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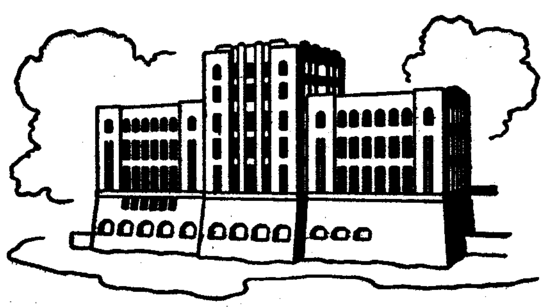

IIHR Report No. 103
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Introduction

In his 1962 paper [1], Eggers presented an ingenius procedure for determining the wave resistance of a ship model in a towing tank from measurements of the wave profile. In the theoretical development of the method, it is assumed that the effect of the wake, and of the contribution of the "near-field" velocity potential to the surface disturbance may be neglected.

A study of the latter assumption for the case of a single source underneath a free surface has been reported [2]. More recently Kobus [3] investigeted the accuracy of the method by applying it to a modified ogival cylinder, and concluded that the error due to the wake is large, but that due to the near-field surface disturbance is small. Because of the computing costs, Kobus's study was restricted to only a few surface profiles. In the present work, by selecting an analytical, vertically constant distribution of sources over rectangular centerplanes, representing approximately a family of vertical struts extending to various depths, it has been possible to extend Kobus's study to a wider range of cases.

Nature of Bodies

We shall consider the flow about a body generated by a distribution of sources on a vertical plane, normal to the undisturbed free surface and parallel to an oncoming stream of velocity U, in a channel of width b and infinite depth. The source plane will be taken as the x-z plane of a right-handed Cartesian coordinate system with the x-axis in the free surface along the centerline of the channel and the z-axis with its positive sense vertically upward. The channel walls will be taken to be the planes $y = \pm b/2$ and the oncoming stream will be treated as in the positive x-direction.

Our procedure will consist of selecting several source distributions as the basis for this study. At a prescribed Froude number, each distribution corresponds to a body form such that both the boundary condition of zero normal velocity component and the linearized free-surface conditions are satisfied. These body forms vary with Froude number; their approximate shapes may be obtained either at zero Froude number or by a further linearization of the boundary condition on the body. In the present case, however, it is not important to know the exact form of the body. Each source distribution serves as a self-contained example of the accuracy of Eggers' method for obtaining the associated wave resistance.

Surface Disturbance

An expression for the surface disturbance $\zeta(x,y)$ for a source of unit strength at a point $(0, 0, -c)$, is from equation (21) of reference [2],

$$\zeta = \frac{4}{Ub}\sum_{m=-\infty}^{\infty} \cos\frac{2\pi my}{b}\left\{\int_{\frac{2\pi|m|}{b}}^{\infty} \frac{e^{-x\xi}\,[\xi^2 \sin c\alpha_m + k_0\alpha_m \cos c\alpha_m]}{\xi^4+k_0{}^2\alpha_m{}^2}\,\xi d\xi\right.$$

$$\left. + \pi\,\frac{k_0+k_m}{k_m}\, e^{-\frac{c}{2}(k_0+k_m)} \cos\omega_m x\right\},\quad x > 0 \qquad (1)$$

where

$$k_0 = \frac{g}{U^2}\,,\quad k_m = \sqrt{k_0{}^2 + \frac{16\pi^2m^2}{b^2}}\,,$$

$$\omega_m = \sqrt{\tfrac{1}{2}k_0(k_0+k_m)}\,,\quad \alpha_m = \sqrt{\xi^2 - \frac{4\pi^2m^2}{b^2}} \qquad (2)$$

and g is the acceleration of gravity. We shall consider a source distribution over the rectangular area of the centerplane $-\ell \le x \le \ell$, $-h \le z \le 0$, of strength $M(x)$ independent of z.

For the above distribution, with the surface disturbance expressed in the form

$$\zeta = \zeta_n + \zeta_f$$

where ζ_n denotes the near-field part and ζ_f the far-field part, we obtain from (1)

$$\zeta_n = \sum_{m=0}^{\infty} A_m(x, k_0) \cos \frac{2\pi my}{b} \tag{3}$$

where

$$A_0 = \frac{4}{Ub} \int_0^{\infty} \int_{-\ell}^{\ell} \frac{\xi(1-\cos \xi h) + k_0 \sin \xi h}{\xi(\xi^2 + k_0^2)} M(t) e^{(t-x)\xi} dt d\xi$$

$$A_m = \frac{8}{Ub} \int_{\frac{2\pi m}{b}}^{\infty} \int_{-\ell}^{\ell} \frac{\xi^3(1-\cos \alpha_m h) + \xi k_0 \alpha_m \sin \alpha_m h}{\alpha_m(\xi^4 + k_0^2 \alpha_m^2)} M(t) e^{(t-x)\xi} dt d\xi, \tag{4}$$

$m > 0$, and

$$\zeta_f = \sum_{m=0}^{\infty} (C_m \cos \omega_m x + S_m \sin \omega_m x) \cos \frac{2\pi my}{b} \tag{5}$$

where

$$\left.\begin{aligned} C_0 &= \frac{8\pi}{Ubk_0} [1 - e^{-hk_0}] \int_{-\ell}^{\ell} M(t) \cos k_0 t dt \\ C_m &= \frac{16\pi}{Ubk_m} [1 - e^{-\frac{h}{2}(k_0+k_m)}] \int_{-\ell}^{\ell} M(t) \cos \omega_m t dt, \quad m > 0 \end{aligned}\right\} \tag{6}$$

$$\left.\begin{aligned} S_0 &= \frac{8\pi}{Ubk_0} [1 - e^{-hk_0}] \int_{-\ell}^{\ell} M(t) \sin k_0 t dt \\ S_m &= \frac{16\pi}{Ubk_m} [1 - e^{-\frac{h}{2}(k_0+k_m)}] \int_{-\ell}^{\ell} M(t) \sin \omega_m t dt, \quad m > 0 \end{aligned}\right\} \tag{7}$$

Eggers' Wave-Resistance Formula

The wave resistance of the body, obtained by applying the Lagally theorem to the source distribution $M(x)$ extending over $-\ell \leq x \leq \ell$ and $-h \leq z \leq 0$ in the x-z plane, may be expressed in the form

$$R_w = -4\pi\rho \int_{-h}^{0} \int_{-\ell}^{\ell} M(x) \left(\frac{\partial\phi}{\partial x}\right)_{y=0} dxdz \tag{8}$$

where ϕ is the disturbance velocity potential, expressible in terms of the disturbance potential of a unit source; see Wehausen [4], eq. (13.36). By examining the contributions of a pair of source elements one can readily show that the only part of the velocity potential which makes a nonzero contribution to the resistance formula (8) is half of the far-field part of the potential, ϕ_f.

Since, as is readily verified,

$$\omega_m^2 + \left(\frac{2\pi m}{b}\right)^2 = \left(\frac{k_0+k_m}{2}\right)^2$$

applying the free-surface boundary condition,

$$\left(\frac{\partial\phi_f}{\partial x}\right)_{z=0} = -\frac{g}{U}\zeta_f$$

and the relation between the coefficients of x, y and z for Laplace's equation to be satisfied, we immediately obtain from (5)

$$\frac{\partial\phi_f}{\partial x} = -\frac{g}{U}\sum_{m=0}^{\infty} (C_m \cos\omega_m x + S_m \sin\omega_m x) \cos\frac{2\pi m y}{b} \cdot e^{\frac{1}{2}(k_0+k_m)z} \tag{9}$$

Hence (8) becomes

$$R_w = \frac{4\pi\rho g}{U}\sum_{m=0}^{\infty} \frac{1-e^{-\frac{1}{2}h(k_0+k_m)}}{k_0+k_m} \left[C_m \int_{-\ell}^{\ell} M(x) \cos\omega_m x dx + S_m \int_{-\ell}^{\ell} M(x) \sin\omega_m x dx\right]$$

and, using (6) and (7), we obtain Eggers' formula

$$R_w = \tfrac{1}{4}\rho g b \left[E_0^2 + \sum_{m=1}^{\infty} \frac{k_m}{k_0+k_m} E_m^2\right], \quad E_m = \sqrt{C_m^2 + S_m^2} \tag{10}$$

<u>Approximate Determination of E_m from $\zeta(x,y)$</u>

In Eggers' method the coefficients C_m and S_m are obtained from a harmonic analysis of the surface profile $\zeta(x,y)$. By (3) and (5) we have

$$\zeta(x,y) = \sum_{m=0}^{\infty} (A_m + C_m \cos \omega_m x + S_m \sin \omega_m x) \cos \frac{2\pi m y}{b} \tag{11}$$

and hence

$$A_m + C_m \cos \omega_m x + S_m \sin \omega_m x = f_m(x) \tag{12}$$

where

$$f_0(x) = \frac{2}{b} \int_0^{b/2} \zeta(x,y)dy; \; f_m(x) = \frac{4}{b} \int_0^{b/2} \zeta(x,y) \cos \frac{2\pi m y}{b} dy, \; m > 0 \tag{13}$$

This may also be written in the form

$$f_m(x) = A_m(x) + E_m \cos (\omega_m x - \gamma_m) \tag{14}$$

where

$$\tan \gamma_m = \frac{S_m}{C_m}$$

If A_m were negligible, as Eggers assumed, one could determine C_m and S_m by writing equation (12) for two values of x. A preferable procedure, which avoids the difficulty that a pair of such equations comes arbitrarily close to being singular for some values of m, is to solve a larger set of equations (12), using several values of x, by the method of least squares. Since the A_m's, which are the "errors" in the least-square calculation, are not a random set of numbers, the least-square method may not give the "best" solution, even if it avoids mathematical difficulties and, possibly, yields a solution with an acceptably small error.

The magnitude of the error due to A_m can be displayed graphically by writing (12) in the form

$$\left.\begin{aligned} C_m \xi + S_m \eta &= 1 - \frac{A_m}{f_m} \\ \xi = \frac{\cos \omega_m x}{f_m}, &\quad \eta = \frac{\sin \omega_m x}{f_m} \end{aligned}\right\} \tag{15}$$

If A_m/f_m were negligible in comparison with unity, (14) indicates that a graph of η against ξ in rectangular coordinates would yield a straight line at a distance $1/E_m$ from the origin. The actual points, however, as is seen from (15), will lie at distances

$$\frac{1}{E'_m} - \frac{1}{E_m} = - \frac{A_m}{E_m f_m} \tag{16}$$

from the desired line where E'_m, obtained from (14) by neglecting $A_m(x)$, is

$$E'_m = f_m(x) \sec (\omega_m x - \gamma_m)$$

From (14) or (16) we then have

$$E'_m = E_m + A_m(x) \sec (\omega_m x - \gamma_m) \tag{17}$$

Since E_m is constant, A_m varies monotonically, and, by (14), (when $A_m << E_m$), f_m varies nearly sinusoidally, (15) indicates that the points (ξ, η) will fall on opposite sides of the line of $A_m = 0$ in successive half cycles of wave length $2\pi/\omega_m$. Thus, if (ξ, η) points are available for several cycles, it should be possible to draw a mean line (ignoring the large deviations from the line when f_m is nearly zero), and then to determine E_m from its distance from the origin.

Selected Distribution

The importance of the near-field coefficients A_m will be studied by means of a simple example. Let us take the vertically constant center-plane source distribution

$$M(x) = - \frac{\lambda U}{\pi \ell} x \tag{18}$$

where λ is a constant and the distribution extends over the area

$$-\ell \leq x \leq \ell, \quad -h \leq z \leq 0$$

For this distribution, together with a uniform stream of velocity U in the positive x direction, the velocity potential at zero Froude number (i.e. for plane z = 0 a rigid boundary) is given by

$$\phi(x,y,z) = Ux + \int_{-\ell}^{\ell} \int_{-h}^{h} \frac{M(\xi)d\zeta d\xi}{\sqrt{(x-\xi)^2+y^2+(z-\zeta)^2}} \tag{19}$$

Let us determine an approximate equation in the plane of the undisturbed free surface, z = 0, of the vertical strut generated by this distribution. Then, setting z = 0 in (19) and integrating with respect to ζ, we obtain

$$\phi(x,y,o) = Ux - 2 \int_{-\ell}^{\ell} M(\xi) \log \frac{\sqrt{(x-\xi)^2+y^2}}{\sqrt{(x-\xi)^2+y^2+h^2} + h} \tag{20}$$

which satisfies the boundary condition

$$\frac{\partial\phi}{\partial n} = U \frac{\partial x}{\partial n} + 2 \int_{-\ell}^{\ell} M(\xi) \left[\frac{r}{r^2+h^2+h\sqrt{r^2+h^2}} - \frac{1}{r}\right] \frac{\partial r}{\partial n} d\xi = 0 \tag{21}$$

where n denotes distance along the outward normal to the strut and $r = \sqrt{(x-\xi)^2+y^2}$. Here we have

$$\frac{\partial r}{\partial n} = \frac{x-\xi}{r}\frac{\partial x}{\partial n} + \frac{y}{r}\frac{\partial y}{\partial n} = -\frac{x-\xi}{r} \sin\gamma + \frac{y}{r}\cos\gamma \tag{22}$$

where $\tan\gamma = dy/dx$ is the slope of the strut section at z = 0.

By substituting (18) and (22) into (21) and performing the integration with respect to ξ, one could now obtain a first order differential equation in y(x), the ordinates of the strut section. We shall be content, however, with the solution for the approximate case of an elongated strut, for which we may assume that γ is small (except near $x = \pm\ell$), and that $h >> y(x)$.

With the foregoing assumptions we now observe that the quantity within brackets in (21) becomes very large in the neighborhood of $\xi = x$. This indicates that the value of the integral is principally determined by the values of $M(\xi)$ and $\partial r/\partial n$ near $\xi = x$. Hence, replacing $M(\xi)$ by $M(x)$ and retaining only the last term of the expression for $\partial r/\partial n$ in (22), and neglecting y^2 in comparison with h^2, equation (21) yields the approximation

$$M(x) \int_{-\ell}^{\ell} \left[\frac{y}{r^2} - \frac{y}{h^2+(x-\xi)^2+h\sqrt{h^2+(x-\xi)^2}}\right] d\xi \doteq \frac{1}{2} U \tan\gamma \tag{23}$$

or, integrating,

$$M(x) \left\{\arctan\frac{\ell-x}{y(x)} + \arctan\frac{\ell+x}{y(x)} - \frac{y}{h}\left[\frac{\sqrt{h^2+(\ell-x)^2}-h}{\ell-x} - \frac{\sqrt{h^2+(\ell+x)^2}+h}{\ell+x}\right]\right\}$$

$$\doteq \frac{1}{2} U \tan\gamma$$

But to the same degree of approximation we can write

$$\arctan\frac{\ell\pm x}{y(x)} \doteq \frac{\pi}{2} - \frac{y(x)}{\ell\pm x}$$

Hence we obtain

$$M(x) \left\{\pi - \frac{y}{h}\left[\frac{\sqrt{h^2+(\ell-x)^2}}{\ell-x} - \frac{\sqrt{h^2+(\ell+x)^2}}{\ell+x}\right]\right\} \doteq \frac{U}{2}\frac{dy}{dx} \tag{24}$$

a linear, first-order differential equation for $y(x)$.

When $h \gg 2\ell$, the term in square brackets in (24) may be expanded by the binomial theorem to give

$$M(x) \left\{\pi - xy\left[\frac{2}{\ell^2-x^2} - \frac{1}{h^2}\right]\right\} \doteq \frac{U}{2}\frac{dy}{dx}$$

and hence, at points x not close to the ends of the body, neglecting the term in brackets because of the smallness of y/ℓ, we obtain

$$M(x) \doteq \frac{U}{2\pi}\frac{dy}{dx} \tag{25}$$

Introducing (18) into (25) and integrating gives for the equation of the strut section

$$y \doteq -\frac{\lambda}{\ell}(\ell'^2-x^2) \tag{26}$$

taking the ends of the strut to be at $x = \pm\ell'$, where ℓ' is slightly greater than ℓ. It is now seen that the constant λ in (18) is approximately the thickness-length ratio of the strut.

For the distribution (18) we readily find

$$\int_{-\ell}^{\ell} M(t)e^{t\xi}dt = -\frac{\lambda U}{\pi\ell\xi^2}\left[(\ell\xi-1)e^{\ell\xi}+(\ell\xi+1)e^{-\ell\xi}\right] \tag{27}$$

$$= -\frac{2\lambda U}{\pi\ell\xi^2}\left[\ell\xi\cosh\ell\xi - \sinh\ell\xi\right] \tag{27a}$$

$$\int_{-\ell}^{\ell} M(t)\cos\omega_m t\,dt = 0 \tag{28}$$

$$\int_{-\ell}^{\ell} M(t)\sin\omega_m t\,dt = \frac{2\lambda U}{\pi\ell\omega_m^2}(\omega_m\ell\cos\omega_m\ell - \sin\omega_m\ell) \tag{29}$$

These will be used to evaluate the near-field coefficients $A_m(h,k_o,x_o)$ and the far-field coefficients $C_m(h,k_o)$ and $S_m(h,k_o)$.

