5a_3a_5 + 6a_5^2) +$$
$$+ 12\beta a_3^2a_5 - (3a_3^2 + a_5^2)(a_3^2 + 5a_5^2)\} \tag{11}$$

where $\beta = 1 - a_1$.

When the parameters λ, σ and a_5 are given, the corresponding values of b, H, a_1, a_3 and η can be computed from (9), (10) and (6). Finally, expressions for C_v and C_H, in terms of those parameters are given by

$$\left.\begin{aligned} C_v &= \frac{1}{2}\{3\alpha^2a_5^2 - \alpha(\alpha - \lambda + 1)a_5 + 2[(\lambda - \alpha)^2 + \lambda - \alpha + 1]\} \\ C_H &= \frac{4}{\pi^2}\{[1 + \frac{4}{3\lambda^2}(1 + \lambda - \alpha)^2] + \frac{\alpha a_5}{45\lambda^2}[5 + 40\lambda + \alpha(40 + 156a_5)]\} \end{aligned}\right\} \tag{12}$$

To use this technique for computing C_v and C_H, the following procedure may be applied:

1) Compute I from (8). If the ship form is given in polar coordinates (r, ϕ), where $x = r(\phi)\cos\phi$, $y = r(\phi)\sin\phi$ then, since $dy = (r\cos\phi + \frac{dr}{d\phi}\sin\phi)d\phi$, (8) yields, after simplification,

$$I = \frac{1}{2}\int_0^{\pi/2} r^4 \sin^2\phi \, d\phi$$

Then obtain η from

$$\eta = \frac{I}{bH^3}$$

2) Assume a set of values of a_5 in the range permitted by the condition given by [2] and compute corresponding values of α from (9) and of a_1 and a_3 from (10).

3) Obtain values of η from (11) for each set of values of a_1, a_3, a_5, α and λ.

4) By interpolation among the values of η from step 3, obtain the value of a_5 corresponding to the actual value of η for the ship form, obtained in step 1.

5) Apply the a_5 computed in step 4 to obtain the corresponding values of α, a_1, a_3 from (9) and (10), and finally C_v and C_H from (12).

A Fortran program following these steps is listed in the appendix. In this program, the input data are the weighing factors for Simpson's rule, A, the polar radii r, the draft-beam ratio, λ, the section-area coefficient σ and the assumed value of a_5. An IBM7044 takes forty seconds for compilation and loading this program and less than a second for computing C_v and C_H for each form.

(c) The conformal-mapping technique

In the first two methods, it is assumed that a ship-section having the same principal geometric characteristics as a member of one of the particular families (the Lewis forms or the more general three-parameters forms) will have the same added-mass coefficients as that member. But, as this assumption is not exact, a third method was derived [3]. In this method, the coefficients of the mapping function (1) are obtained directly by conformal mapping of the given section in the z-plane into a unit circle in the ζ-plane. With the availability of high-speed computers it is now possible to perform conformal transformations for functions with a rather large number of coefficients.

Following a method proposed by Bieberbach for conformal mapping of the interior of regions of arbitrary form, corresponding results were derived by Landweber and Macagno [3] for exterior regions. To improve the accuracy, the conformal transformation is applied twice, because it was found that a single direct application of the Bieberbach method gave results for the added masses that were not sufficiently accurate.

The expressions obtained for these added-mass coefficients are as follows:

$$C_v = \frac{1}{b^2}\left[2(1 + a_1) - \frac{S}{\pi}\right] \tag{13}$$

$$C_H = \frac{16}{\pi^2 H^2}\left[\alpha_{11}(a_1 - 1)^2 + 2\alpha_{12}a_3 + 2\alpha_{13}(a_1 - 1)a_5 + \ldots \right.$$
$$\left. + \zeta_{22}{a_3}^2 + 2\alpha_{23}a_3a_5 + \ldots \right]$$

or, with $a_1' = a_1 - 1, \quad a_m' = a_m, \quad m = 2, 3, 4, \ldots$

$$C_H = \frac{16}{\pi^2 H^2}\sum\sum \alpha_{rs}\, a_r'\, a_s' \tag{14}$$

where α_{rs} is defined by the finite series

$$\alpha_{rs} = -\frac{(2r - 1)(2s - 1)}{2(s - r)(2s + 2r - 1)}\left[\frac{1}{2(2r - 1)} + \frac{1}{2r + 1} + \frac{1}{2r + 3} + \ldots \right.$$
$$\left. \ldots + \frac{1}{2s - 3} + \frac{1}{2(2s - 1)}\right], \quad r < s$$

As it is seen from (13) and (14), only one coefficient, a_1, is required for calculating C_v, but all the coefficients of the mapping function are required for the calculation of C_H.