Evaluation of A_o

We obtain from (27a)

$$A_o = -\frac{8\lambda}{\pi\ell b}\int_0^\infty \frac{(1-\cos\xi h + \frac{k_0}{\xi}\sin\xi h)(\ell\xi\cosh\ell\xi - \sinh\ell\xi)e^{-x\xi}}{\xi^2(\xi^2+k_0^2)}d\xi \tag{30}$$

But

$$\frac{1}{\xi^2(\xi^2+k_0^2)} = \frac{1}{k_0^2}\left(\frac{1}{\xi^2} - \frac{1}{\xi^2+k_0^2}\right)$$

The part of the integral associated with $1/\xi^2$ can be evaluated as follows. We have

$$\frac{\partial}{\partial\ell}(\ell\xi\cosh\ell\xi - \sinh\ell\xi) = \ell\xi^2\sinh\ell\xi$$

Then (30) may be written in the form

$$A_0 = -\frac{4\lambda}{\pi \ell b k_0^2}\int_0^{\ell}\int_0^{\infty}(1 - \cos\xi h + \frac{k_0}{\xi}\sin\xi h)(e^{-x_1\xi} - e^{-x_2\xi})\ \ell d\xi d\ell$$

$$+ \frac{4\lambda}{\pi \ell b k_0^2}\int_0^{\infty}\frac{(1 - \cos\xi h + \frac{k_0}{\xi}\sin\xi h)[(\ell\xi-1)e^{-x_1\xi} + (\ell\xi+1)e^{-x_2\xi}]}{\xi^2+k_0^2}\, d\xi$$

where

$$x_1 = x - \ell, \qquad x_2 = x + \ell$$

But from tables of Laplace transforms we find

$$\int_0^{\infty}(1 - \cos\xi h + \frac{k_0}{\xi}\sin\xi h)(e^{-x_1\xi} - e^{-x_2\xi})\, d\xi$$

$$= \frac{1}{x_1} - \frac{1}{x_2} - \frac{x_1}{x_1^2+h^2} + \frac{x_2}{x_2^2+h^2} + k_0(\arctan\frac{x_2}{h} - \arctan\frac{x_1}{h})$$

and hence we obtain from the previous expression for $\mathbf{A_0}$

$$A_0 = -\frac{4\lambda}{\pi \ell b k_0^2}\Big\{x\ \ell n\frac{x_2}{x_1} - (1-k_0 h)\frac{x}{2}\ \ell n\frac{x_2{}^2+h^2}{x_1{}^2+h^2} + [\frac{k_0}{2}(h^2 - x_1x_2) - h]\arctan\frac{2\ell h}{h^2+x_1x_2}$$

$$-k_0 h\ell - \int_0^{\infty}\frac{(1-\cos\xi h + \frac{k_0}{\xi}\sin\xi h)[(\ell\xi-1)e^{-x_1\xi} + (\ell\xi+1)e^{-x_2\xi}]}{\xi^2+k_0^2}\, d\xi \qquad (31)$$

When h is infinite, we obtain from (31)

$$(A_0)_\infty = -\frac{4\lambda}{\pi \ell b k_0^2}[x\ \ell n\frac{x_2}{x_1} - 2\ell - \int_0^{\infty}\frac{(\ell\xi-1)e^{-x_1\xi} + (\ell\xi+1)e^{-x_2\xi}}{\xi^2+k_0^2}\, d\xi]$$

$$= -\frac{4\lambda}{\pi \ell b k_0^2}\Big\{x\ \ell n\frac{x_2}{x_1} - 2\ell - \frac{1}{k_0}[k_0\ell G(k_0x_1) - F(k_0x_1)$$

$$+ k_0\ell G(k_0x_2) + F(k_0x_2)]\Big\} \qquad (32)$$

where, by [5], p. 57,

$$\left.\begin{aligned} F(x) &= \text{Ci } x \sin x - \text{si } x \cos x \\ G(x) &= -\text{Ci } x \cos x - \text{si } x \sin x \end{aligned}\right\} \tag{33}$$

and si x and Ci x are the sine - and cosine-integral functions

$$\left.\begin{aligned} \text{si } x = \text{Si } x - \frac{\pi}{2} &= -\int_x^\infty \frac{\sin x}{x}\,dx \\ \text{Ci } x &= -\int_x^\infty \frac{\text{Cos } x}{x}\,dx \end{aligned}\right\} \tag{34}$$

When $k_0x>>1$, the functions F(x) and G(x) may be computed from the asymptotic formulas

$$\left.\begin{aligned} F(x) &\sim \frac{1}{x}\left[1 - \frac{2!}{x^2} + \frac{4!}{x^4} - \cdots\right] \\ G(x) &\sim \frac{1}{x^2}\left[1 - \frac{3!}{x^2} + \frac{5!}{x^4} - \cdots\right] \end{aligned}\right\} \tag{35}$$

If also $\ell/x<<1$, we have the expansion

$$x \,\ell n \frac{x_2}{x_1} - 2\ell = 2\ell \left(\frac{\ell^2}{3x^2} + \frac{\ell^4}{5x^4} + \cdots\right) \tag{36}$$

Substituting (35) and (36) into (32) yields the asymptotic expansion

$$(A_0)_\infty \sim -\frac{8\lambda\ell^2}{3\pi b k_0^2}\cdot\frac{1}{x^2}\left[1 + \frac{3\ell^2}{5x^2} - \frac{6x^2}{k_0^2 x_1^2 x_2^2}\right] \tag{37}$$

Under the same conditions, k_0x 1, /x 1, the expression for A_0 in (30) may be approximated by

$$A_0 \doteq (A_0)_\infty - \frac{8\lambda\ell^2}{3\pi b k_0^2}\int_0^\infty (k_0 \sin \xi h - \xi \cos \xi h)e^{-x\xi}d\xi$$

$$\doteq (A_0)_\infty - \frac{8\lambda\ell^2}{3\pi b k_0^2}\left[\frac{k_0 h+1}{x^2+h^2} - \frac{2x^2}{(x^2+h^2)^2}\right] \tag{38}$$

Hence, by (37), we obtain

$$\frac{A_0}{(A_0)_\infty} \doteq 1 + \frac{x^2}{x^2+h^2}\left[k_0h+1 - \frac{2x}{x^2+h^2}\right] \tag{39}$$

From (39) one can deduce the following rather interesting variation of A_0 with h:

$$h \ll x, \qquad \frac{A_0}{(A_0)_\infty} \doteq k_0 h$$

$$h = x, \qquad \frac{A_0}{(A_0)_\infty} \doteq 1+\tfrac{1}{2}k_0 x$$

$$h \gg x, \qquad \frac{A_0}{(A_0)_\infty} \doteq 1 + \frac{k_0 x^2}{h}$$

Thus, A_0 increases linearly for small values of h to a maximum value which may be many times greater than $(A_0)_\infty$, and then decreases slowly according to the inverse first power of h to its limiting value $(A_0)_\infty$.

<u>Evaluation of A_m</u>

Substituting (27) into (4) gives

$$A_m = -\frac{8\lambda}{\pi b \ell}\int_{\nu_m}^{\infty} \frac{\xi^2(1-\cos\alpha_m h) + k_0\alpha_m \sin\alpha_m h}{\alpha_m \xi(\xi^4+k_0^2\alpha_m^2)}\left[(\ell\xi-1)e^{-x_1\xi}+(\ell\xi+1)e^{-x_2\xi}\right] d\xi, \tag{40}$$

$$\nu_m = 2\pi m/b$$

When $h\nu_m<1$ and $x\nu_m \gg 1$, (40) becomes approximately

$$A_m \doteq -\frac{4\lambda}{\pi b\ell}\int_{\nu_m}^{\infty} \frac{(2k_0h+\xi^2h^2)\alpha_m}{\xi(\xi^2+k_0^2\alpha_m^2)}\left[(\ell\xi-1)e^{-x_1\xi}+(\ell\xi+1)e^{-x_2\xi}\right] d\xi \tag{41}$$

When $h = \infty$, (40) yields

$$(A_m)_\infty = -\frac{8\lambda}{\pi b\ell}\int_{\nu_m}^{\infty} \frac{\xi\left[(\ell\xi-1)e^{-x_1\xi}+(\ell\xi+1)e^{-x_2\xi}\right]}{\alpha_m(\xi^4+k_0^2\alpha_m^2)}\, d\xi \tag{42}$$

For large values of m, the expressions for A_m and $(A_m)_\infty$ can be approximately expressed in terms of the modified Bessel functions. Let us substitute $\xi = \nu_m \xi'$ in (41) and (42) and then introduce the approximation $\xi' = 1$, except in α_m and in the exponents. We then have

$$A_m \doteq -\frac{4\lambda}{\pi b \ell \nu_m^3} \int_1^\infty (2k_0 h + \nu_m^2 h^2)\sqrt{\xi'^2 - 1}\; [(\nu_m \ell - 1)e^{-\nu_m x_1 \xi'} + (\nu_m \ell + 1)e^{-\nu_m x_2 \xi'}]\, d\xi' \quad (41a)$$

$$(A_m)_\infty \doteq -\frac{8\lambda}{\pi b \ell \nu_m^3} \int_1^\infty \frac{[(\nu_m \ell - 1)e^{-\nu_m x_1 \xi'} + (\nu_m \ell + 1)e^{-\nu_m x_2 \xi'}]}{\sqrt{\xi'^2 - 1}}\, d\xi' \quad (42a)$$

Then, from the forms for the modified Bessel functions,

$$\left.\begin{aligned} &\int_1^\infty e^{-ax}(x^2-1)^{-\frac{1}{2}}\, dx = K_0(a) \\ &\int_1^\infty e^{-ax}(x^2-1)^{\frac{1}{2}}\, dx = \frac{1}{a} K_1(a) \end{aligned}\right\} \quad (43)$$

we obtain

$$A_m \doteq -\frac{4\lambda}{\pi b \ell \nu_m^3} (2k_o h + \nu_m^2 h^2)[(\nu_m \ell - 1)\frac{K_1(\nu_m x_1)}{\nu x_1} + (\nu_m \ell + 1)\frac{K_1(\nu_m x_2)}{\nu x_2}] \quad (44)$$

and

$$(A_m)_\infty \doteq -\frac{8\lambda}{\pi b \ell \nu_m^3} [(\nu \ell - 1)K_0(\nu x_1) + (\nu \ell + 1)K_0(\nu x_2)] \quad (45)$$

The dominant terms in (44) and (45) are

$$A_m \sim -\frac{4\lambda}{\pi b \ell \nu_m^3} (2k_0 h + \nu_m^2 h^2)(\nu_m \ell - 1) \cdot \frac{K_1(\nu_m x_1)}{\nu_m x_1}$$

$$(A_m)_\infty \sim -\frac{8\lambda}{\pi b \ell \nu_m^3} (\nu_m \ell - 1)\, K_0\, (\nu_m x_1)$$

Since also

$$K_0\, (\nu_m x_1) \sim K_1\, (\nu_m x_1)$$

we obtain

$$\frac{A_m}{(A_m)_\infty} \sim \frac{2k_0 h + \nu_m{}^2 h^2}{2\nu_m x_1} \tag{46}$$

Evaluation of C_m and S_m

Since M(x) is an odd function for the linear distribution, it is seen from (6) that $C_m = 0$. For S_m we obtain

$$S_0 = -\frac{16\lambda(\sin k_0\ell - k_0\ell \cos k_0\ell)}{b\ell k_0{}^3}\,[1-e^{-hk_0}] \tag{47}$$

$$S_m = -\frac{64\lambda(\sin \omega_m\ell - \omega_m\ell \cos \omega_m\ell)}{b\ell k_0 k_m(k_0+k_m)}\,[1-e^{-\frac{h}{2}(k_0+k_m)}],\ m > 0 \tag{48}$$

For all m, we have

$$\frac{S_m}{(S_m)_\infty} = 1 - e^{-\frac{h}{2}(k_0+k_m)} < 1 \tag{49}$$

Values of S_m and Resistance Coefficients C_w

In order to compare the results of the present work with those of Kobus [3], the values $\ell = 3$ feet, $\lambda = 0.04\pi$ and $b = 10$ feet were selected. This corresponds to a strut of about 6-foot chord and chord-thickness ratio of about 8. The strut used by Kobus was a modified ogive of the same chord-thickness ratio, while the present strut has a small non-zero radius of curvature at the ends, which is less, however, than the radius at the ends of the ellipse of the same thickness ratio.

Values of S_m and the corresponding values of C_w at a speed of 5.0 feet per second ($k_0 = 1.288$ per foot) were computed for drafts of 0.5, 1.0, 2.0 and 4.0 feet, and also for infinity. The most interesting features of the results, given in Table 1, are that the sensitivity of

S_m to variation in the draft h becomes rapidly less with increasing values of m, and that the changes in S_m for $h \geq 4$ feet are negligible. It will be seen, on the contrary, that the coefficients A_0 remain sensitive to variations in h at much larger values of the draft.

Graphs of the resistance coefficients C_w versus k_0 for the struts of drafts, in feet, of $h = 0.5$, 1.0, and ∞, are shown in Fig. 1. The resistance coefficients

$$C_w = \frac{R_w}{\frac{1}{2}\rho \ell^2 U^2}$$

were obtained by calculating S_m for $m = 0$ to 49 and applying the formula, derived from (10) with $C_m = 0$,

$$C_w = \frac{k_0 b}{2\ell^2} [S_0{}^2 + \sum_{m=1}^{\infty} \frac{k_m}{k_0+k_m} S_m{}^2] \tag{50}$$

Three of the values of k_0, $k_0 = 1.40$, 1.85, 2.50 per foot, corresponding to successive extrema of C_w, were selected as the constant speeds at which to study the variations of the errors in C_w with x and h.

Curves of S_0 and S_1 versus k_0 for the same strut drafts, $h \geq 4$ feet, are shown in Fig. 2. Data for S_2, S_3. . . were also calculated in the course of obtaining the values of C_w in Fig. 1, but these are not presented since, as will be seen, only S_0 and S_1 values will be required to calculate errors in C_w. Also of interest is the variation of S_0 and S_1 with draft h, shown in Fig. 3 for $k_0 = 1.40$, 1.85, and 2.50 per foot.

Values of A_m; Calculation of ζ_m

A family of curves showing the variation of A_m with downstream distance x is shown in Fig. 4 for a strut of infinite draft at a speed of 4.80 feet per second ($k_0 = 1.40$ per foot). One sees from the figure that, for $x \geq 6$ feet (half the strut length), A_0 and A_1 are much larger than A_2, A_3. . . This characteristic, which was confirmed in all sets of A_m calculated, is the basis for selecting A_0, A_1 and the associated values of S_0 and S_1 in the subsequent analysis.

Since A_0 and A_1 depend on the three variables h, k_0 and x, while S_0 and S_1 depend on only h and k_0, it was necessary in the main, to restrict the study of the former variables to only a few values of x. In Fig. 5, the variation of A_0 and A_1 with k_0 is shown for a strut of infinite draft at x = 6.0 and 9.0 feet. In Fig. 6, the variation of A_0 and A_1 with draft h at a constant speed of 4.80 feet per second (k_0 = 1.40 per foot) is given at x = 6.0, 8.0, 10.0, 12.0 and 15.0 feet. The occurrence of a maximum in the variation of A_0 with h had already been indicated in the previous discussion based on its asymptotic formula. One sees from Fig. 6 that these maxima occur when h is approximately equal to x, the downstream distance measured from the midsection of the strut. Furthermore, the magnitudes of these maxima are several times greater than the corresponding values of A_0 for a strut of infinite draft.

Actual values of A_0, A_1, S_0, S_1 and C_w used in the analysis of errors are given in Tables 2, 3, and 4. In Table 2 the parameters are h = 0.5, 1.0, 2.0, 4.0, 8.0, ∞ in feet and k_0 = 1.40, 1.85, 2.50 per foot, and x is varied in small increments from x = 6 to 15 feet. In Table 3 the parameters are h = 4.0 and 8.0 feet, x = 6.0 and 9.0 feet, and k_0 is varied in small increments. In Table 4, the parameters are k_0 = 1.40 per foot, x = 6.0, 8.0, 10.0, 12.0 and 15.0 feet, and h is varied in small increments.

Using values of A_m and S_m for a strut of infinite draft at a speed of 5.0 feet per second (k_0 = 1.288 per foot), the surface profile at a transverse section one model length behind the strut (x = 9 feet), shown in Fig. 8, was calculated from equations (3) and (5). Also given in this figure is the surface profile at the same transverse section behind the modified ogive of Kobus [3]. The agreement between these profiles is seen to be very good.

Errors in Determination of S_m and C_w

Since C_m is zero in the present case, the previous analysis of the error in the determination of S_m due to A_m can be simplified. From

(12) we have

$$A_m + S_m \sin \omega_m x = f_m(x) \tag{51}$$

Put

$$S'_m = f_m(x) \csc \omega_m x = S_m + A_m \csc \omega_m x \tag{52}$$

Then S'_m is the quantity one would obtain from the harmonic analysis of a transverse surface profile given by (11), and $S'_m - S_m$ is the error in the determination of S_m. The relation (52) is a special case of (17) with $S_m = E_m$ and $S'_m = E'_m$. Typical graphs showing the variations of S'_0 and S'_1 with downstream distance x are shown in Figs. 7a and 7b for the case of a strut of infinite draft at $k_0 = 1.40$ per foot.

For a symmetrical strut, Eggers' formula for the wave-resistance coefficient has been given in (50). As is well known, a good approximation to C_w cannot be calculated from this formula by replacing the S_m by S'_m, since, for some m, S'_m becomes arbitrarily large. This difficulty can be circumvented by applying the method of least squares to obtain a mean of the values of $S'_m(x)$ at several values of x. One obtains by this procedure the mean value

$$\bar{S}_m = S_m + \frac{\sum_i A_m(x_i) \sin \omega_m x_i}{\sum_i \sin^2 \omega_m x_i} \tag{53}$$

Conceivably (53) could also yield a large error if, for some value of m, the x_i's were selected so that all the $\omega_m x_i$ were nearly integral multiples of π. One could ensure that this would not occur by employing nonuniform intervals between the successive x_i. Furthermore, in contrast with the expression for S'_m in (52), which gives a very large error when $\omega_m x$ is nearly an integral multiple of π, the contribution of such a value in (53) would be small. On the other hand, since the errors due to the A_m's are not random, one could not expect that the least-square solutions would be the "best" ones, or as good as those obtained by the procedure recommended in the section "Approximate Determination of E_m from $\xi(x,y)$."

In the present case it is simpler and more convenient to continue to use the expression for the error ΔS_m given by (52), discarding those values of x at which either $\csc k_0x$ or $\csc \omega_1 x$ becomes large. It is unnecessary to consider the values of $\csc \omega_m x$ for $m > 1$ since the associated values of A_m are very small, as was shown in the previous section, and the resulting term would certainly make a negligible contribution in the least-square formula (53).

Assuming that only the errors in S_0 and S_1 need to be considered, expression (50) yields for the error in C_w

$$\Delta C_w \doteq \frac{k_0 b}{2\ell^2} [S_0'^2 - S_0{}^2 + \frac{k_1}{k_0+k_1} (S_1'^2 - S_1{}^2)] \tag{54}$$

or, by (52),

$$\Delta C_w \doteq \frac{k_0 b}{2\ell^2}[2S_0 A_0 \csc k_0 x + A_0{}^2 \csc^2 k_0 x + \frac{k_1}{k_0+k_1}(2S_1 A_1 \csc \omega_1 x + A_1{}^2 \csc^2 \omega_1 x)] \tag{55}$$

The variations of $\Delta C_w/C_w$ with x, k_0 and h, calculated from the values of S_0, S_1, A_0, A_1 and C_w given in Tables 2, 3 and 4, are shown in Figs. 9, 10 and 11. Figures 9a, b, and c show the variation with downstream distance x for various drafts h. Vertical lines extending over the range of ordinates, $-6.0 \leqq 100\Delta C_w/C_w \leqq 6.0$, indicate the locations of the singularities of $\csc k_0 x$, i.e. the values $x = n\pi/k_0$. The vertical lines extending alternately over the positive and negative half of the range of ordinates indicate the locations of the singularities of $\csc \omega_1 x$, at $x = n\pi/\omega_1$. In the neighborhood of each of these singularities the terms $\csc^2 k_0 x$ or $\csc^2 \omega_1 x$ will eventually dominate and the error will approach $+\infty$. The apparent tendency of the error to approach $-\infty$, shown in two of the ranges in Figure 9a, is due to the fact that S_0 and S_1 are very much greater than A_0 and A_1 respectively, so that when $\csc k_0 x$, or $\csc \omega_1 x$ is negative, one sees from (55) that ΔC_w may at first assume large negative values near a singularity; but eventually $A_0{}^2 \csc^2 k_0 x$ or $A_1{}^2 \csc^2 \omega_1 x$ must dominate and ΔC_w oscillates wildly between large negative values and $+\infty$ in a very small neighborhood of each singular point. For this reason the portions of the curves of $\Delta C_w/C_w$ near the singular points have been omitted, as is indicated by hatching or simply by joining portions of curves on either side of a singular point by dashed lines. Rapid variations in the neighborhood of the singular values of k_0 are treated similarly in Figures 10a and 10b.

One sees from Figures 9a, b, and c that the orders of magnitude of the errors $\Delta C_w/C_w$ are about the same for the three values of k_0, a rather surprising result, since $k_0 = 1.40$ and 2.5 are associated with minima of C_w, and $k_0 = 1.85$ with a maximum. At a draft of 0.5 feet, which would yield a form most nearly of ship-like dimensions, the mean error would be less than one percent in absolute value. With increasing draft the mean error increases to about 5 percent at $h = 8$ feet. As x increases, the error decreases rapidly for $h = 0.5$ feet, but more slowly at the larger drafts.

Because of the double set of singular values of k_0, it was necessary to hatch numerous intervals of k_0 in Figures 10a and b, in which the variation of $\Delta C_w/C_w$ was too rapid to be of practical interest. The results for drafts of 4.0 and 8.0 feet are almost identical. Mean errors of the order of magnitude of 10 percent are indicated at $x = 6$ feet, a half model length downstream from the strut, but the mean error is reduced to about 5 percent at $x = 9$ feet, one model length downstream.

Finally, we see in Figure 11 an interesting variation of the error $\Delta C_w/C_w$ with drafts similar to that of A_0 in Figure 6. As for A_0, the figure indicates that the error $\Delta C_w/C_w$ is largest in absolute value when $x \doteq h$. A maximum error slightly greater than 8 percent at $x = 6$ feet, reduces quickly to an absolute value of about 4 percent at $x = 10$ feet. At all values of x the error reduces to zero as h approaches zero, confirming that one would expect a negligible error for shiplike drafts.

Conclusions

1. At downstream distances greater than half the model length, only the first two coefficients of the expression for the

near-field surface disturbance, A_0 and A_1 contribute significantly to the error in the determination of wave resistance by Eggers' transverse-cut method.

2. The far-field coefficients, which are required for the calculation of the wave resistance, increase monotonically with strut draft and become very nearly equal to the values for a strut of infinite draft at moderate strut drafts. In contrast, the first near-field coefficient A_0 is proportional to the draft for small values of the draft, increases (in absolute value) to a maximum value which is much greater than the asymptotic value for infinite draft, to which it slowly approaches with further increase in draft. Although the second near-field coefficient A_1 varies monotonically with draft, it approaches the asymptotic value for infinite draft more slowly than the far-field coefficients.

3. The increase in wave resistance of a vertical strut with increasing draft is very small for drafts greater than half the strut length.

4. For ship forms, the error in Eggers' transverse cut method due to the neglect of the near-field disturbance is less than one percent, if profiles at downstream distances greater than one model length are used.

5. For vertical struts, the error increases with the draft to a maximum in absolute value when the draft is approximately equal to the downstream distance, measured from the midsection of the strut, and then decreases slowly with increasing distance. Errors of about 5 percent would be expected if Eggers' method were applied to struts of drafts equal to about half to three times the model length at downstream distances between one and two model lengths.

6. The multiplicity of singular values in the neighborhoods of which the errors may become very large emphasizes the importance of measuring the surface profiles at many downstream sections. From these measurements the coefficients required for calculating the wave resistance can be obtained either by the method of least squares or, preferably, by the proposed graphical procedure.

Although the present study has indicated that the errors due to the near-field term neglected in Eggers' method for determining wave resistance can be minimized by appropriate procedures, there still remains to consider the error due to the presence of a wake, which, according to Kobus [3], is far from negligible.

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TABLE 1 VALUES OF C_w AND S_m FOR k_o = 1.288/FT

h(FT)	0.50000	1.00000	2.00000	4.00000	INFINITE
C_w	0.00302	0.00527	0.00690	0.00746	0.00750
m	S_m	S_m	S_m	S_m	S_m
0	-0.03333	-0.05083	-0.06485	-0.06978	-0.07019
1	-0.02167	-0.03169	-0.03845	-0.04021	-0.04029
2	0.02083	0.02828	0.03190	0.03243	0.03243
3	0.03418	0.04333	0.04643	0.04667	0.04667
4	0.03083	0.03694	0.03839	0.03845	0.03845
5	0.02077	0.02380	0.02431	0.02432	0.02432
6	0.00979	0.01084	0.01097	0.01097	0.01097
7	0.00072	0.00078	0.00078	0.00078	0.00078
8	-0.00548	-0.00579	-0.00581	-0.00581	-0.00581
9	-0.00882	-0.00919	-0.00920	-0.00920	-0.00920
10	-0.00980	-0.01011	-0.01011	-0.01011	-0.01011
11	-0.00909	-0.00930	-0.00930	-0.00930	-0.00930
12	-0.00733	-0.00745	-0.00745	-0.00745	-0.00745
13	-0.00506	-0.00512	-0.00512	-0.00512	-0.00512
14	-0.00269	-0.00271	-0.00271	-0.00271	-0.00271
15	-0.00052	-0.00052	-0.00052	-0.00052	-0.00052
16	0.00128	0.00128	0.00128	0.00128	0.00128
17	0.00261	0.00262	0.00262	0.00262	0.00262
18	0.00346	0.00347	0.00347	0.00347	0.00347
19	0.00385	0.00386	0.00386	0.00386	0.00386
20	0.00386	0.00386	0.00386	0.00386	0.00386
21	0.00355	0.00355	0.00355	0.00355	0.00355
22	0.00301	0.00301	0.00301	0.00301	0.00301
23	0.00232	0.00233	0.00233	0.00233	0.00233
24	0.00157	0.00157	0.00157	0.00157	0.00157
25	0.00080	0.00080	0.00080	0.00080	0.00080
26	0.00008	0.00008	0.00008	0.00008	0.00008
27	-0.00056	-0.00056	-0.00056	-0.00056	-0.00056
28	-0.00108	-0.00108	-0.00108	-0.00108	-0.00108
29	-0.00148	-0.00148	-0.00148	-0.00148	-0.00148
30	-0.00175	-0.00175	-0.00175	-0.00175	-0.00175
31	-0.00190	-0.00190	-0.00190	-0.00190	-0.00190
32	-0.00192	-0.00192	-0.00192	-0.00192	-0.00192
33	-0.00184	-0.00184	-0.00184	-0.00184	-0.00184
34	-0.00167	-0.00167	-0.00167	-0.00167	-0.00167
35	-0.00144	-0.00144	-0.00144	-0.00144	-0.00144
36	-0.00116	-0.00116	-0.00116	-0.00116	-0.00116
37	-0.00085	-0.00085	-0.00085	-0.00085	-0.00085
38	-0.00052	-0.00052	-0.00052	-0.00052	-0.00052
39	-0.00021	-0.00021	-0.00021	-0.00021	-0.00021
40	0.00009	0.00009	0.00009	0.00009	0.00009
41	0.00037	0.00037	0.00037	0.00037	0.00037
42	0.00060	0.00060	0.00060	0.00060	0.00060
43	0.00079	0.00079	0.00079	0.00079	0.00079
44	0.00094	0.00094	0.00094	0.00094	0.00094
45	0.00103	0.00103	0.00103	0.00103	0.00103
46	0.00108	0.00108	0.00108	0.00108	0.00108
47	0.00109	0.00109	0.00109	0.00109	0.00109
48	0.00105	0.00105	0.00105	0.00105	0.00105
49	0.00098	0.00098	0.00098	0.00098	0.00098

TABLE 2a-1 VARIATION OF C_W, S_o, S_1, A_o AND A_1 WITH DOWNSTREAM DISTANCE X(FT) FOR k_o =1.40/FT

h(FT)	0.50000		1.00000		2.00000	
C_W	0.00256		0.00397		0.00468	
S_o	-0.01460		-0.02185		-0.02724	
S_1	0.00410		0.00591		0.00705	
X(FT)	A_o	A_1	A_o	A_1	A_o	A_1
6.00	-0.00105	-0.00035	-0.00212	-0.00075	-0.00408	-0.00115
6.20	-0.00098	-0.00030	-0.00197	-0.00063	-0.00381	-0.00131
6.40	-0.00091	-0.00025	-0.00184	-0.00053	-0.00356	-0.00111
6.60	-0.00085	-0.00021	-0.00172	-0.00045	-0.00334	-0.00094
6.80	-0.00080	-0.00018	-0.00161	-0.00038	-0.00313	-0.00080
7.00	-0.00075	-0.00015	-0.00151	-0.00032	-0.00295	-0.00068
7.20	-0.00071	-0.00013	-0.00142	-0.00027	-0.00278	-0.00058
7.40	-0.00067	-0.00011	-0.00134	-0.00023	-0.00262	-0.00049
7.60	-0.00063	-0.00009	-0.00126	-0.00020	-0.00248	-0.00042
7.80	-0.00060	-0.00008	-0.00120	-0.00017	-0.00235	-0.00036
8.00	-0.00057	-0.00007	-0.00113	-0.00014	-0.00223	-0.00031
8.20	-0.00054	-0.00006	-0.00108	-0.00012	-0.00212	-0.00026
8.40	-0.00051	-0.00005	-0.00102	-0.00011	-0.00202	-0.00022
8.60	-0.00049	-0.00004	-0.00097	-0.00009	-0.00192	-0.00019
8.80	-0.00046	-0.00004	-0.00093	-0.00008	-0.00183	-0.00016
9.00	-0.00044	-0.00003	-0.00089	-0.00007	-0.00175	-0.00014
9.20	-0.00042	-0.00003	-0.00085	-0.00006	-0.00167	-0.00012
9.40	-0.00040	-0.00002	-0.00081	-0.00005	-0.00160	-0.00010
9.60	-0.00039	-0.00002	-0.00077	-0.00004	-0.00153	-0.00009
9.80	-0.00037	-0.00002	-0.00074	-0.00004	-0.00147	-0.00008
10.00	-0.00035	-0.00001	-0.00071	-0.00003	-0.00141	-0.00007
10.20	-0.00034	-0.00001	-0.00068	-0.00003	-0.00135	-0.00006
10.40	-0.00033	-0.00001	-0.00066	-0.00002	-0.00130	-0.00005
10.60	-0.00031	-0.00001	-0.00063	-0.00002	-0.00125	-0.00004
10.80	-0.00030	-0.00001	-0.00061	-0.00002	-0.00120	-0.00004
11.00	-0.00029	-0.00001	-0.00058	-0.00001	-0.00116	-0.00003
11.20	-0.00028	-0.00001	-0.00056	-0.00001	-0.00112	-0.00003
11.40	-0.00027	-0.00000	-0.00054	-0.00001	-0.00108	-0.00002
11.60	-0.00026	-0.00000	-0.00052	-0.00001	-0.00104	-0.00002
11.80	-0.00025	-0.00000	-0.00051	-0.00001	-0.00100	-0.00002
12.00	-0.00024	-0.00000	-0.00049	-0.00001	-0.00097	-0.00001
12.20	-0.00023	-0.00000	-0.00047	-0.00001	-0.00094	-0.00001
12.40	-0.00023	-0.00000	-0.00046	-0.00000	-0.00091	-0.00001
12.60	-0.00022	-0.00000	-0.00044	-0.00000	-0.00088	-0.00001
12.80	-0.00021	-0.00000	-0.00043	-0.00000	-0.00085	-0.00001
13.00	-0.00021	-0.00000	-0.00041	-0.00000	-0.00082	-0.00001
13.20	-0.00020	-0.00000	-0.00040	-0.00000	-0.00080	-0.00001
13.40	-0.00019	-0.00000	-0.00039	-0.00000	-0.00078	-0.00000
13.60	-0.00019	-0.00000	-0.00038	-0.00000	-0.00075	-0.00000
13.80	-0.00018	-0.00000	-0.00037	-0.00000	-0.00073	-0.00000
14.00	-0.00018	-0.00000	-0.00036	-0.00000	-0.00071	-0.00000
14.20	-0.00017	-0.00000	-0.00035	-0.00000	-0.00069	-0.00000
14.40	-0.00017	-0.00000	-0.00034	-0.00000	-0.00067	-0.00000
14.60	-0.00016	-0.00000	-0.00033	-0.00000	-0.00065	-0.00000
14.80	-0.00016	-0.00000	-0.00032	-0.00000	-0.00063	-0.00000
15.00	-0.00015	-0.00000	-0.00031	-0.00000	-0.00062	-0.00000

TABLE 2a-2 VARIATION OF C_w, S_o, S_1, A_o AND A_1 WITH DOWNSTREAM DISTANCE X(FT) FOR k_o =1.40/FT

h(FT)	4.00000		8.00000		INFINITE	
C_w	0.00481		0.00481		0.00481	
S_o	-0.02890		-0.02900		-0.02900	
S_1	0.00731		0.00733		0.00733	
X(FT)	A_o	A_1	A_o	A_1	A_o	A_1
6.00	-0.00655	-0.00254	-0.00711	-0.00294	-0.00146	-0.00279
6.20	-0.00619	-0.00218	-0.00686	-0.00255	-0.00136	-0.00243
6.40	-0.00586	-0.00188	-0.00663	-0.00221	-0.00127	-0.00211
6.60	-0.00555	-0.00161	-0.00641	-0.00192	-0.00119	-0.00183
6.80	-0.00527	-0.00138	-0.00620	-0.00167	-0.00112	-0.00159
7.00	-0.00501	-0.00119	-0.00600	-0.00146	-0.00105	-0.00139
7.20	-0.00476	-0.00103	-0.00580	-0.00127	-0.00099	-0.00121
7.40	-0.00454	-0.00088	-0.00562	-0.00110	-0.00094	-0.00105
7.60	-0.00432	-0.00076	-0.00544	-0.00096	-0.00089	-0.00092
7.80	-0.00412	-0.00066	-0.00528	-0.00083	-0.00083	-0.00080
8.00	-0.00394	-0.00056	-0.00511	-0.00073	-0.00079	-0.00070
8.20	-0.00376	-0.00049	-0.00496	-0.00063	-0.00076	-0.00061
8.40	-0.00360	-0.00042	-0.00481	-0.00055	-0.00072	-0.00053
8.60	-0.00345	-0.00036	-0.00467	-0.00048	-0.00068	-0.00046
8.80	-0.00330	-0.00031	-0.00453	-0.00042	-0.00065	-0.00040
9.00	-0.00317	-0.00027	-0.00440	-0.00037	-0.00062	-0.00035
9.20	-0.00304	-0.00023	-0.00427	-0.00032	-0.00060	-0.00031
9.40	-0.00292	-0.00020	-0.00415	-0.00028	-0.00057	-0.00027
9.60	-0.00281	-0.00017	-0.00403	-0.00024	-0.00055	-0.00024
9.80	-0.00270	-0.00015	-0.00392	-0.00021	-0.00052	-0.00021
10.00	-0.00260	-0.00013	-0.00381	-0.00019	-0.00050	-0.00018
10.20	-0.00251	-0.00011	-0.00370	-0.00016	-0.00048	-0.00016
10.40	-0.00241	-0.00010	-0.00360	-0.00014	-0.00046	-0.00014
10.60	-0.00233	-0.00008	-0.00350	-0.00012	-0.00045	-0.00012
10.80	-0.00225	-0.00007	-0.00341	-0.00011	-0.00043	-0.00011
11.00	-0.00217	-0.00006	-0.00332	-0.00009	-0.00041	-0.00009
11.20	-0.00210	-0.00005	-0.00323	-0.00008	-0.00040	-0.00008
11.40	-0.00203	-0.00005	-0.00315	-0.00007	-0.00038	-0.00007
11.60	-0.00196	-0.00004	-0.00307	-0.00006	-0.00037	-0.00006
11.80	-0.00190	-0.00004	-0.00299	-0.00005	-0.00036	-0.00005
12.00	-0.00184	-0.00003	-0.00291	-0.00005	-0.00035	-0.00005
12.20	-0.00178	-0.00003	-0.00284	-0.00004	-0.00033	-0.00004
12.40	-0.00172	-0.00002	-0.00277	-0.00004	-0.00032	-0.00004
12.60	-0.00167	-0.00002	-0.00270	-0.00003	-0.00031	-0.00003
12.80	-0.00162	-0.00002	-0.00263	-0.00003	-0.00030	-0.00003
13.00	-0.00157	-0.00001	-0.00257	-0.00002	-0.00029	-0.00002
13.20	-0.00153	-0.00001	-0.00251	-0.00002	-0.00028	-0.00002
13.40	-0.00148	-0.00001	-0.00245	-0.00002	-0.00028	-0.00002
13.60	-0.00144	-0.00001	-0.00239	-0.00002	-0.00027	-0.00002
13.80	-0.00140	-0.00001	-0.00233	-0.00001	-0.00026	-0.00001
14.00	-0.00136	-0.00001	-0.00228	-0.00001	-0.00025	-0.00001
14.20	-0.00133	-0.00001	-0.00223	-0.00001	-0.00025	-0.00001
14.40	-0.00129	-0.00001	-0.00218	-0.00001	-0.00024	-0.00001
14.60	-0.00126	-0.00000	-0.00213	-0.00001	-0.00023	-0.00001
14.80	-0.00122	-0.00000	-0.00208	-0.00001	-0.00023	-0.00001
15.00	-0.00119	-0.00000	-0.00204	-0.00001	-0.00022	-0.00001

TABLE 2b-1 VARIATION OF C_w, S_o, S_1, A_o AND A_1 WITH DOWNSTREAM DISTANCE X(FT) FOR k_o =1.85/FT

h (FT)	0.50000		1.00000		2.00000	
C_w	0.00486		0.00831		0.01026	
S_o	0.03062		0.04276		0.04948	
S_1	0.05770		0.07847		0.08864	
X(FT)	A_o	A_1	A_o	A_1	A_o	A_1
6.00	-0.00082	-0.00034	-0.00164	-0.00070	-0.00311	-0.00139
6.20	-0.00076	-0.00028	-0.00152	-0.00059	-0.00290	-0.00118
6.40	-0.00071	-0.00024	-0.00142	-0.00050	-0.00271	-0.00100
6.60	-0.00066	-0.00020	-0.00132	-0.00042	-0.00254	-0.00085
6.80	-0.00062	-0.00017	-0.00124	-0.00036	-0.00238	-0.00073
7.00	-0.00058	-0.00015	-0.00116	-0.00031	-0.00224	-0.00062
7.20	-0.00055	-0.00012	-0.00109	-0.00026	-0.00211	-0.00053
7.40	-0.00051	-0.00011	-0.00103	-0.00022	-0.00199	-0.00045
7.60	-0.00049	-0.00009	-0.00097	-0.00019	-0.00189	-0.00039
7.80	-0.00046	-0.00008	-0.00092	-0.00016	-0.00179	-0.00033
8.00	-0.00043	-0.00007	-0.00087	-0.00014	-0.00169	-0.00028
8.20	-0.00041	-0.00006	-0.00082	-0.00012	-0.00161	-0.00024
8.40	-0.00039	-0.00005	-0.00078	-0.00010	-0.00153	-0.00021
8.60	-0.00037	-0.00004	-0.00074	-0.00009	-0.00146	-0.00018
8.80	-0.00035	-0.00004	-0.00071	-0.00007	-0.00139	-0.00015
9.00	-0.00034	-0.00003	-0.00068	-0.00006	-0.00133	-0.00013
9.20	-0.00032	-0.00003	-0.00065	-0.00005	-0.00127	-0.00011
9.40	-0.00031	-0.00002	-0.00062	-0.00005	-0.00121	-0.00010
9.60	-0.00030	-0.00002	-0.00059	-0.00004	-0.00116	-0.00008
9.80	-0.00028	-0.00002	-0.00057	-0.00003	-0.00111	-0.00007
10.00	-0.00027	-0.00001	-0.00054	-0.00003	-0.00107	-0.00006
10.20	-0.00026	-0.00001	-0.00052	-0.00003	-0.00103	-0.00005
10.40	-0.00025	-0.00001	-0.00050	-0.00002	-0.00099	-0.00005
10.60	-0.00024	-0.00001	-0.00048	-0.00002	-0.00095	-0.00004
10.80	-0.00023	-0.00001	-0.00046	-0.00002	-0.00091	-0.00004
11.00	-0.00022	-0.00001	-0.00045	-0.00001	-0.00088	-0.00003
11.20	-0.00021	-0.00001	-0.00043	-0.00001	-0.00085	-0.00003
11.40	-0.00021	-0.00000	-0.00041	-0.00001	-0.00082	-0.00002
11.60	-0.00020	-0.00000	-0.00040	-0.00001	-0.00079	-0.00002
11.80	-0.00019	-0.00000	-0.00038	-0.00001	-0.00076	-0.00002
12.00	-0.00019	-0.00000	-0.00037	-0.00001	-0.00074	-0.00001
12.20	-0.00018	-0.00000	-0.00036	-0.00001	-0.00071	-0.00001
12.40	-0.00017	-0.00000	-0.00035	-0.00000	-0.00069	-0.00001
12.60	-0.00017	-0.00000	-0.00034	-0.00000	-0.00067	-0.00001
12.80	-0.00016	-0.00000	-0.00033	-0.00000	-0.00065	-0.00001
13.00	-0.00016	-0.00000	-0.00032	-0.00000	-0.00063	-0.00001
13.20	-0.00015	-0.00000	-0.00031	-0.00000	-0.00061	-0.00001
13.40	-0.00015	-0.00000	-0.00030	-0.00000	-0.00059	-0.00000
13.60	-0.00014	-0.00000	-0.00029	-0.00000	-0.00057	-0.00000
13.80	-0.00014	-0.00000	-0.00028	-0.00000	-0.00055	-0.00000
14.00	-0.00014	-0.00000	-0.00027	-0.00000	-0.00054	-0.00000
14.20	-0.00013	-0.00000	-0.00026	-0.00000	-0.00052	-0.00000
14.40	-0.00013	-0.00000	-0.00026	-0.00000	-0.00051	-0.00000
14.60	-0.00012	-0.00000	-0.00025	-0.00000	-0.00049	-0.00000
14.80	-0.00012	-0.00000	-0.00024	-0.00000	-0.00048	-0.00000
15.00	-0.00012	-0.00000	-0.00023	-0.00000	-0.00047	-0.00000