A Fortran program written for this method requires as input data the weighing factors for Simpson's rule, A, and the polar radii of a ship section. In the 7044 computer it takes fifty-four seconds for compilation and loading the program, and ten seconds of execution time to give the coefficients C_v and C_H for each form.

Application to Selected Ship Sections

In order to compare the three methods, four transverse sections of a cargo ship were used, applying data furnished by the David Taylor Model Basin. Their characteristics are given in Table 1, and the polar radii in Table 2.

TABLE 1

Geometric Properties and Characteristics
of Four Sections of a Cargo Ship

Frame No.	Half-Beam	Draft	S	λ	σ	η
61	31.12	19.27	938.52	.6192	.7825	.40896
91	39.00	19.99	1430.22	.5126	.9171	.55586
170	35.64	21.89	1111.41	.6143	.7121	.34245
203	11.30	22.69	290.09	2.0080	.5657	.24517

Fortran programs written for the IBM 7044 for the three-parameters and conformal-mapping methods were applied. The values obtained for S, λ, σ and η are also listed in Table 1. Values of the coefficients of the transformations for the first two methods a_1, a_3 and a_1, a_3, a_5 respectively are listed in Table 3.

Introducing these coefficients in the equations of the forms, given by (2) and (5) respectively, yields the approximating ship sections. The given ship sections as well as the approximations are graphed in Figs. 1a, b, c, d.

In Table 4 the values of C_v and C_H computed by the three methods are listed.

Discussion and Conclusions

The conformal-mapping method has the advantage of making use of finding directly the coefficients of the conformal transformation of an arbitrary ship form. In the other methods it is assumed that the ship form is obtained by a transformation with two or three fixed coefficients.

Evaluation of the results presented in Table 4 on the basis that the conformal-mapping method gives the most nearly exact values (as has been shown for sections where exact values are known, see Table 3, ref. [3]), shows that for C_v, the three-parameter is preferable to the two-parameter method. For C_H, however, one method is not consistently superior to the other, and the deviations of either from the conformal mapping values may be quite large.

Consequently, it is recommended that the conformal-mapping procedure be used for obtaining the added-mass coefficients of ship sections.

Acknowledgments

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[2] L. Landweber and M. Macagno, "Added Mass of a Three-Parameter Family of Two-Dimensional Forms Oscillating in a Free Surface," Journal of Ship Research, Vol. 2, No. 4, 1959.

[3] L. Landweber and M. Macagno, "Added Masses of Two-Dimensional Forms by Conformal Mapping," to be published in the Journal of Ship Research.

[4] C. W. Prohaska, "Vibrations Verticales du Navire," Bulletin de l'Association Technique Maritime et Aeronautique, 1947, p. 171.

TABLE 2

Radii of the Ship-Sections for $0 < \phi < 90°$

Frame 61

Deg.	Radius ft	Deg.	Radius ft
0	31.12	46	23.50
1	30.92	47	23.38
2	30.76	48	23.18
3	30.60	49	23.04
4	30.38	50	22.88
5	30.24	51	22.72
6	30.06	52	22.58
7	29.90	53	22.42
8	29.78	54	22.26
9	29.62	55	22.12
10	29.48	56	22.00
11	29.30	57	21.84
12	29.14	58	21.70
13	29.02	59	21.56
14	28.88	60	21.44
15	28.70	61	21.30
16	28.54	62	21.20
17	28.38	63	21.08
18	28.24	64	20.96
19	28.10	65	20.82
20	27.94	66	20.74
21	27.76	67	20.62
22	27.58	68	20.54
23	27.42	69	20.42
24	27.28	70	20.32
25	27.10	71	20.22
26	26.94	72	20.14
27	26.78	73	20.06
28	26.60	74	20.00
29	26.42	75	19.92
30	26.24	76	19.84
31	26.08	77	19.78
32	25.90	78	19.72
33	25.70	79	19.66
34	25.54	80	19.62
35	25.36	81	19.54
36	25.16	82	19.50
37	25.02	83	19.44
38	24.84	84	19.40
39	24.68	85	19.36
40	24.52	86	19.34
41	24.34	87	19.32
42	24.16	88	19.30
43	24.00	89	19.28
44	23.86	90	19.27
45	23.68		