TABLE 2b-2 VARIATION OF C_w, S_o, S_1, A_o AND A_1 WITH DOWNSTREAM DISTANCE X(FT) FOR k_o =1.85/FT

h (FT)	4.00000		8.00000		INFINITE	
C_w	0.01057		0.01057		0.01057	
S_o	0.05070		0.05074		0.05074	
S_1	0.09013		0.09016		0.09016	
X(FT)	A_o	A_1	A_o	A_1	A_o	A_1
6.00	-0.00485	-0.00220	-0.00513	-0.00250	-0.00087	-0.00235
6.20	-0.00459	-0.00189	-0.00496	-0.00217	-0.00081	-0.00204
6.40	-0.00435	-0.00163	-0.00480	-0.00189	-0.00075	-0.00178
6.60	-0.00413	-0.00141	-0.00465	-0.00165	-0.00070	-0.00155
6.80	-0.00392	-0.00121	-0.00450	-0.00143	-0.00066	-0.00135
7.00	-0.00373	-0.00104	-0.00436	-0.00125	-0.00062	-0.00117
7.20	-0.00355	-0.00090	-0.00423	-0.00109	-0.00058	-0.00103
7.40	-0.00338	-0.00078	-0.00410	-0.00095	-0.00055	-0.00089
7.60	-0.00323	-0.00067	-0.00398	-0.00083	-0.00052	-0.00078
7.80	-0.00308	-0.00058	-0.00386	-0.00072	-0.00049	-0.00068
8.00	-0.00294	-0.00050	-0.00374	-0.00063	-0.00047	-0.00059
8.20	-0.00281	-0.00043	-0.00364	-0.00055	-0.00044	-0.00052
8.40	-0.00269	-0.00037	-0.00353	-0.00048	-0.00042	-0.00045
8.60	-0.00258	-0.00032	-0.00343	-0.00042	-0.00040	-0.00040
8.80	-0.00247	-0.00028	-0.00333	-0.00037	-0.00038	-0.00035
9.00	-0.00237	-0.00024	-0.00324	-0.00032	-0.00036	-0.00030
9.20	-0.00228	-0.00021	-0.00315	-0.00028	-0.00035	-0.00026
9.40	-0.00219	-0.00018	-0.00306	-0.00024	-0.00033	-0.00023
9.60	-0.00211	-0.00016	-0.00298	-0.00021	-0.00032	-0.00020
9.80	-0.00203	-0.00014	-0.00289	-0.00019	-0.00030	-0.00018
10.00	-0.00195	-0.00012	-0.00282	-0.00016	-0.00029	-0.00016
10.20	-0.00188	-0.00010	-0.00274	-0.00014	-0.00028	-0.00014
10.40	-0.00181	-0.00009	-0.00267	-0.00012	-0.00027	-0.00012
10.60	-0.00175	-0.00008	-0.00260	-0.00011	-0.00026	-0.00010
10.80	-0.00169	-0.00007	-0.00253	-0.00010	-0.00025	-0.00009
11.00	-0.00163	-0.00006	-0.00246	-0.00008	-0.00024	-0.00008
11.20	-0.00158	-0.00005	-0.00240	-0.00007	-0.00023	-0.00007
11.40	-0.00152	-0.00004	-0.00234	-0.00006	-0.00022	-0.00006
11.60	-0.00147	-0.00004	-0.00228	-0.00005	-0.00021	-0.00005
11.80	-0.00143	-0.00003	-0.00222	-0.00005	-0.00021	-0.00005
12.00	-0.00138	-0.00003	-0.00217	-0.00004	-0.00020	-0.00004
12.20	-0.00134	-0.00003	-0.00211	-0.00004	-0.00019	-0.00004
12.40	-0.00130	-0.00002	-0.00206	-0.00003	-0.00019	-0.00003
12.60	-0.00126	-0.00002	-0.00201	-0.00003	-0.00018	-0.00003
12.80	-0.00122	-0.00002	-0.00196	-0.00003	-0.00017	-0.00002
13.00	-0.00119	-0.00001	-0.00192	-0.00002	-0.00017	-0.00002
13.20	-0.00115	-0.00001	-0.00187	-0.00002	-0.00016	-0.00002
13.40	-0.00112	-0.00001	-0.00183	-0.00002	-0.00016	-0.00002
13.60	-0.00109	-0.00001	-0.00178	-0.00001	-0.00015	-0.00001
13.80	-0.00106	-0.00001	-0.00174	-0.00001	-0.00015	-0.00001
14.00	-0.00103	-0.00001	-0.00170	-0.00001	-0.00015	-0.00001
14.20	-0.00100	-0.00001	-0.00167	-0.00001	-0.00014	-0.00001
14.40	-0.00097	-0.00000	-0.00163	-0.00001	-0.00014	-0.00001
14.60	-0.00095	-0.00000	-0.00159	-0.00001	-0.00013	-0.00001
14.80	-0.00092	-0.00000	-0.00156	-0.00001	-0.00013	-0.00001
15.00	-0.00090	-0.00000	-0.00152	-0.00001	-0.00013	-0.00001

TABLE 2c-1 VARIATION OF C_w, S_o, S_1, A_o AND A_1 WITH DOWNSTREAM DISTANCE X(FT) FOR k_o = 2.50/FT

h(FT)	0.50000		1.00000		2.00000	
C_w	0.00151		0.00194		0.00204	
S_o	0.00509		0.00654		0.00708	
S_1	0.00020		0.00025		0.00027	
X(FT)	A_o	A_1	A_o	A_1	A_o	A_1
6.00	-0.00062	-0.00030	-0.00122	-0.00062	-0.00229	-0.00118
6.20	-0.00057	-0.00025	-0.00114	-0.00052	-0.00214	-0.00101
6.40	-0.00053	-0.00022	-0.00106	-0.00044	-0.00200	-0.00086
6.60	-0.00050	-0.00018	-0.00099	-0.00037	-0.00187	-0.00073
6.80	-0.00046	-0.00015	-0.00092	-0.00032	-0.00176	-0.00062
7.00	-0.00044	-0.00013	-0.00087	-0.00027	-0.00166	-0.00053
7.20	-0.00041	-0.00011	-0.00081	-0.00023	-0.00156	-0.00046
7.40	-0.00038	-0.00010	-0.00077	-0.00020	-0.00147	-0.00039
7.60	-0.00036	-0.00008	-0.00072	-0.00017	-0.00139	-0.00033
7.80	-0.00034	-0.00007	-0.00068	-0.00014	-0.00132	-0.00029
8.00	-0.00033	-0.00006	-0.00065	-0.00012	-0.00125	-0.00025
8.20	-0.00031	-0.00005	-0.00061	-0.00011	-0.00119	-0.00021
8.40	-0.00029	-0.00004	-0.00058	-0.00009	-0.00113	-0.00018
8.60	-0.00028	-0.00004	-0.00055	-0.00008	-0.00108	-0.00016
8.80	-0.00026	-0.00003	-0.00053	-0.00007	-0.00103	-0.00013
9.00	-0.00025	-0.00003	-0.00050	-0.00006	-0.00098	-0.00012
9.20	-0.00024	-0.00002	-0.00048	-0.00005	-0.00094	-0.00010
9.40	-0.00023	-0.00002	-0.00046	-0.00004	-0.00090	-0.00009
9.60	-0.00022	-0.00002	-0.00044	-0.00004	-0.00086	-0.00007
9.80	-0.00021	-0.00001	-0.00042	-0.00003	-0.00082	-0.00006
10.00	-0.00020	-0.00001	-0.00040	-0.00003	-0.00079	-0.00005
10.20	-0.00019	-0.00001	-0.00039	-0.00002	-0.00076	-0.00005
10.40	-0.00019	-0.00001	-0.00037	-0.00002	-0.00073	-0.00004
10.60	-0.00018	-0.00001	-0.00036	-0.00002	-0.00070	-0.00004
10.80	-0.00017	-0.00001	-0.00034	-0.00001	-0.00067	-0.00003
11.00	-0.00016	-0.00001	-0.00033	-0.00001	-0.00065	-0.00003
11.20	-0.00016	-0.00000	-0.00032	-0.00001	-0.00063	-0.00002
11.40	-0.00015	-0.00000	-0.00031	-0.00001	-0.00060	-0.00002
11.60	-0.00015	-0.00000	-0.00030	-0.00001	-0.00058	-0.00002
11.80	-0.00014	-0.00000	-0.00028	-0.00001	-0.00056	-0.00001
12.00	-0.00014	-0.00000	-0.00028	-0.00001	-0.00054	-0.00001
12.20	-0.00013	-0.00000	-0.00027	-0.00000	-0.00053	-0.00001
12.40	-0.00013	-0.00000	-0.00026	-0.00000	-0.00051	-0.00001
12.60	-0.00013	-0.00000	-0.00025	-0.00000	-0.00049	-0.00001
12.80	-0.00012	-0.00000	-0.00024	-0.00000	-0.00048	-0.00001
13.00	-0.00012	-0.00000	-0.00023	-0.00000	-0.00046	-0.00001
13.20	-0.00011	-0.00000	-0.00023	-0.00000	-0.00045	-0.00001
13.40	-0.00011	-0.00000	-0.00022	-0.00000	-0.00043	-0.00000
13.60	-0.00011	-0.00000	-0.00021	-0.00000	-0.00042	-0.00000
13.80	-0.00010	-0.00000	-0.00021	-0.00000	-0.00041	-0.00000
14.00	-0.00010	-0.00000	-0.00020	-0.00000	-0.00040	-0.00000
14.20	-0.00010	-0.00000	-0.00020	-0.00000	-0.00039	-0.00000
14.40	-0.00010	-0.00000	-0.00019	-0.00000	-0.00037	-0.00000
14.60	-0.00009	-0.00000	-0.00018	-0.00000	-0.00036	-0.00000
14.80	-0.00009	-0.00000	-0.00018	-0.00000	-0.00036	-0.00000
15.00	-0.00009	-0.00000	-0.00017	-0.00000	-0.00035	-0.00000

TABLE 2c-2 VARIATION OF C_w, S_o, S_1, A_o AND A_1 WITH DOWNSTREAM DISTANCE X(FT) FOR k_o = 2.50/FT

h (FT)	4.00000		8.00000		INFINITE	
C_w	0.00205		0.00205		0.00205	
S_o	0.00713		0.00713		0.00713	
S_1	0.00027		0.00027		0.00027	
X(FT)	A_o	A_1	A_o	A_1	A_o	A_1
6.00	-0.00350	-0.00181	-0.00363	-0.00202	-0.00049	-0.00187
6.20	-0.00332	-0.00156	-0.00351	-0.00176	-0.00045	-0.00163
6.40	-0.00315	-0.00135	-0.00341	-0.00154	-0.00042	-0.00142
6.60	-0.00299	-0.00116	-0.00330	-0.00134	-0.00039	-0.00124
6.80	-0.00284	-0.00100	-0.00321	-0.00117	-0.00037	-0.00108
7.00	-0.00271	-0.00087	-0.00311	-0.00102	-0.00035	-0.00094
7.20	-0.00258	-0.00075	-0.00302	-0.00089	-0.00032	-0.00083
7.40	-0.00246	-0.00065	-0.00293	-0.00078	-0.00031	-0.00072
7.60	-0.00235	-0.00056	-0.00285	-0.00068	-0.00029	-0.00063
7.80	-0.00225	-0.00049	-0.00277	-0.00059	-0.00027	-0.00055
8.00	-0.00215	-0.00042	-0.00269	-0.00052	-0.00026	-0.00048
8.20	-0.00205	-0.00036	-0.00261	-0.00046	-0.00024	-0.00042
8.40	-0.00197	-0.00032	-0.00254	-0.00040	-0.00023	-0.00037
8.60	-0.00189	-0.00027	-0.00247	-0.00035	-0.00022	-0.00032
8.80	-0.00181	-0.00024	-0.00240	-0.00030	-0.00021	-0.00028
9.00	-0.00174	-0.00021	-0.00234	-0.00027	-0.00020	-0.00025
9.20	-0.00167	-0.00018	-0.00227	-0.00023	-0.00019	-0.00022
9.40	-0.00160	-0.00015	-0.00221	-0.00020	-0.00018	-0.00019
9.60	-0.00154	-0.00013	-0.00215	-0.00018	-0.00017	-0.00017
9.80	-0.00149	-0.00012	-0.00210	-0.00016	-0.00017	-0.00015
10.00	-0.00143	-0.00010	-0.00204	-0.00014	-0.00016	-0.00013
10.20	-0.00138	-0.00009	-0.00199	-0.00012	-0.00015	-0.00011
10.40	-0.00133	-0.00008	-0.00193	-0.00010	-0.00015	-0.00010
10.60	-0.00128	-0.00007	-0.00189	-0.00009	-0.00014	-0.00009
10.80	-0.00124	-0.00006	-0.00184	-0.00008	-0.00014	-0.00007
11.00	-0.00120	-0.00005	-0.00179	-0.00007	-0.00013	-0.00007
11.20	-0.00116	-0.00004	-0.00175	-0.00006	-0.00013	-0.00006
11.40	-0.00112	-0.00004	-0.00170	-0.00005	-0.00012	-0.00005
11.60	-0.00108	-0.00003	-0.00166	-0.00005	-0.00012	-0.00004
11.80	-0.00105	-0.00003	-0.00162	-0.00004	-0.00011	-0.00004
12.00	-0.00102	-0.00003	-0.00158	-0.00004	-0.00011	-0.00003
12.20	-0.00099	-0.00002	-0.00154	-0.00003	-0.00011	-0.00003
12.40	-0.00096	-0.00002	-0.00150	-0.00003	-0.00010	-0.00003
12.60	-0.00093	-0.00002	-0.00147	-0.00002	-0.00010	-0.00002
12.80	-0.00090	-0.00001	-0.00143	-0.00002	-0.00010	-0.00002
13.00	-0.00087	-0.00001	-0.00140	-0.00002	-0.00009	-0.00002
13.20	-0.00085	-0.00001	-0.00137	-0.00002	-0.00009	-0.00002
13.40	-0.00082	-0.00001	-0.00133	-0.00001	-0.00009	-0.00001
13.60	-0.00080	-0.00001	-0.00130	-0.00001	-0.00008	-0.00001
13.80	-0.00078	-0.00001	-0.00128	-0.00001	-0.00008	-0.00001
14.00	-0.00076	-0.00001	-0.00125	-0.00001	-0.00008	-0.00001
14.20	-0.00074	-0.00000	-0.00122	-0.00001	-0.00008	-0.00001
14.40	-0.00072	-0.00000	-0.00119	-0.00001	-0.00008	-0.00001
14.60	-0.00070	-0.00000	-0.00116	-0.00001	-0.00007	-0.00001
14.80	-0.00068	-0.00000	-0.00114	-0.00001	-0.00007	-0.00001
15.00	-0.00066	-0.00000	-0.00112	-0.00000	-0.00007	-0.00000

TABLE 3a VARIATION OF C_w, S_o, S_1, A_o AND A_1 WITH SPEED (k_o) FOR h = 4.0FT

				X= 6.0FT		X= 9.0FT	
k_o (/FT)	C_w	S_o	S_1	A_o	A_1	A_o	A_1
1.20	0.01306	-0.10716	-0.08333	-0.00772	-0.00271	-0.00371	-0.00208
1.30	0.00696	-0.06502	-0.03469	-0.00709	-0.00262	-0.00342	-0.00028
1.40	0.00481	-0.02890	0.00732	-0.00655	-0.00254	-0.00317	-0.00027
1.50	0.00542	0.00057	0.04123	-0.00608	-0.00246	-0.00295	-0.00026
1.60	0.00737	0.02313	0.06627	-0.00567	-0.00238	-0.00276	-0.00026
1.70	0.00935	0.03888	0.08224	-0.00531	-0.00231	-0.00259	-0.00025
1.80	0.01047	0.04823	0.08951	-0.00499	-0.00224	-0.00244	-0.00024
1.90	0.01035	0.05185	0.08893	-0.00471	-0.00217	-0.00231	-0.00024
2.00	0.00909	0.05059	0.08172	-0.00445	-0.00210	-0.00219	-0.00023
2.10	0.00712	0.04545	0.06934	-0.00422	-0.00204	-0.00208	-0.00023
2.20	0.00502	0.03751	0.05342	-0.00402	-0.00197	-0.00198	-0.00022
2.30	0.00331	0.02782	0.03559	-0.00383	-0.00192	-0.00189	-0.00022
2.40	0.00230	0.01739	0.01741	-0.00366	-0.00186	-0.00181	-0.00021
2.50	0.00205	0.00713	0.00027	-0.00350	-0.00181	-0.00174	-0.00021
2.60	0.00239	-0.00220	-0.01468	-0.00335	-0.00176	-0.00167	-0.00020
2.70	0.00303	-0.01002	-0.02658	-0.00322	-0.00171	-0.00160	-0.00020
2.80	0.00363	-0.01593	-0.03491	-0.00309	-0.00166	-0.00155	-0.00019
2.90	0.00394	-0.01972	-0.03944	-0.00298	-0.00162	-0.00149	-0.00019
3.00	0.00384	-0.02138	-0.04027	-0.00287	-0.00158	-0.00144	-0.00018
3.10	0.00336	-0.02104	-0.03774	-0.00277	-0.00154	-0.00139	-0.00018
3.20	0.00267	-0.01898	-0.03242	-0.00268	-0.00150	-0.00134	-0.00017
3.30	0.00196	-0.01556	-0.02503	-0.00259	-0.00146	-0.00130	-0.00017
3.40	0.00142	-0.01123	-0.01636	-0.00251	-0.00143	-0.00126	-0.00017
3.50	0.00116	-0.00643	-0.00724	-0.00233	-0.00139	-0.00123	-0.00016
3.60	0.00118	-0.00161	0.00157	-0.00236	-0.00136	-0.00119	-0.00016
3.70	0.00142	0.00285	0.00938	-0.00229	-0.00133	-0.00116	-0.00016
3.80	0.00172	0.00660	0.01568	-0.00223	-0.00130	-0.00113	-0.00015
3.90	0.00196	0.00942	0.02008	-0.00217	-0.00127	-0.00110	-0.00015
4.00	0.00204	0.01117	0.02241	-0.00211	-0.00125	-0.00107	-0.00015
4.10	0.00193	0.01180	0.02266	-0.00206	-0.00122	-0.00104	-0.00015
4.20	0.00165	0.01136	0.02100	-0.00200	-0.00120	-0.00102	-0.00014
4.30	0.00131	0.01000	0.01773	-0.00196	-0.00117	-0.00099	-0.00014
4.40	0.00099	0.00790	0.01324	-0.00191	-0.00115	-0.00097	-0.00014
4.50	0.00078	0.00532	0.00801	-0.00186	-0.00113	-0.00095	-0.00014
4.60	0.00073	0.00249	0.00254	-0.00182	-0.00111	-0.00093	-0.00013
4.70	0.00080	-0.00031	-0.00272	-0.00178	-0.00108	-0.00091	-0.00013
4.80	0.00096	-0.00285	-0.00734	-0.00174	-0.00106	-0.00089	-0.00013
4.90	0.00112	-0.00495	-0.01098	-0.00170	-0.00104	-0.00087	-0.00013
5.00	0.00122	-0.00646	-0.01342	-0.00167	-0.00103	-0.00085	-0.00013
5.10	0.00122	-0.00730	-0.01454	-0.00163	-0.00101	-0.00083	-0.00012
5.20	0.00112	-0.00744	-0.01435	-0.00160	-0.00099	-0.00082	-0.00012
5.30	0.00094	-0.00694	-0.01295	-0.00157	-0.00098	-0.00080	-0.00012
5.40	0.00074	-0.00588	-0.01055	-0.00154	-0.00096	-0.00079	-0.00012
5.50	0.00059	-0.00438	-0.00742	-0.00151	-0.00094	-0.00077	-0.00012
5.60	0.00051	-0.00261	-0.00387	-0.00148	-0.00093	-0.00076	-0.00011
5.70	0.00052	-0.00074	-0.00022	-0.00145	-0.00091	-0.00074	-0.00011
5.80	0.00060	0.00106	0.00320	-0.00143	-0.00090	-0.00073	-0.00011
5.90	0.00070	0.00266	0.00613	-0.00140	-0.00089	-0.00072	-0.00011
6.00	0.00079	0.00392	0.00835	-0.00138	-0.00087	-0.00071	-0.00011