TABLE 2 Continued

Frame 91

Deg.	Radius ft	Deg.	Radius ft
0	39.00	46	27.44
1	38.98	47	27.16
2	38.98	48	26.76
3	38.96	49	26.40
4	38.96	50	25.98
5	38.94	51	25.64
6	38.94	52	25.24
7	38.92	53	24.96
8	38.92	54	24.62
9	38.90	55	24.32
10	38.90	56	24.02
11	38.86	57	23.76
12	38.82	58	23.50
13	38.74	59	23.24
14	38.68	60	23.04
15	38.60	61	22.80
16	38.54	62	22.60
17	38.42	63	22.40
18	38.32	64	22.22
19	38.20	65	22.04
20	38.04	66	21.88
21	37.90	67	21.72
22	37.72	68	21.56
23	37.54	69	21.40
24	37.32	70	21.28
25	37.06	71	21.14
26	36.82	72	21.02
27	36.52	73	20.90
28	36.22	74	20.82
29	35.90	75	20.72
30	35.56	76	20.62
31	35.12	77	20.54
32	34.74	78	20.46
33	34.28	79	20.40
34	33.78	80	20.34
35	33.36	81	20.30
36	32.78	82	20.22
37	32.36	83	20.18
38	31.78	84	20.14
39	31.24	85	20.10
40	30.76	86	20.08
41	30.20	87	20.04
42	29.62	88	20.02
43	29.10	89	20.00
44	28.56	90	19.99
45	28.10		

TABLE 2 Continued

Frame 170

Deg.	Radius ft	Deg.	Radius ft
0	35.64	46	25.04
1	35.32	47	24.90
2	34.98	48	24.80
3	34.64	49	24.64
4	34.32	50	24.56
5	34.00	51	24.40
6	33.68	52	24.30
7	33.36	53	24.16
8	33.04	54	24.04
9	32.72	55	23.90
10	32.40	56	23.80
11	32.14	57	23.70
12	31.86	58	23.56
13	31.60	59	23.48
14	31.32	60	23.38
15	31.06	61	23.30
16	30.78	62	23.22
17	30.50	63	23.14
18	30.28	64	23.04
19	30.02	65	22.96
20	29.76	66	22.90
21	29.54	67	22.80
22	29.28	68	22.74
23	29.08	69	22.68
24	28.86	70	22.62
25	28.64	71	22.54
26	28.42	72	22.50
27	28.24	73	22.44
28	28.02	74	22.40
29	27.82	75	22.34
30	27.64	76	22.28
31	27.42	77	22.24
32	27.26	78	22.18
33	27.10	79	22.16
34	26.88	80	22.12
35	26.70	81	22.08
36	26.52	82	22.08
37	26.40	83	22.04
38	26.20	84	22.00
39	26.08	85	21.96
40	25.96	86	21.94
41	25.76	87	21.93
42	25.62	88	21.92
43	25.46	89	21.91
44	25.32	90	21.90
45	25.18		

TABLE 2 Continued

Frame 203

Deg.	Radius ft	Deg.	Radius ft
0	11.30	46	10.22
1	11.18	47	10.34
2	11.00	48	10.44
3	10.86	49	10.56
4	10.72	50	10.68
5	10.58	51	10.82
6	10.46	52	10.98
7	10.34	53	11.10
8	10.24	54	11.28
9	10.12	55	11.42
10	10.00	56	11.60
11	9.94	57	11.80
12	9.86	58	12.04
13	9.80	59	12.28
14	9.72	60	12.50
15	9.68	61	12.76
16	9.62	62	13.04
17	9.58	63	13.36
18	9.54	64	13.68
19	9.50	65	14.00
20	9.48	66	14.34
21	9.42	67	14.72
22	9.40	68	15.16
23	9.38	69	15.64
24	9.34	70	16.08
25	9.32	71	16.54
26	9.32	72	17.02
27	9.32	73	17.52
28	9.32	74	18.00
29	9.32	75	18.50
30	9.32	76	19.04
31	9.32	77	19.60
32	9.36	78	20.08
33	9.40	79	20.58
34	9.42	80	20.96
35	9.44	81	21.36
36	9.52	82	21.74
37	9.54	83	22.06
38	9.60	84	22.36
39	9.66	85	22.60
40	9.72	86	22.76
41	9.76	87	22.72
42	9.82	88	22.70
43	9.94	89	22.70
44	10.02	90	22.69
45	10.12		

TABLE 3

Coefficients of the Two and Three Parameter Sections

Frame	Two Parameter Form		Three Parameter Form		
	a_1	a_3	a_1	a_3	a_5
61	0.235739	0.001735	0.214840	0.005425	0.021767
91	0.297847	-0.075660	0.292302	-0.073591	0.006212
170	0.249422	0.044059	0.228974	0.047698	0.021318
203	-0.377228	0.125725	-0.462398	0.059641	0.107315