TABLE 3b VARIATION OF C_w, S_o, S_1, A_o AND A_1 WITH SPEED (k_o) FOR h = 8.0FT

				X= 6.0FT		X= 9.0FT	
k_o (/FT)	C_w	S_o	S_1	A_o	A_1	A_o	A_1
1.20	0.01320	-0.10804	-0.08356	-0.00854	-0.00316	-0.00522	-0.00039
1.30	0.00699	-0.06538	-0.03475	-0.00776	-0.00305	-0.00477	-0.00038
1.40	0.00481	-0.02900	0.00733	-0.00711	-0.00294	-0.00440	-0.00037
1.50	0.00542	0.00057	0.04127	-0.00655	-0.00283	-0.00408	-0.00036
1.60	0.00537	0.02317	0.06631	-0.00607	-0.00273	-0.00379	-0.00034
1.70	0.00935	0.03893	0.08228	-0.00566	-0.00263	-0.00355	-0.00033
1.80	0.01048	0.04827	0.08954	-0.00529	-0.00254	-0.00333	-0.00032
1.90	0.01035	0.05187	0.08895	-0.00497	-0.00245	-0.00314	-0.00032
2.00	0.00909	0.05060	0.08173	-0.00468	-0.00237	-0.00297	-0.00031
2.10	0.00712	0.04546	0.06935	-0.00443	-0.00229	-0.00282	-0.00030
2.20	0.00502	0.03751	0.05342	-0.00420	-0.00222	-0.00268	-0.00029
2.30	0.00331	0.02782	0.03559	-0.00399	-0.00215	-0.00256	-0.00028
2.40	0.00230	0.01739	0.01741	-0.00380	-0.00208	-0.00244	-0.00027
2.50	0.00205	0.00713	0.00027	-0.00363	-0.00202	-0.00234	-0.00027
2.60	0.00239	-0.00220	-0.01468	-0.00347	-0.00196	-0.00224	-0.00026
2.70	0.00303	-0.01002	-0.02658	-0.00332	-0.00191	-0.00215	-0.00025
2.80	0.00363	-0.01593	-0.03491	-0.00319	-0.00185	-0.00207	-0.00025
2.90	0.00394	-0.01972	-0.03944	-0.00307	-0.00180	-0.00199	-0.00024
3.00	0.00384	-0.02138	-0.04027	-0.00295	-0.00175	-0.00192	-0.00023
3.10	0.00336	-0.02104	-0.03774	-0.00285	-0.00171	-0.00186	-0.00023
3.20	0.00267	-0.01898	-0.03242	-0.00275	-0.00166	-0.00180	-0.00022
3.30	0.00196	-0.01556	-0.02503	-0.00265	-0.00162	-0.00174	-0.00022
3.40	0.00142	-0.01123	-0.01636	-0.00257	-0.00158	-0.00168	-0.00021
3.50	0.00116	-0.00643	-0.00724	-0.00249	-0.00154	-0.00163	-0.00021
3.60	0.00118	-0.00161	0.00157	-0.00241	-0.00151	-0.00159	-0.00021
3.70	0.00142	0.00285	0.00938	-0.00234	-0.00147	-0.00154	-0.00020
3.80	0.00172	0.00660	0.01568	-0.00227	-0.00144	-0.00150	-0.00020
3.90	0.00196	0.00942	0.02008	-0.00221	-0.00141	-0.00146	-0.00019
4.00	0.00204	0.01117	0.02241	-0.00215	-0.00138	-0.00142	-0.00019
4.10	0.00193	0.01180	0.02266	-0.00209	-0.00135	-0.00138	-0.00019
4.20	0.00165	0.01136	0.02100	-0.00204	-0.00132	-0.00135	-0.00018
4.30	0.00131	0.01000	0.01773	-0.00198	-0.00129	-0.00132	-0.00018
4.40	0.00099	0.00790	0.01324	-0.00193	-0.00127	-0.00128	-0.00017
4.50	0.00078	0.00532	0.00801	-0.00189	-0.00124	-0.00125	-0.00017
4.60	0.00073	0.00249	0.00254	-0.00184	-0.00122	-0.00122	-0.00017
4.70	0.00080	-0.00031	-0.00272	-0.00180	-0.00120	-0.00120	-0.00017
4.80	0.00096	-0.00285	-0.00734	-0.00176	-0.00117	-0.00117	-0.00016
4.90	0.00112	-0.00495	-0.01098	-0.00172	-0.00115	-0.00115	-0.00016
5.00	0.00122	-0.00646	-0.01342	-0.00168	-0.00113	-0.00112	-0.00016
5.10	0.00122	-0.00730	-0.01454	-0.00165	-0.00111	-0.00110	-0.00015
5.20	0.00112	-0.00744	-0.01435	-0.00161	-0.00109	-0.00108	-0.00015
5.30	0.00094	-0.00694	-0.01295	-0.00158	-0.00108	-0.00106	-0.00015
5.40	0.00074	-0.00588	-0.01055	-0.00155	-0.00106	-0.00104	-0.00015
5.50	0.00059	-0.00438	-0.00742	-0.00152	-0.00104	-0.00102	-0.00015
5.60	0.00051	-0.00261	-0.00387	-0.00149	-0.00102	-0.00100	-0.00014
5.70	0.00052	-0.00074	-0.00022	-0.00146	-0.00101	-0.00098	-0.00014
5.80	0.00060	0.00106	0.00320	-0.00144	-0.00099	-0.00096	-0.00014
5.90	0.00070	0.00266	0.00613	-0.00141	-0.00098	-0.00095	-0.00014
6.00	0.00079	0.00392	0.00835	-0.00138	-0.00096	-0.00093	-0.00014

TABLE 4 VARIATION OF C_w, S_o, S_1, A_o AND A_1 WITH STRUT DRAFT h(FT)
FOR k_o =1.40/FT

h (FT)	C_w	S_o	S_1	X= 6.0FT		X= 8.0FT	
				A_o	A_1	A_o	A_1
0.50	0.00256	-0.01460	0.00410	-0.00105	-0.00035	-0.00057	-0.00007
1.00	0.00397	-0.02185	0.00591	-0.00212	-0.00075	-0.00113	-0.00014
2.00	0.00468	-0.02724	0.00705	-0.00408	-0.00155	-0.00223	-0.00031
4.00	0.00481	-0.02890	0.00731	-0.00655	-0.00254	-0.00394	-0.00056
6.00	0.00481	-0.02900	0.00733	-0.00726	-0.00286	-0.00483	-0.00069
8.00	0.00481	-0.02900	0.00733	-0.00711	-0.00294	-0.00511	-0.00073
10.00	0.00481	-0.02900	0.00733	-0.00668	-0.00295	-0.00506	-0.00074
12.00	0.00481	-0.02900	0.00733	-0.00619	-0.00296	-0.00485	-0.00074
14.00	0.00481	-0.02900	0.00733	-0.00574	-0.00296	-0.00458	-0.00074
16.00	0.00481	-0.02900	0.00733	-0.00534	-0.00296	-0.00431	-0.00074
18.00	0.00481	-0.02900	0.00733	-0.00499	-0.00296	-0.00406	-0.00074
20.00	0.00481	-0.02900	0.00733	-0.00470	-0.00296	-0.00382	-0.00074
22.00	0.00481	-0.02900	0.00733	-0.00444	-0.00296	-0.00361	-0.00074
24.00	0.00481	-0.02900	0.00733	-0.00422	-0.00296	-0.00343	-0.00074

h(FT)	C_w	X=10.0FT		X=12.0FT		X=15.0FT	
		A_o	A_1	A_o	A_1	A_o	A_1
0.50	0.00256	-0.00035	-0.00001	-0.00024	-0.00000	-0.00015	-0.00000
1.00	0.00397	-0.00071	-0.00003	-0.00049	-0.00001	-0.00031	-0.00000
2.00	0.00468	-0.00141	-0.00007	-0.00097	-0.00001	-0.00062	-0.00000
4.00	0.00481	-0.00260	-0.00013	-0.00184	-0.00003	-0.00119	-0.00000
6.00	0.00481	-0.00340	-0.00017	-0.00249	-0.00004	-0.00167	-0.00000
8.00	0.00481	-0.00381	-0.00019	-0.00291	-0.00005	-0.00204	-0.00001
10.00	0.00481	-0.00395	-0.00019	-0.00313	-0.00005	-0.00228	-0.00001
12.00	0.00481	-0.00392	-0.00019	-0.00321	-0.00005	-0.00243	-0.00001
14.00	0.00481	-0.00380	-0.00019	-0.00320	-0.00005	-0.00250	-0.00001
16.00	0.00481	-0.00364	-0.00019	-0.00313	-0.00005	-0.00251	-0.00001
18.00	0.00481	-0.00347	-0.00019	-0.00303	-0.00005	-0.00249	-0.00001
20.00	0.00481	-0.00330	-0.00019	-0.00291	-0.00005	-0.00244	-0.00001
22.00	0.00481	-0.00314	-0.00019	-0.00279	-0.00005	-0.00237	-0.00001
24.00	0.00481	-0.00299	-0.00019	-0.00267	-0.00005	-0.00230	-0.00001

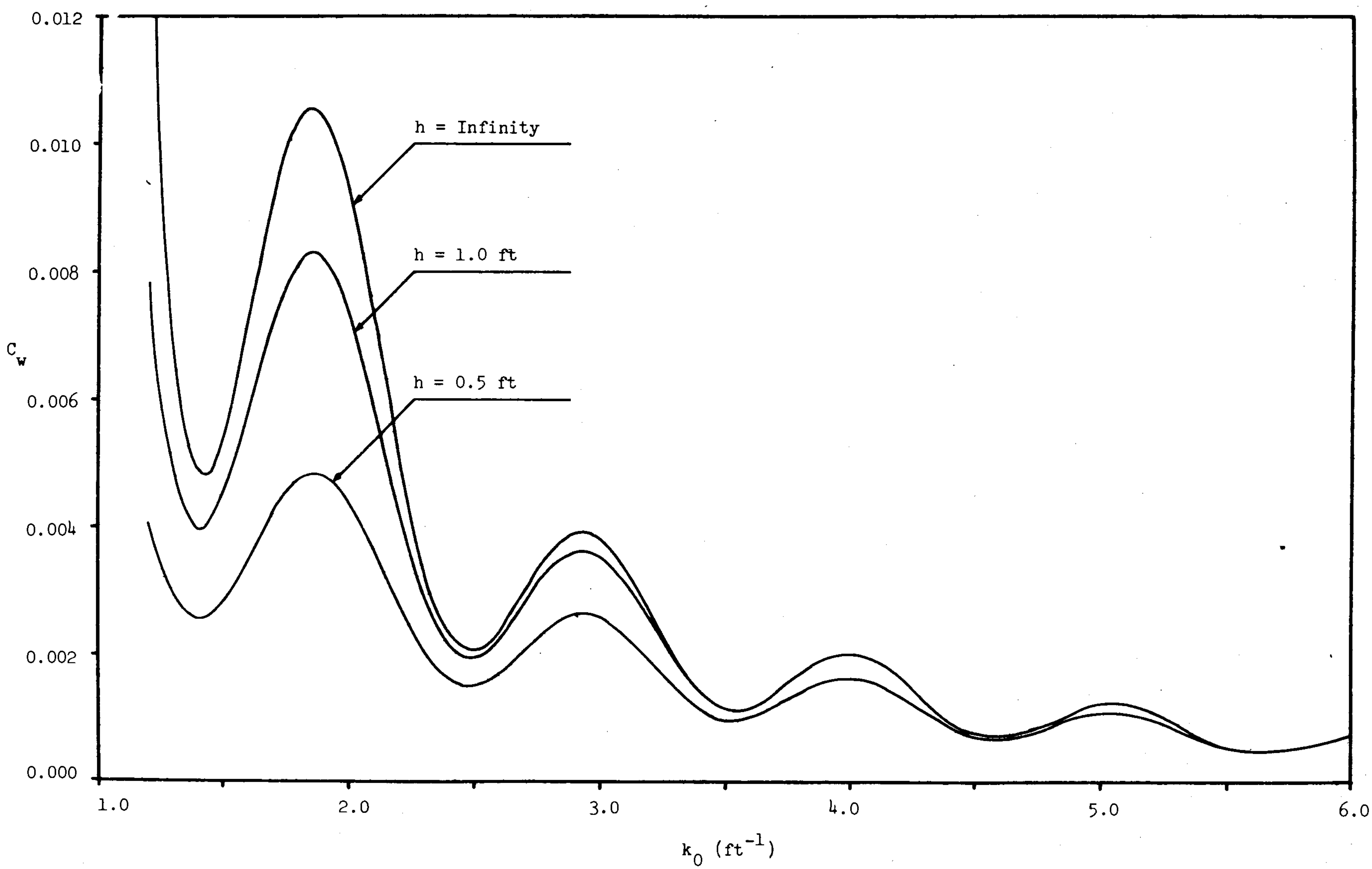


Fig. 1. Wave-Resistance Coefficient C_w versus k_0 for Struts of Various Drafts.

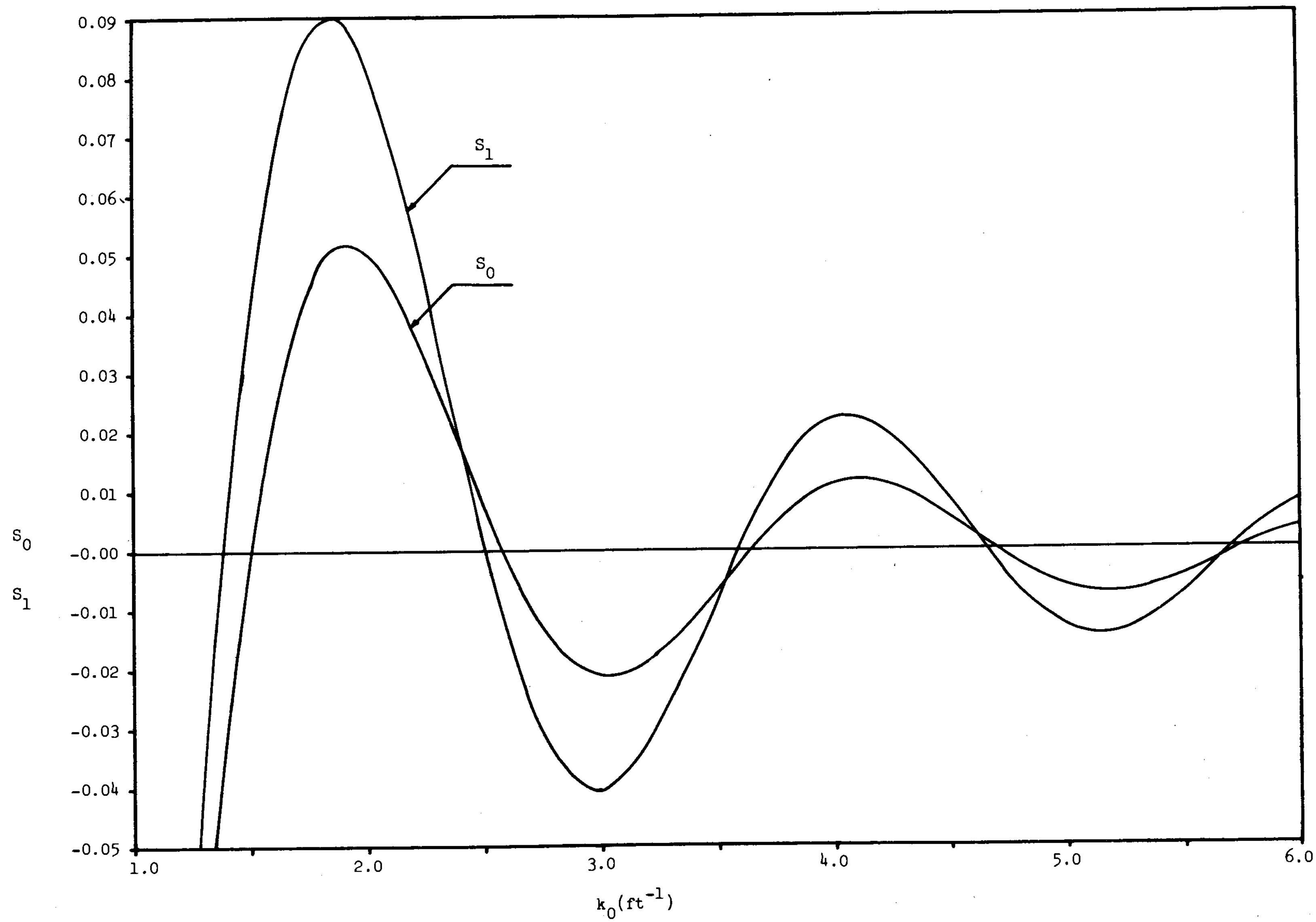


Fig. 2. Variation of Far-Field Coefficients S_0 and S_1 with k_0 for Struts of Large Draft ($h \geq 4$ ft)

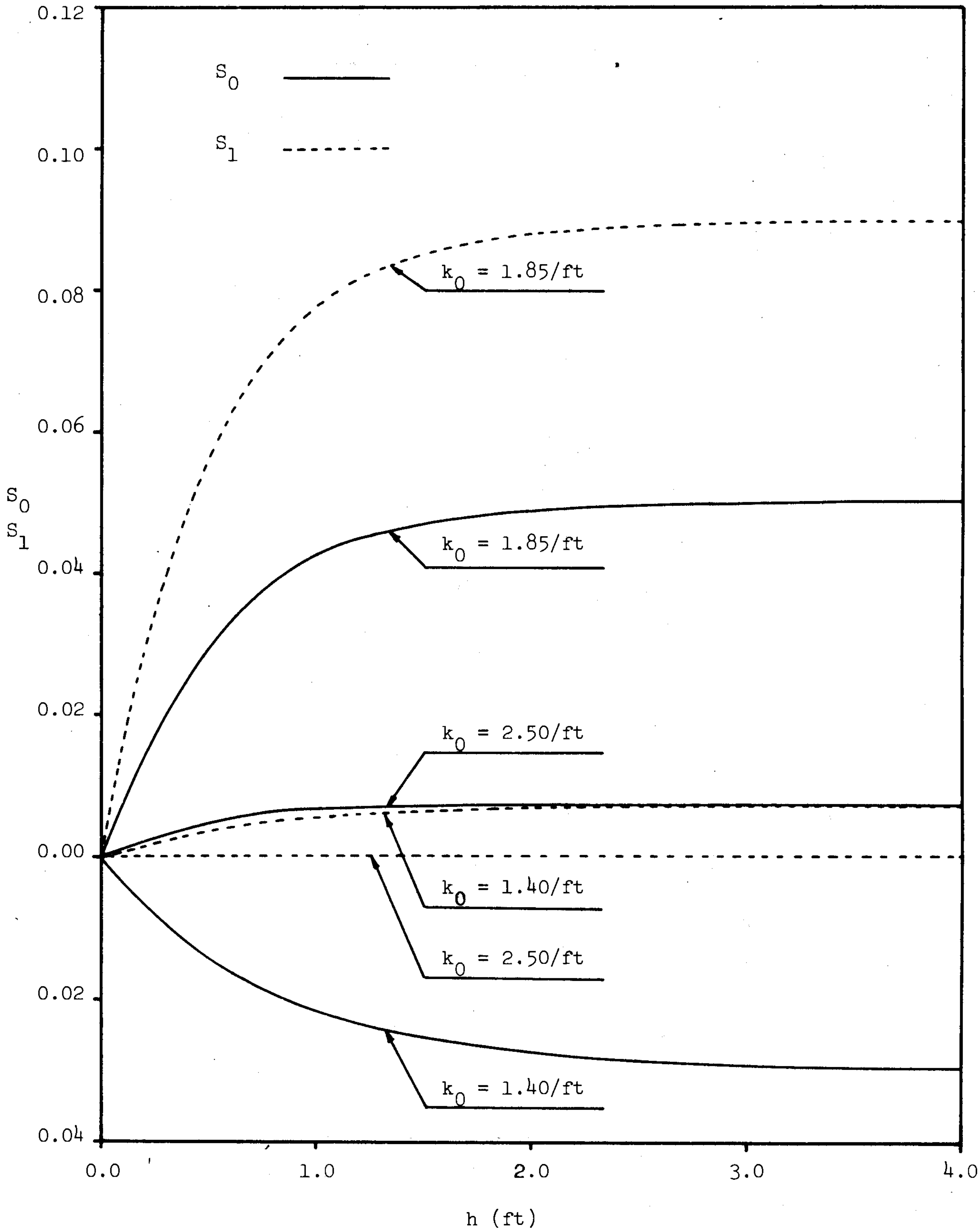


Fig. 3. Variation of Far-Field Coefficients S_0, S_1 with Draft h for Various Values of k_0

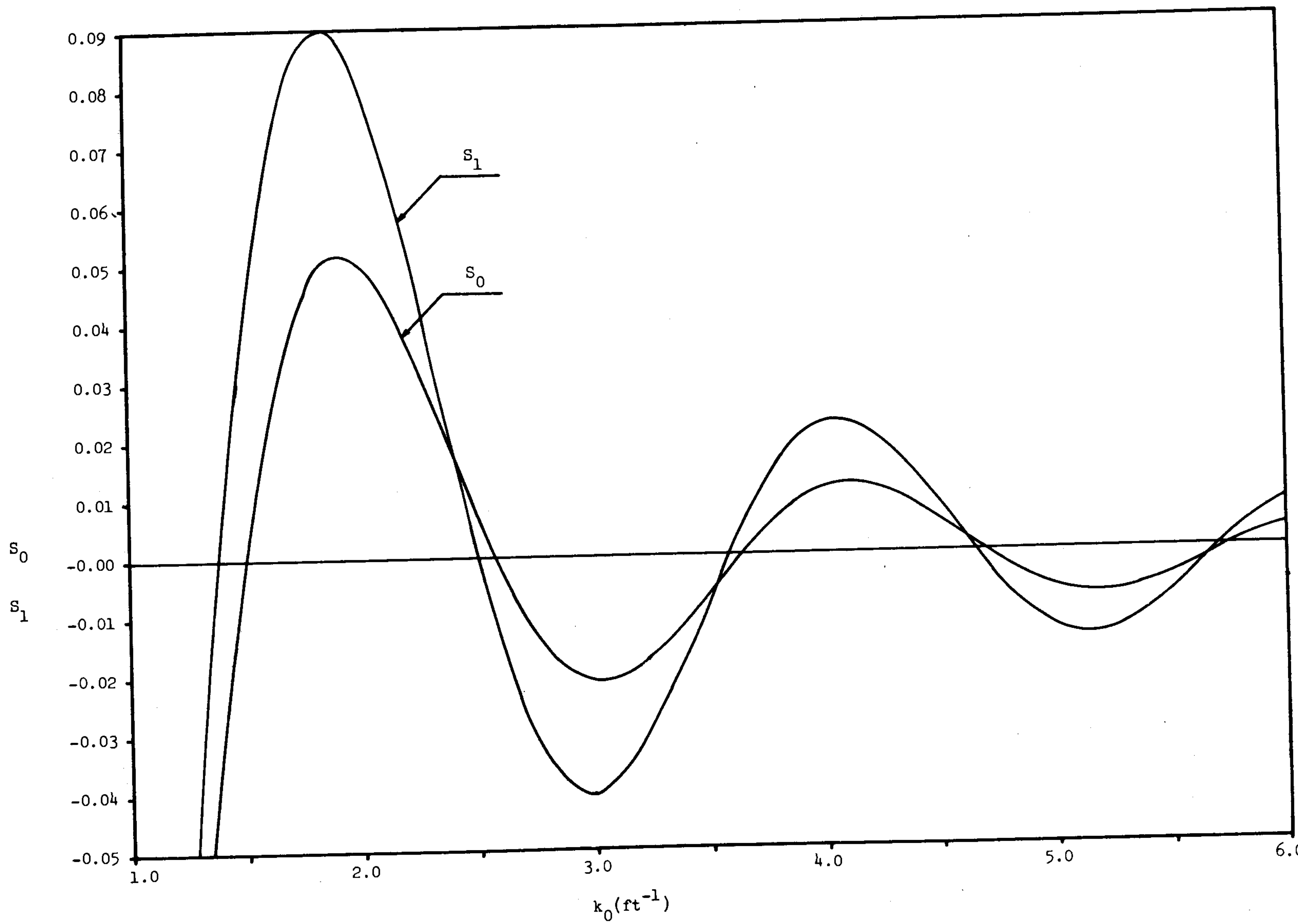


Fig. 2. Variation of Far-Field Coefficients S_0 and S_1 with k_0 for Struts of Large Draft ($h \geq 4$ ft)

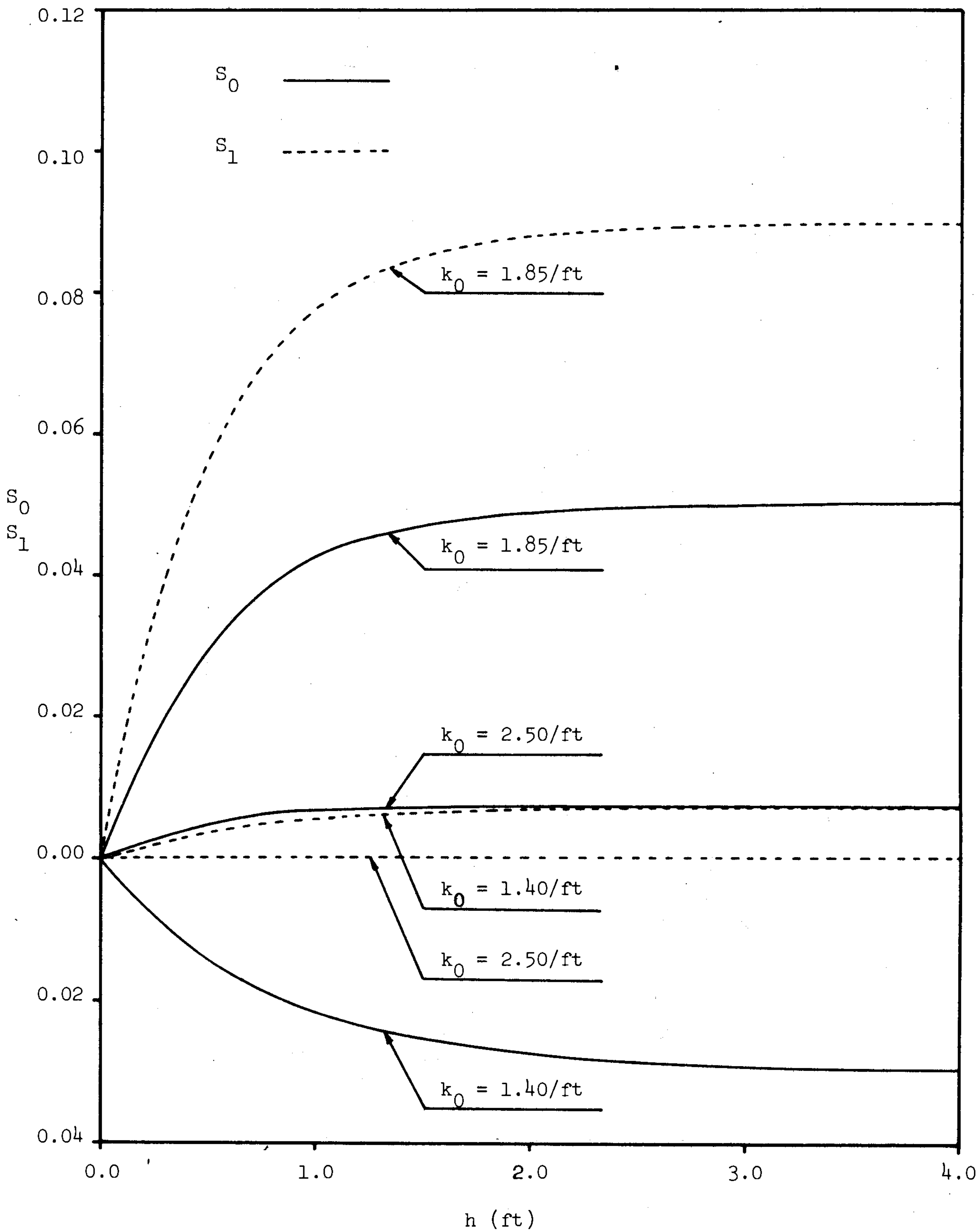
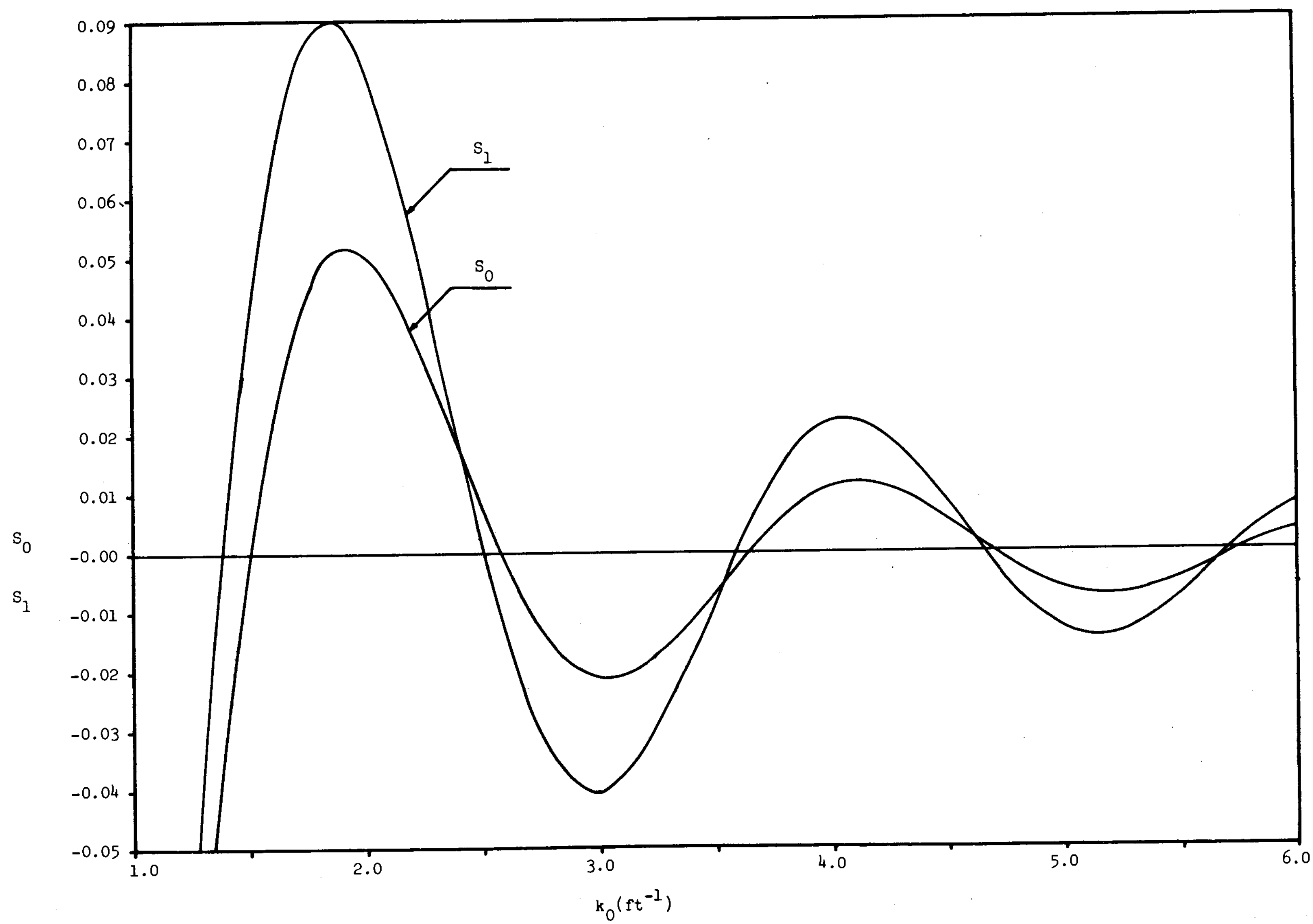


Fig. 3. Variation of Far-Field Coefficients S_0, S_1 with Draft h for Various Values of k_0



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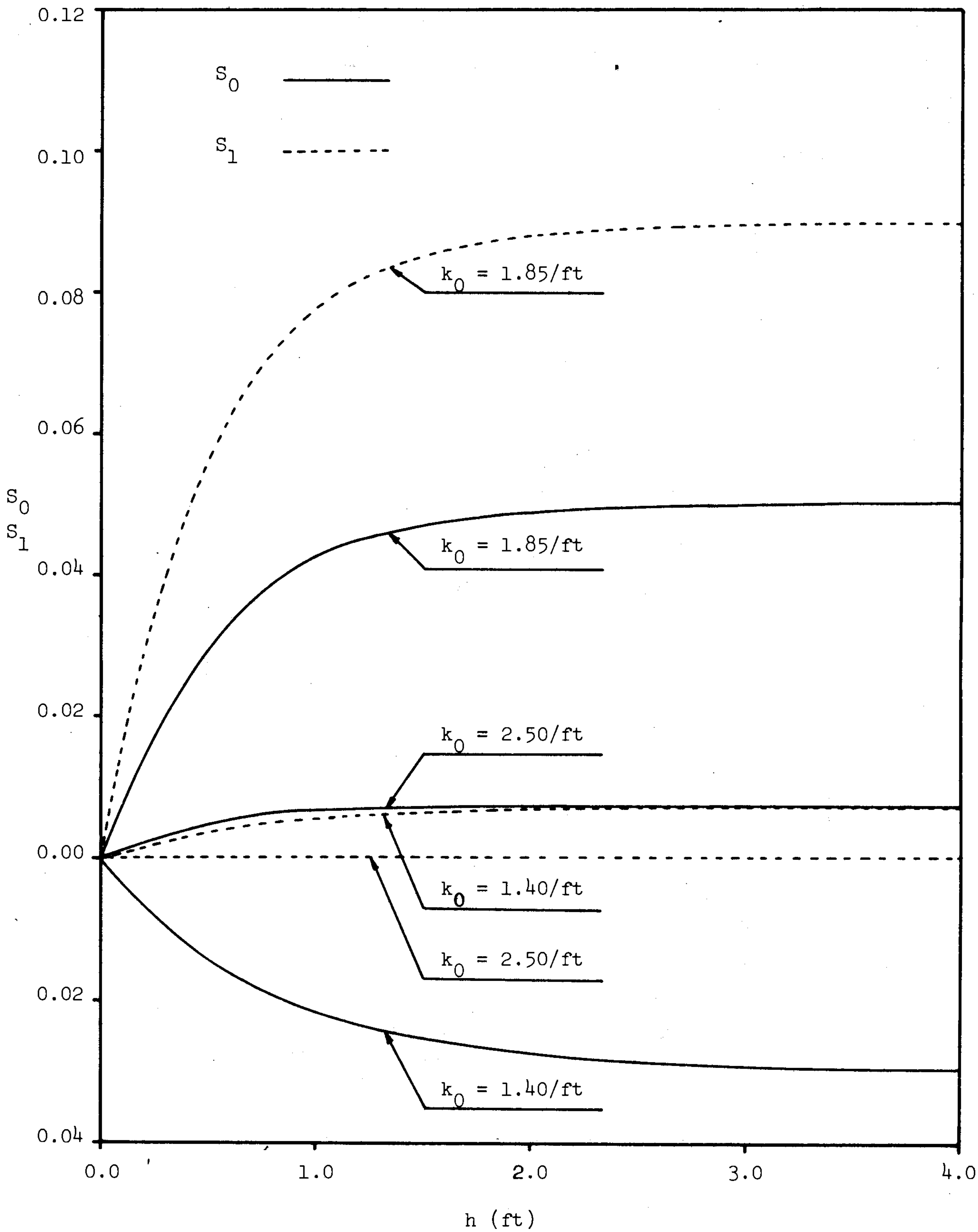

Fig. 3. Variation of Far-Field Coefficients S_0, S_1 with Draft h for Various Values of k_0

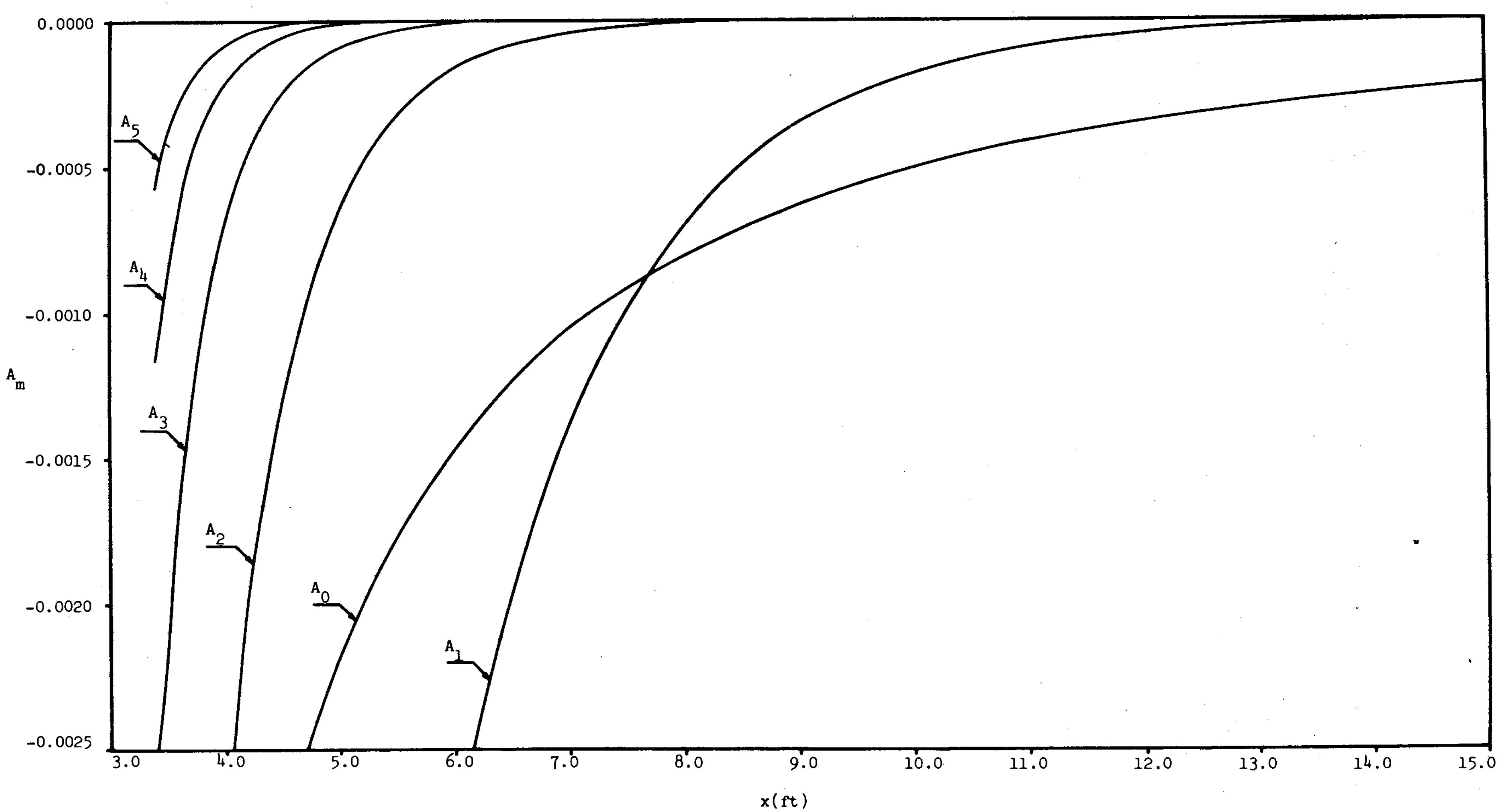

Fig. 4. Variation of A_m with Downstream Distance x for a Strut of Infinite Draft, with $k_0 = 1.40/\text{ft}$.