TABLE 4

Added-Mass Coefficients of four Sections of a Cargo Ship

Method	Frame 61		Frame 91		Frame 170		Frame 203	
	C_v	C_H	C_v	C_H	C_v	C_H	C_v	C_H
Two-Parameter	0.997	0.405	1.139	0.434	0.937	0.413	0.777	0.409
Three-Parameter	0.955	0.487	1.124	0.466	0.902	0.489	0.720	0.577
Conformal-Mapping	0.951	0.439	1.126	0.453	0.883	0.442	0.751	0.611

```
      IF(DIF.LT.0.0) GO TO 16
   15 CONTINUE
   16 DIF = ETA(J) - W
      AA5 = B(J) - DIF*(B(J)-B(J-1))/(ETA(J) - ETA(J-1))
      T1 = 1.0 + 3.0 * AA5 * AA5
      T2 = 3.0 * TE + AA5 * TI
      T3 = 2.0*(1.0+CL*TE ) + 8.0 * CL * SIGMA / P
      RAD = SQRT(T2*T2 - 4.0 * T1 * T3)
      AL = (T2 - RAD) / (2.0 * T1)
      AA1 = (TI/AL) - AA5
      AA3 = (TE/AL) - 1.0
      Z = 3.0 * (AL * AA5)**2
      Z1 = AL*(AL - CL + 1.)*AA5
      ZZ = CL - AL + 1.0
      Z2 = 2.0*(ZZ + (ZZ-1.0)**2)
      CV = 0.5 * (Z - Z1 + Z2)
      ONE = 1.0+ 4.*ZZ * ZZ/(3. * CL * CL)
      TWO =(5. + 40. * CL + AL*(40.+156.*AA5))*AL*AA5/(45.*CL*CL)
      CH = 4.0 * (ONE + TWO) / (P*P)
      WRITE(6,17)AA1,AA3,AA5,CV,CH
   17 FORMAT(34H  VALUES OF A1,A3,A5,CV AND CH ARE/(5F20.8))
      CALL EXIT
      END
```

```
C     ADDED MASS FOR TWO DIMENSIONAL FORMS BY CONFORMAL MAPPING METHOD
      DIMENSION A(16),B(16),CI(16),CE(8),D(8),DI(16),F(8),G(8),AA(16),
     1R(8),ANG(91),R2(91),RO(91),RS(91),XI(91),ETA(91),Z(91),DZ(91),DRO
     2(91),TA(91),TE(91),BI(8,9),C(16,16),T(91,16),BB(16,16),CC(16,16),
     3GG(16,16),GA(16,16),P(16),Q(16),T1(91,16),AU(91)
      CALL TRAPS (-1,-1)
      PP = 3.14159265
      DANG = 0.0174533
      READ(5,1)((C(I,J),J=1,8),I = 1,16)
    1 FORMAT(8F10.6)
      READ(5,1)((C(I,J),J=9,16),I=1,16)
      READ(5,2) ((GA(I,J),J = 1,8),I=1,16)
    2 FORMAT(8F10.0)
      READ(5,2 )((GA(I,J),J =9,16),I=1,16)
C     COMPUTATION  OF COEFF.  B
      READ(5,149) AU
  149 FORMAT(40 F2.1)
  100 READ(5,4) RS
    4 FORMAT(7F11.7)
      BE = RS(1)
      HE = RS(91)
      H3 = DANG / 3.0
      SO = 0.0
      DO 77 I = 1,91
      RS(I) = RS(I) * RS(I)
   77 SO = SO + RS(I) * AU(I)* H3
      WRITE(6,150) BE , HE , SO
  150 FORMAT(41H VALUES OF BEAM,DRAFT AND AREA OF SECTION/(3F20.8))
      Z(1) = 0.0
      DO 7 I = 1,90
    7 Z(I+1) = Z(I) + DANG
      DO 8 I = 1,91
    8 RS(I) = RS(I) * PP / SO
      ES = SQRT(PP/ SO)
      BE = BE * ES
      HE = HE * ES
      SO = PP
    9 DO 10 J = 1,8
      AJ=J
      E(J)=0.
      DO 20 I=1,91
      IF((J-1).NE.0) GO TO 11
      TE(I) =-ALOG(SQRT(RS(I))) * COS(2.0 * Z(I))
      GO TO 20
   11 TE(I) =((2.*AJ-1.)/(2.*AJ-2.))*COS(2.*AJ*Z(I))/RS(I)**(J-1)
   20 CONTINUE
      DO 12 L = 1,90
   12 E(J) = E(J) + 2.*(TE(L)+TE(L+1))* DANG
      DO 14 K = 1,8
      AK = K
      TT= (2.*AJ-1.)*(2.*AK-1.)/(2.*(AJ+AK-1.))
      BI(J,K) = 0.0
```

```
      DO 13 I = 1,90
   13 TA(I) = TT*COS(2.*(AJ-AK)*Z(I))/RS(I)**(J+K-1)
      TA(91) = TT* COS(PP*(AJ-AK))/RS(91)**(J+K-1)
      DO 14 L = 1,90
      BI(J,K) = BI(J,K)+2.*(TA(L)+TA(L+1))*DANG
   14 CONTINUE
   10 CONTINUE
      NEWSH=0
      DO 15 J = 1,8
   15 BI(J,9) = E(J)
      DO 18 K = 2,9
      DA = BI(K-1,K-1)
      IF(DA.NE.0.0) GO TO 16
      NEWSH=1
      GO TO 999
   16 DO 17 I = K,9
   17 BI(K-1,I) = BI(K-1,I) / DA
      DO 18  J = 1,8
      IF((K-1).EQ.J) GO TO 18
      F = BI(J,K-1)
      DO 19 I = K,9
   19 BI(J,I) = BI(J,I) - F * BI(K-1,I)
   18 CONTINUE
      DO 28 I=1,8
   28 B(I) = BI(I,9)
      DO 47 I = 1,91