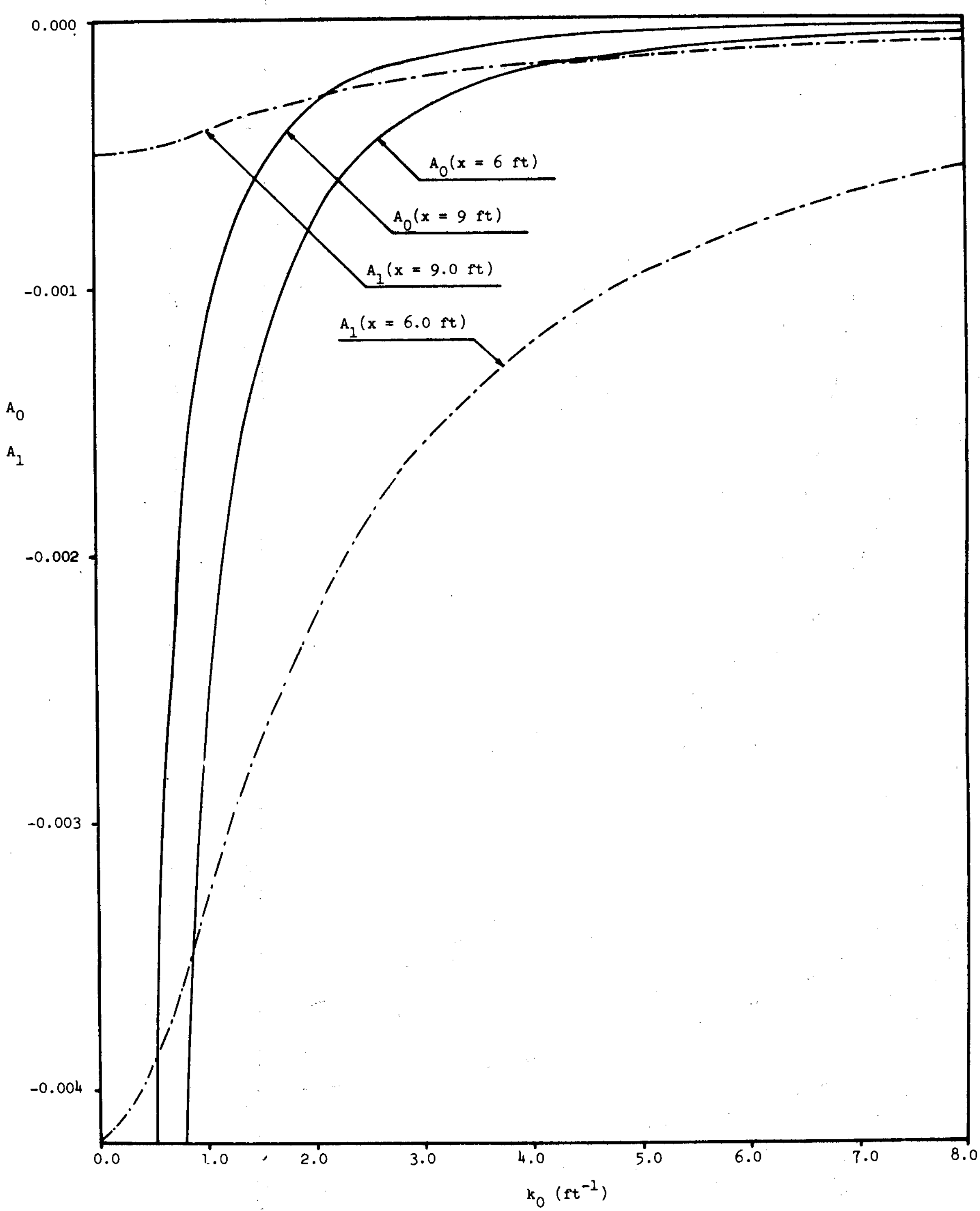

Fig. 5. Variation of Near-Field Coefficients A_0 and A_1 with k_0 for x = 6 and 9 ft

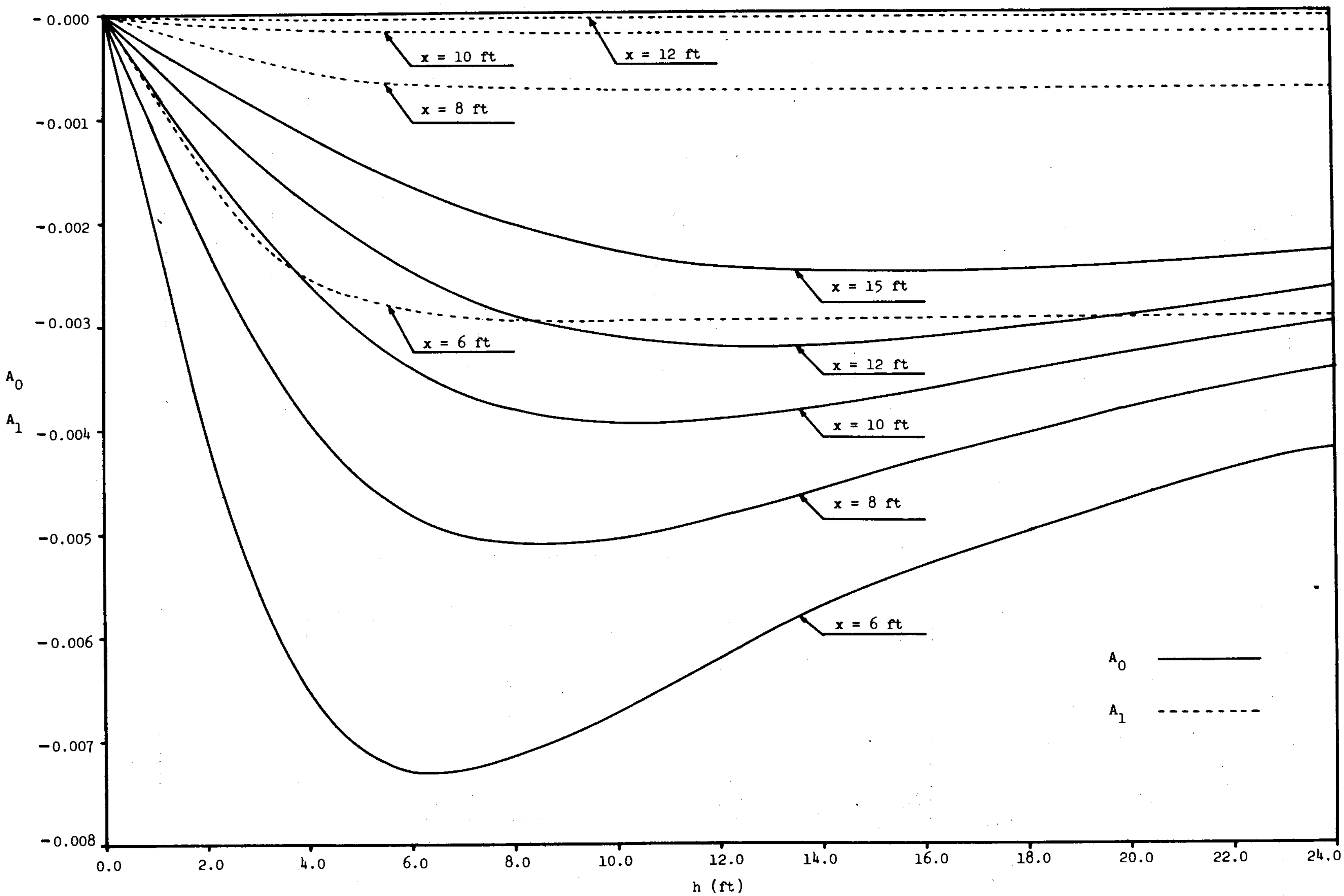

Fig. 6. Variation of Near-Field Coefficients A_0 and A_1 with Draft at Several Values of x for $k_0 = 1.40/ft$

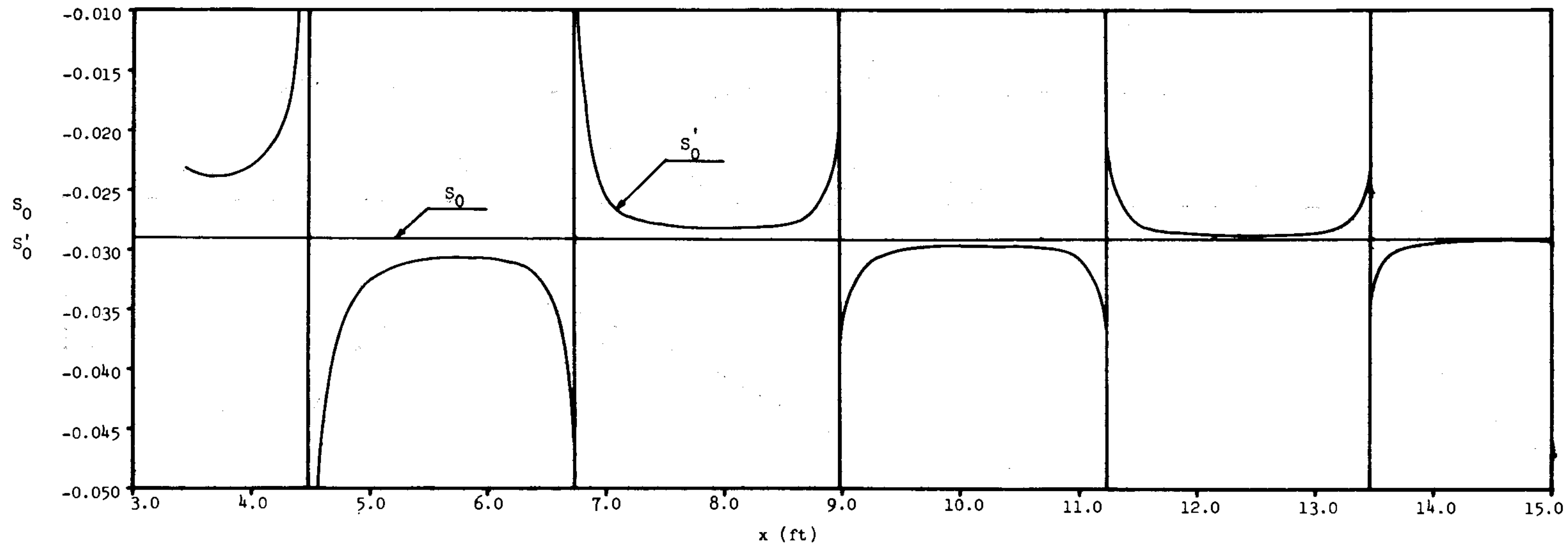

Fig. 7a. Variation of S_0' with downstream distance x for a Strut of Infinite Draft with k_0 = 1.40/ft

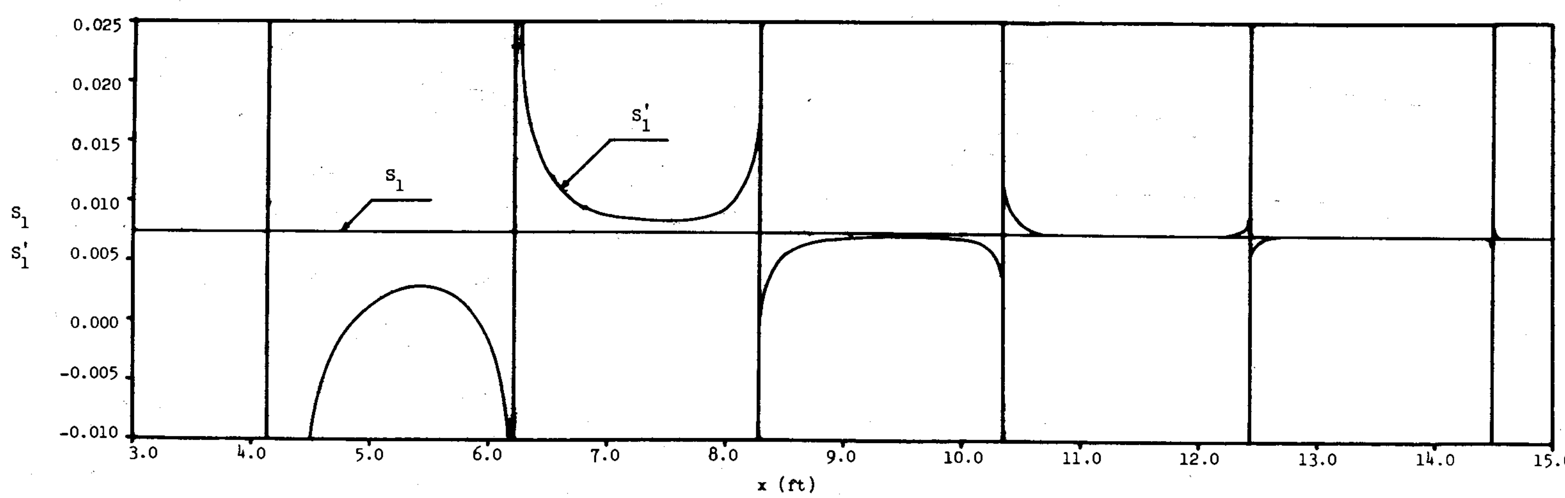

Fig. 7b. Variation of S_1' with downstream distance x for a Strut of Infinite Draft with k_0 = 1.40/ft

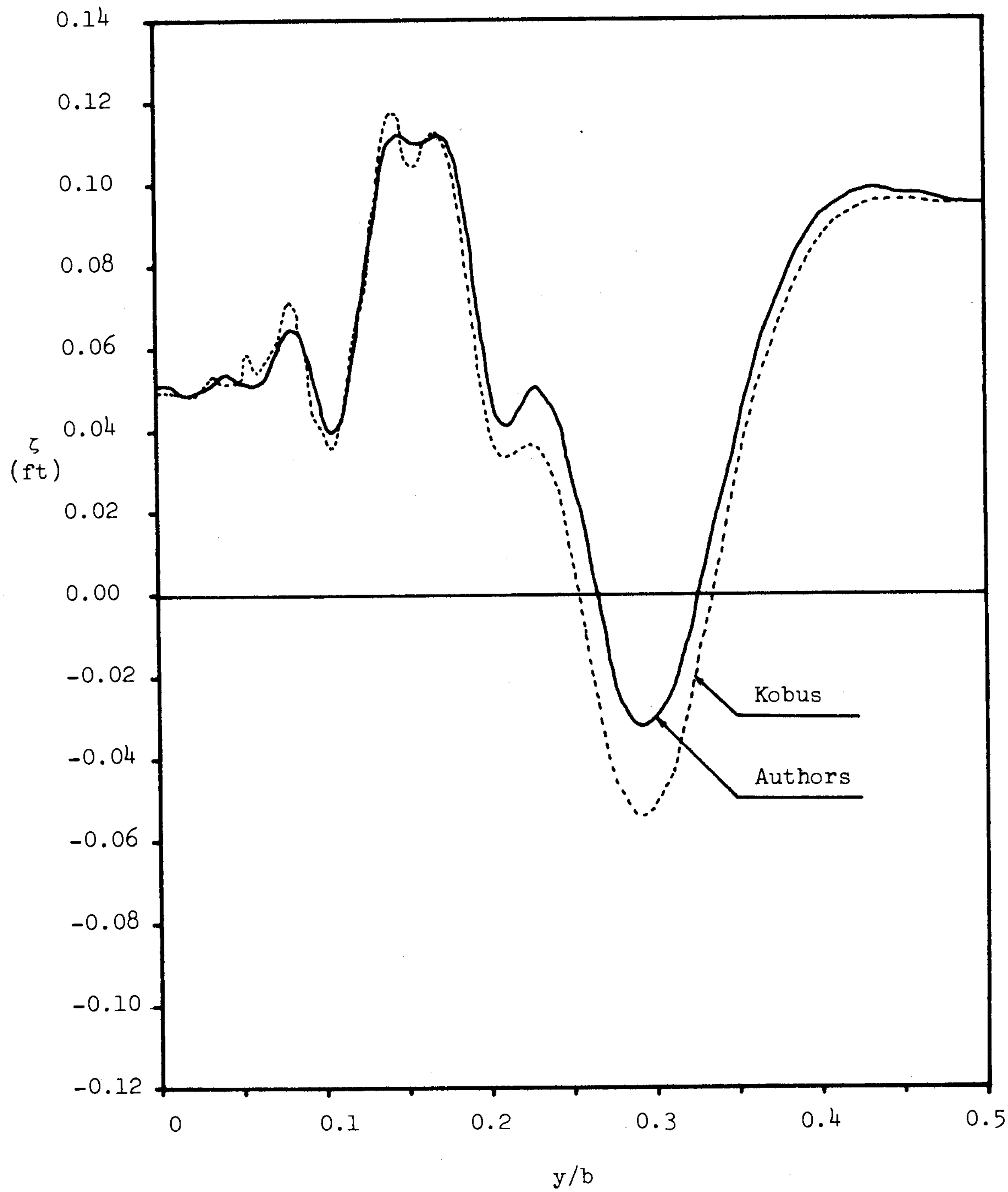

Fig. 8. Transverse Surface Profile at One Model Length behind Strut - Comparison with Profile for Modified Ogive of Kobus.