      R(1) = SQRT(RS(I))
      CE(1) =-B(1) / R(1)
      D(1) = COS(Z(I))
      G(1) = SIN(Z(I))
      DO 45 J = 2,8
      R(J) = R(J - 1) * RS(I)
      CE(J) =-B(J) / R(J)
      AP = 2 * J - 1
      D(J) = COS(AP * Z(I))
   45 G(J) = SIN(AP * Z(I))
      XI(I) = R(1) * D(1)
      ETA(I) = R(1) * G(1)
      DO 46 K = 1,8
      XI(I) = XI(I) - CE(K) * D(K)
   46 ETA(I) = ETA(I) + CE(K) * G(K)
      RO(I) = SQRT(XI(I)*XI(I) + ETA(I) * ETA(I))
   47 Z(I) = ATAN2 (ETA(I),XI(I))
      S1 = SO
      DO 48 J = 1,8
   48 S1 = S1 + B(J) * E(J)
      ROC12 = S1/PP
      RM = 0.0
      W = 0.63661977
      DO 49 I = 1,90
      DZ(I) = Z(I+1) - Z(I)
   49 RM = RM+DZ(I) * (RO(I) + RO(I+1)) * 0.5
      RM = RM * W
```

```
      DO 21 I = 1,8
   21 B(I) = B(I) / (RM**( 2 * I ))
      RM2 = RM * RM
      S1 = S1 / RM2
      DO 50 I = 1,90
      DRO(I) = (RO(I) - RM)/RM
      DO 50 J = 1,16
      A(J) = 0.0
      Q(J) = 0.0
      P(J) = 0.0
      H = J * 2
      T(I,J) = DRO(I) * COS(H * Z(I))
   50 T1(I,J) = T(I,J) * DRO(I)
      DO 51 J = 1,16
      DO 51 I = 1,90
      Q(J) = Q(J) + DZ(I) * W * (T1(I,J) + T1(I+1,J))
   51 P(J) = P(J) + DZ(I) * W * (T(I,J) + T(I+1,J))
C      COMPUTATION OF C(J)
      DO 53 J = 1,16
      SA = 0.0
      DO 54 K = 1,16
      IF(J.EQ.K) GO TO 54
      J1 = ABS(J - K)
      AL = 2*K - 1
      SA = SA + P(K) * P(J1) * AL
   54 CONTINUE
   52 DO 53 K = 1,16
      AJ =(J * 2 - 1 )/ 2
   53 CI(J) = -(P(J) + SA - Q(J) * AJ)
      S2 = 0.0
      DO 60 J = 1,16
      AJ = 2 * J - 1
   60 S2 = S2 + AJ * P(J) * P(J)
      S2 = S1 + S2 * PP * RM * RM
      ROC22 = S2 / PP
      DO 62 I = 1,16
      DO 62 J = 1,16
      BB(I,J) = 0.0
   62 GG(I,J) = 0.0
      DO 73 N = 1,16
      BB(1,N) = B(N)
      A(N) = -B(N)
   73 D1(N) = - CI(N)
C      COMPUTATION OF B(K,N)
      DO 63 K = 2,16
      DO 63 N = 1,16
      DO 63 M = 1,N
      L = N+1-M
   63 BB(K,N) = BB(K,N) + B(M) * BB(K-1,L)
C     COMPUTATION OF G(J,M)
      DO 64 M = 2,16
      M1 = M - 1
      DO 64 J = 1,M1
      L = M - J
      DO 64 K = 1,L
```

```
      N = L + 1 - K
   64 GG(J,M) = GG(J,M) + GA(J,K) * BB(K,N)
C      COMPUTATION OF A(I)  FOR FIRST MAPPING
      DO 65 M = 2,16
      M1 = M - 1
      DO 65 J = 1,M1
   65 A(M) = A(M) - A(J) * GG(J,M)
      DO 199  I = 1,16
      DO 199  J = 1,16
  199 GG(I,J) = 0.0
C     COMPUTATION OF G'(J,M)
      GG(1,2) = -CI(1)
      DO 67 M = 3,16
      N1 = M - 1
      EA = 2 * N1 - 1
      GG(N1,M) = - EA * CI(1)
      N2 = M - 2
      DO 67 J = 1,N2
      AJ = J
      FA = 2 * J - 1
      SE = 0.0
      L = M - J
      L1 = L - 1
      DO 66 K = 1,L1
      K1 = L - K
   66 SE = SE +CI(K) * CI(K1)
   67 GG(J,M) = FA *(-CI(L) + AJ * SE)
C      COMPUTATION OF D(I)
      DO 68 M = 2,16
      M1 = M - 1
      DO 68 J = 1,M1
   68 D1(M) = D1(M) - D1(J) * GG(J,M)
C      COMPUTATION OF CORRECTED A
      DO 69 M = 1,16
   69 AA(M) = A(M) + D1(M)
      DO 70 M = 2,16
      M1 = M-1
      DO 70 J = 1,M1
   70 AA(M) = AA(M) - (AA(J) - D1(J)) * GG(J,M)
      AH = 0.0
      CV = (2.0 * RM2*(ROC22 + AA(1)) - 1.0) / BE**2
      AA(1) = AA(1) - ROC22
      DO 71 I = 1,16
      DO 71 J = 1,16
   71 AH = AH+ C(I,J)*AA(I) * AA(J) / ROC22 **(I+J-1)
      CH = 16.0 * RM2 * AH / (PP * HE) ** 2
      WRITE(6,227) CH,CV
  227 FORMAT(4H CH= F20.8/4H CV= F20.8)
  999 WRITE(6,228) NEWSH,K
  228 FORMAT(1H ,2I5)
      CALL EXIT
      END
```