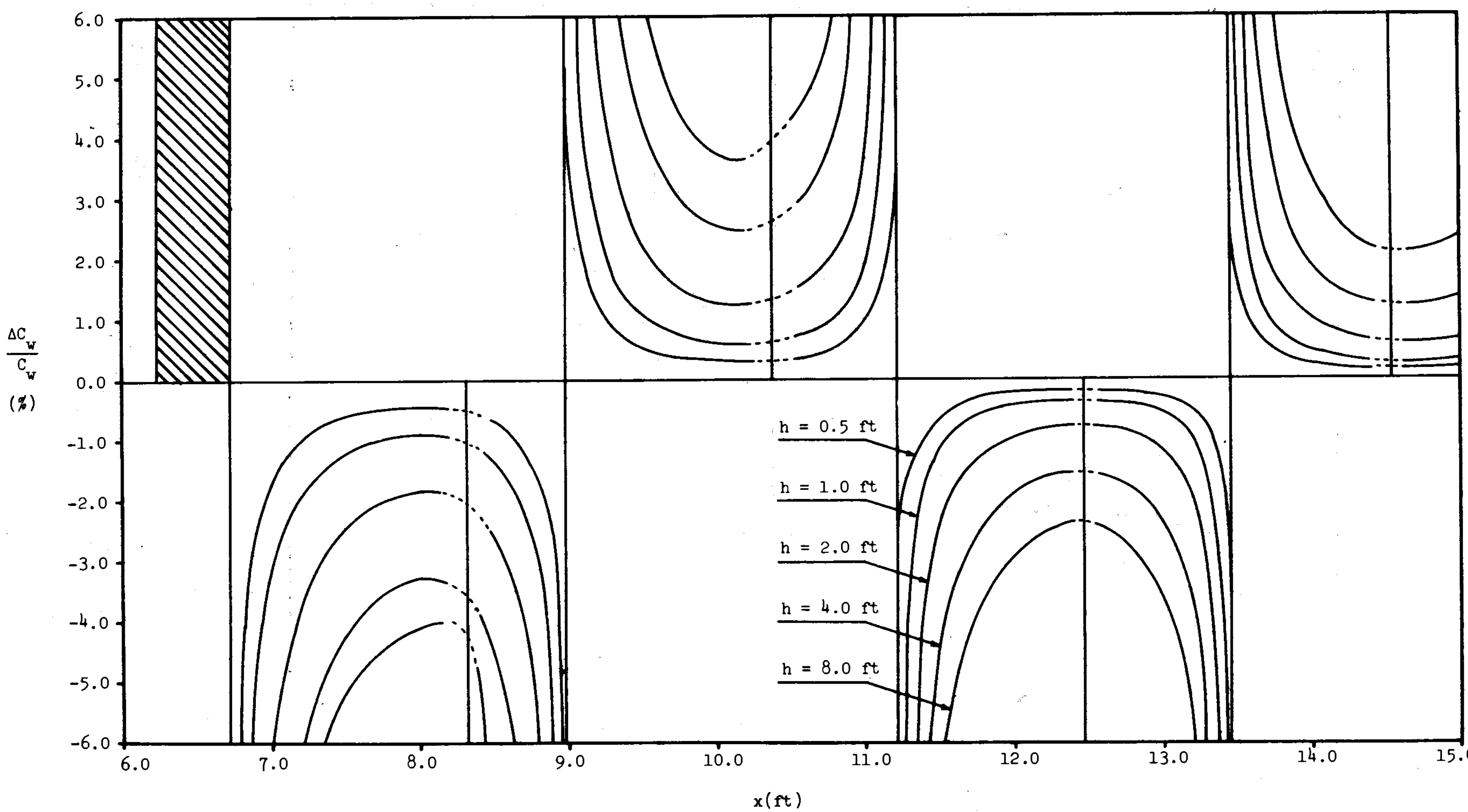

Fig. 9a. Variation of Error in Eggers' Method with Downstream Distance x for $k_0 = 1.40/\text{ft}$

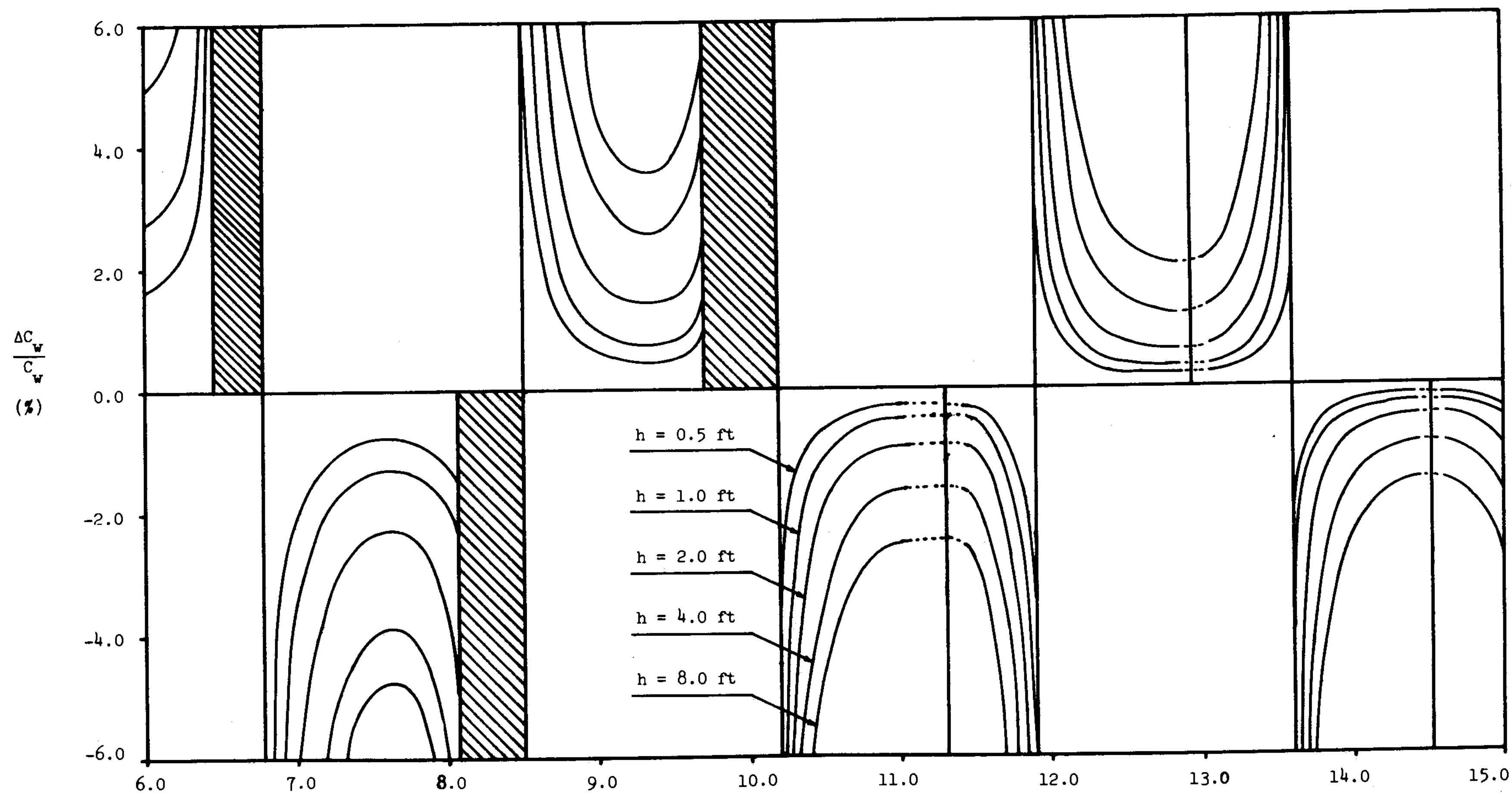

Fig. 9b. Variation of Error in Eggers' Method with Downstream Distance x for $k_0 = 1.85/ft$

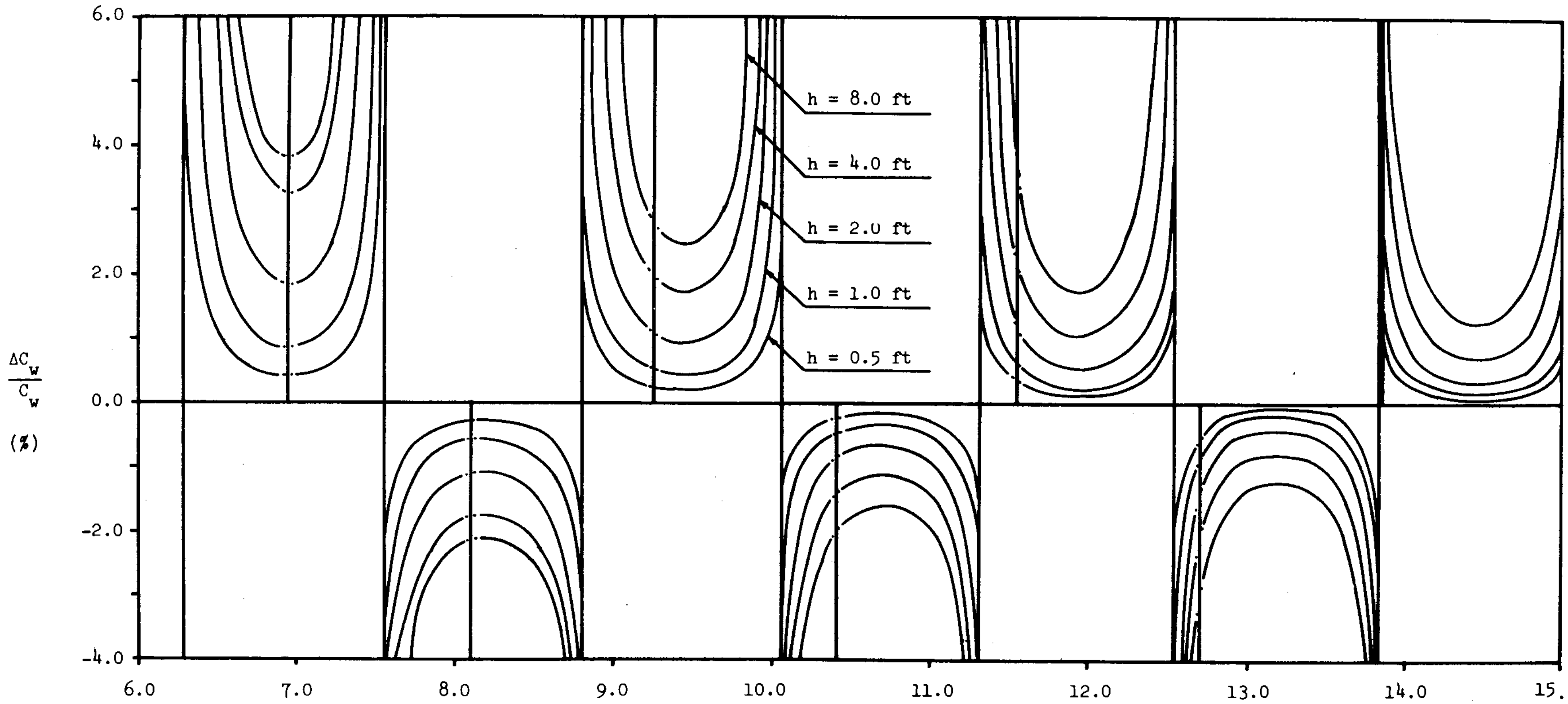

Fig. 9c. Variation of Error in Eggers' Method with Downstream Distance x for $k_0 = 2.5/\text{ft}$

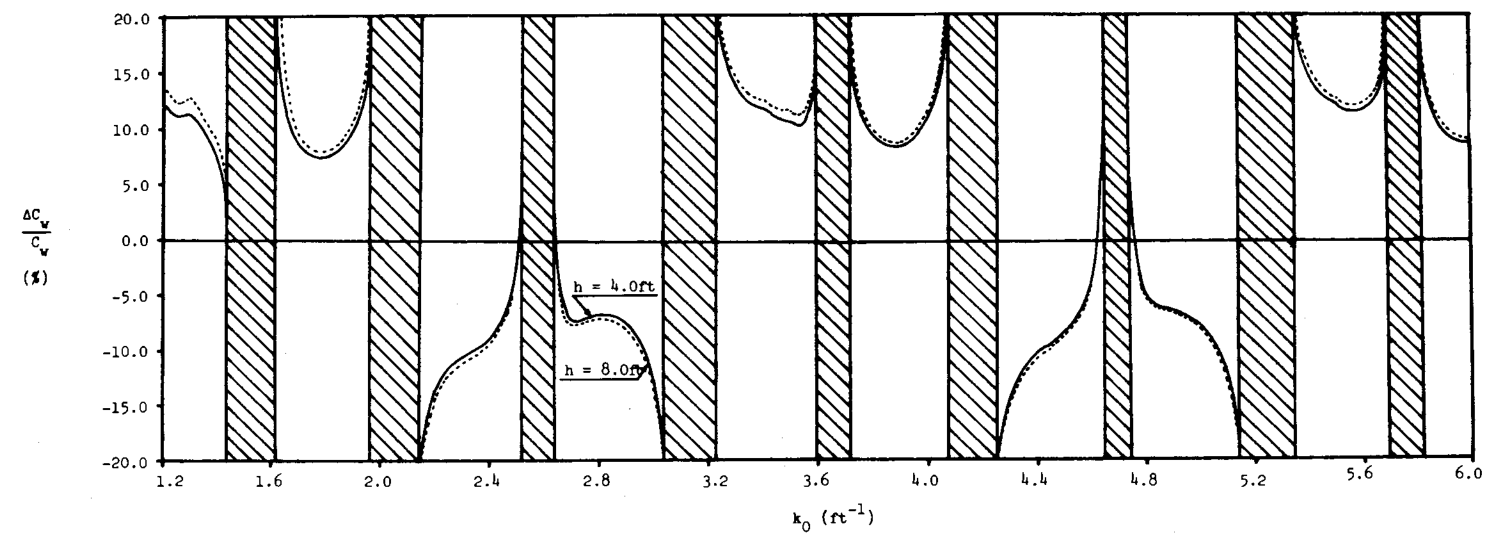

Fig. 10a. Variation of Error in Eggers' Method with $k_0 = g/U^2$ for x = 6.0 ft

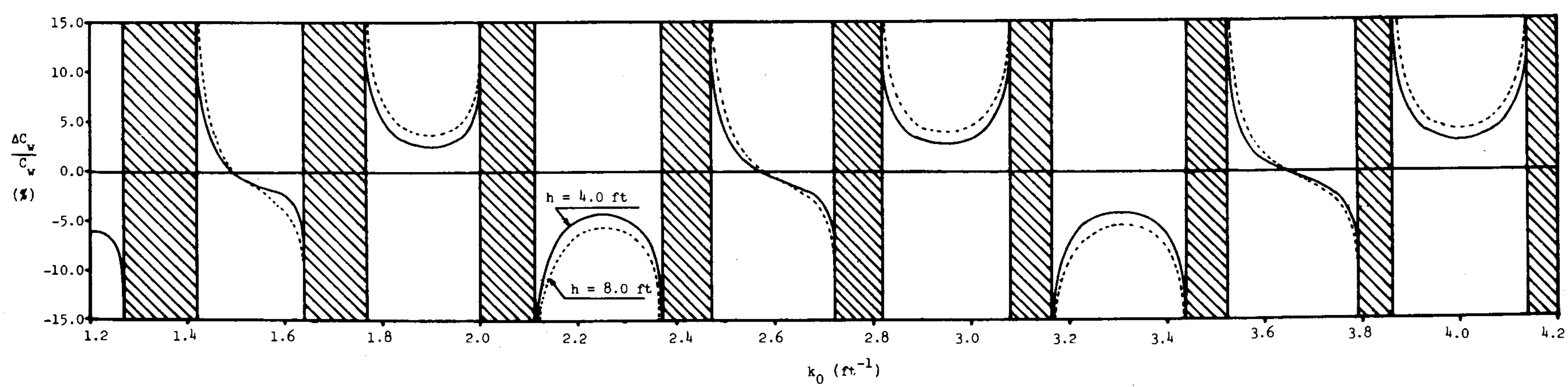

Fig. 10b. Variation of Error in Eggers' Method with $k_0 = g/U^2$ for x = 9.0 ft

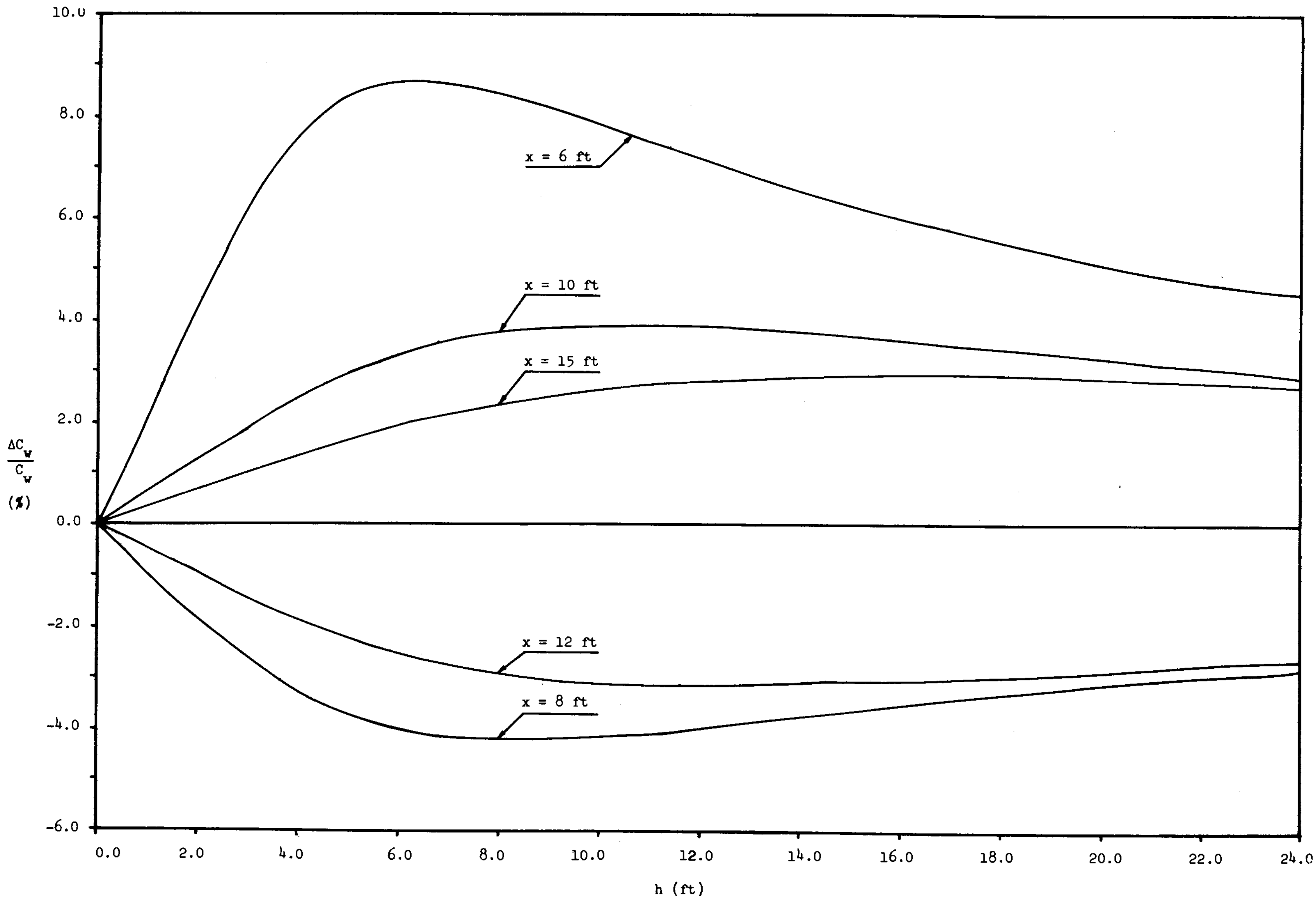

Fig. 11. Variation of Error in Eggers' Method with Strut Draft h for $k_0 = 1.40/ft$

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13. ABSTRACT

One of the assumptions in Eggers' method for the determination of the wave resistance of a ship form from transverse surface-profile measurements is that a "near-field" term in the expression for the surface disturbance may be neglected. Application to a family of vertical struts, with drafts varying from shiplike dimensions to infinity, indicates that, by measuring the transverse profiles at many sections at least one model length downstream, the error due to this assumption can be reduced to less than one percent for forms of shiplike draft, but may be as large as 5 percent for drafts of the order of magnitude of the model length. A procedure for reducing these errors further is suggested.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Ship resistance						
Wave resistance						
Surface waves						