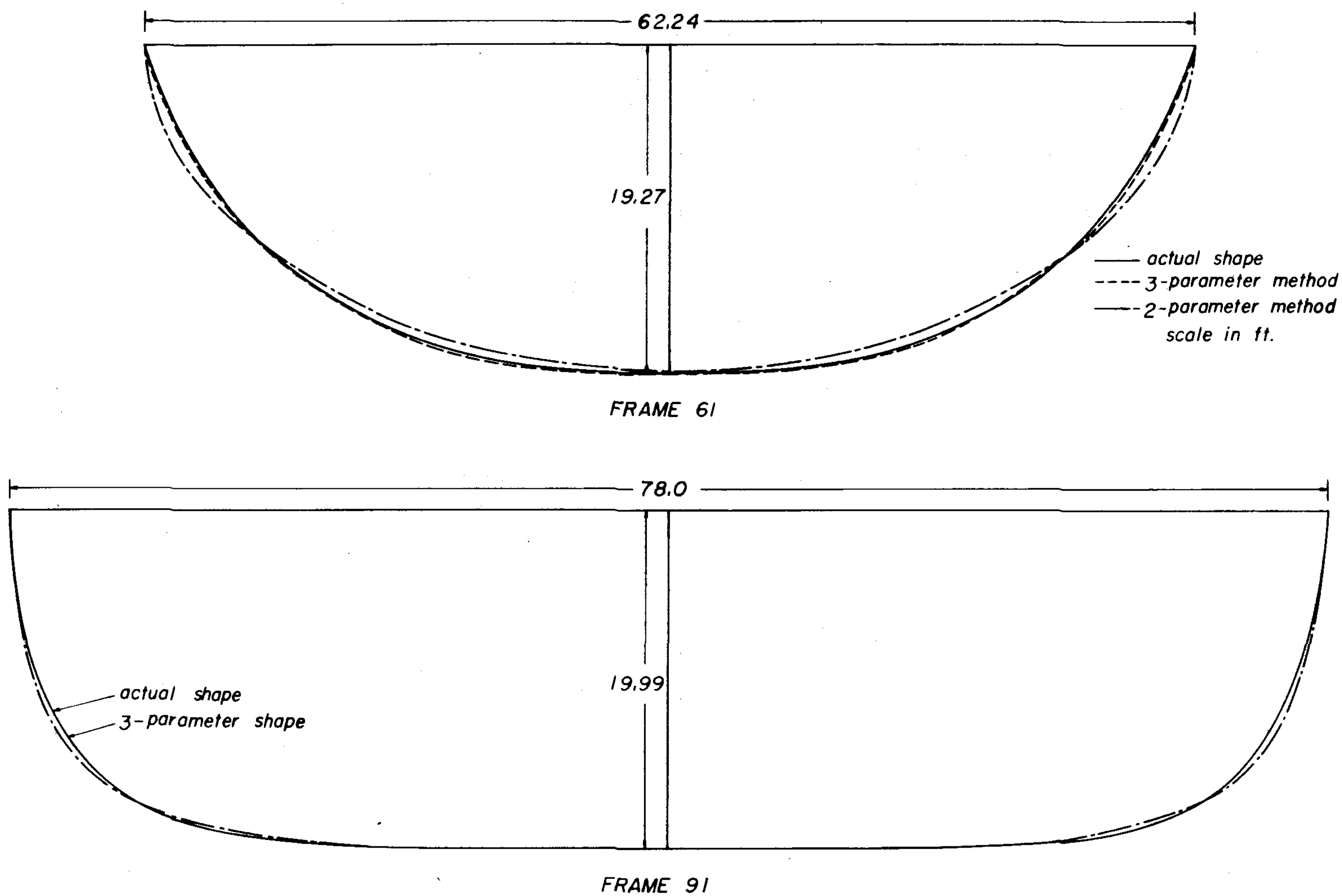

FIG. 1a, b Shapes of actual and approximate ship sections

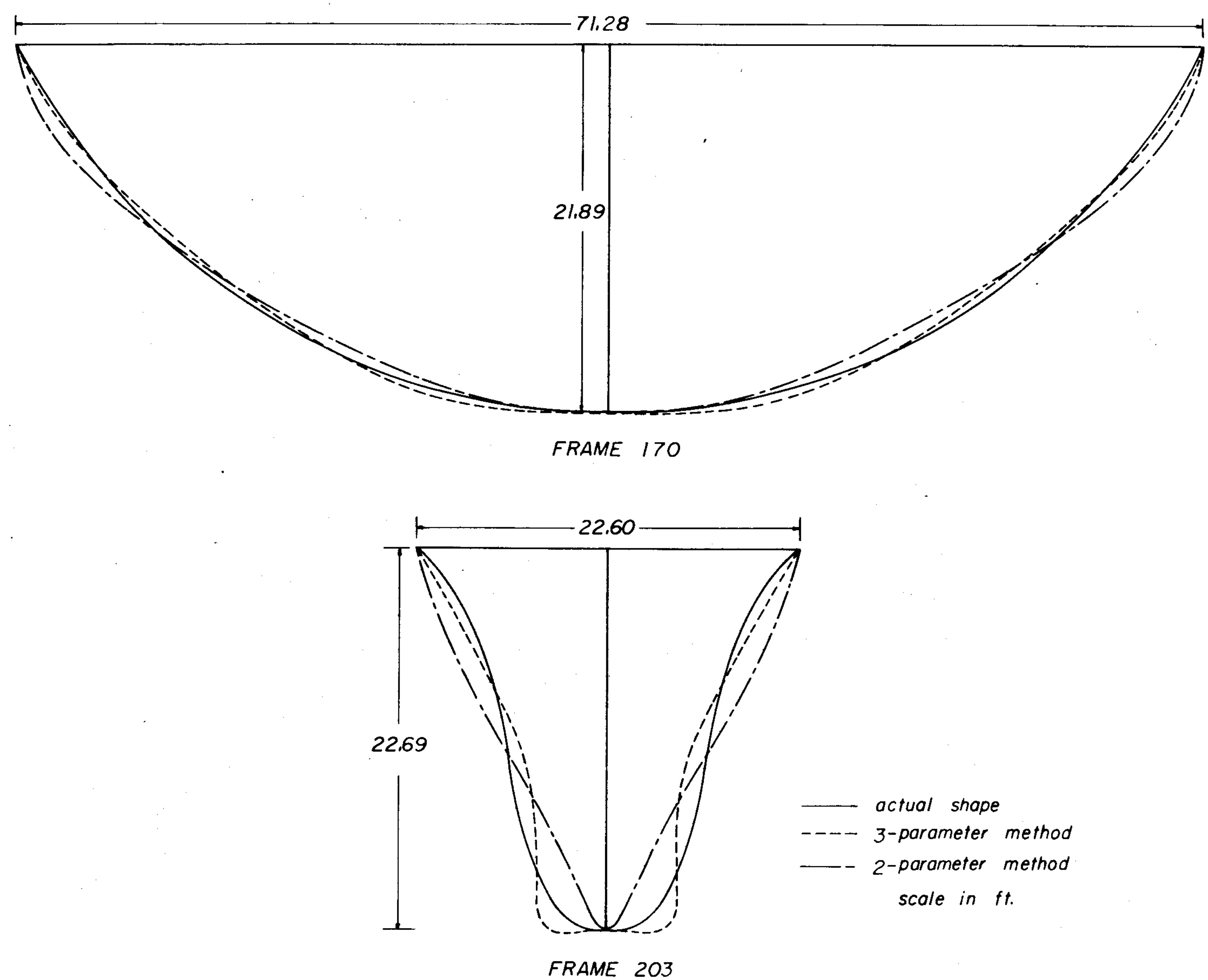

FIG 1c,d Shapes of actual and approximate ship sections

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13. ABSTRACT

Three methods that have been developed for computing added-mass coefficients of two-dimensional forms, oscillating horizontally or vertically at a free surface are compared by application to a set of four cargo-ship sections. Computer programs are listed for two of these methods. The method employing conformal mapping is recommended as the best of the three.

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