

Script of the Film
 CHARACTERISTICS OF LAMINAR AND TURBULENT FLOW

by

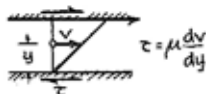
Hunter Rouse

Institute of Hydraulic Research
 University of Iowa
 Iowa City

Beakers of heavy oil,
 light oil, water,
 and mercury, with
 stirring rods

When we speak of heavy oils and light oils, we are really referring to their viscosity or resistance to shear. Now the tangential stress of shear is also confused with the normal stress of adhesion, which causes wetting. As we shall soon see, the fact that mercury does not wet the glass stirring rod does not mean that it has no viscosity. Whether a fluid wets a boundary or not, the interaction of the respective molecules is such that there can be no slip either between the fluid and the boundary or between different parts of the fluid. Thus, if one boundary is moved parallel to another, a viscous fluid contained between them will undergo a continuous shear or angular deformation as delineated by the velocity gradient. This is known as Couette flow. If we were to measure the force per unit area τ required to move the one boundary or to hold the other stationary, its ratio to the rate of deformation would be a direct indication of the dynamic viscosity μ of the fluid. Lines of hydrogen bubbles produced along a fine wire by periodic electrolysis of a glycerine-water solution show the typical Couette velocity distribution.

Animation



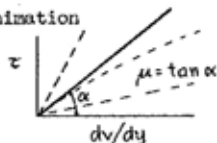
Parallel wall displaced, lines of hydrogen bubbles showing velocity distribution

Rotating-cylinder viscosimeter

A device using the Couette principle for the measurement of viscosity is formed by two nested cylinders so closely spaced that their curvature can be neglected. By moving the outer cylinder at any desired speed and by measuring the force on the inner one through the torque on the suspension, the viscosity can be determined. Here the fluid is water, and the prevailing speed produces a certain deflection of the inner cylinder. Now the fluid is mercury, which is seen to be more viscous than water, in that even a lower speed would still produce a greater deflection. Finally, the fluid is simply the air of the atmosphere, which is also seen to be viscous, though very much less so than either water or mercury.

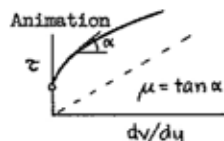
operates with
 water,
 mercury,
 and air

Animation

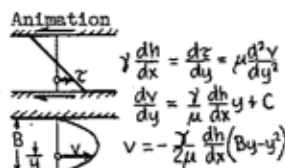


Paint brush dipped in
 can and stroked on
 wall

If the measurements of shearing stress and rate of angular deformation were plotted against each other, for what is known as a Newtonian fluid (such as water, mercury, or air) all points would fall on a straight line, the constant slope of which would represent constancy of the viscosity. A line with a steeper slope would thus indicate a fluid with a higher viscosity, and a line with a less-steep slope one with a lower viscosity. A non-Newtonian fluid, on the contrary, is one for which the plotted points follow a curve rather than a straight line. A good example is a liquid suspension



Flow between stationary walls, hydrogen bubbles showing velocity distribution



Animation $\tau = \mu \frac{dv}{dy}$

$$\tau_{xy} = \mu \left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right)$$

$$\omega_z = \frac{1}{2} \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right)$$

Rotating tank with floating cross

Rotating cylinders with floating cross

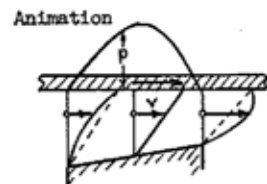
Hele-Shaw water table, foil, and dye crystals

of finely divided solids, some of the new dripless paints illustrating this behavior to an extreme degree. Such a non-Newtonian fluid is characterized by a curve which indicates an initial stress at zero rate of deformation and a viscosity that decreases with increasing deformation rate.

Whereas Couette flow is produced by a moving boundary, what is known as Poiseuille flow involves the movement of a fluid between stationary boundaries as the result of pumping or the action of gravity. For reasons of force equilibrium, the intensity of shear must vary across the flow at the same rate that the piezometric head varies along the flow. Since the longitudinal piezometric gradient must be a constant. And since in a Newtonian fluid the shear must vary with the velocity gradient, the velocity must be distributed parabolically across the flow section, as use of the hydrogen-bubble method in a solution of glycerine and water indeed shows it to do.

Angular deformation in uniform Poiseuille flow was expressed as a simple velocity gradient. In general nonuniform flow it is the sum of the velocity gradients in two perpendicular directions. Now rotation was seen in the second film to be the difference of the same two gradients. It is thus possible to have rotational flow without deformation, as in a tank of any liquid turning on its axis at constant speed; conversely, the motion of a viscous liquid between cylinders having tangential velocities that are inversely proportional to their radii is, despite its deformation, completely irrotational. Flow between closely spaced parallel boundaries will be rotational in any plane perpendicular to the boundary but essentially irrotational in the plane of motion. This gives rise to the Hele-Shaw method of showing patterns of irrotational flow through use - paradoxically - of a viscous fluid, as approximated by sprinkling dye crystals over the thin sheet of free-surface flow on this water table. By and large, however, the motion of a viscous fluid is likely to be rotational in all planes.

Flow between walls with relative motion and pressure gradient, bubbles showing three velocity conditions



The linear and parabolic velocity profiles of Couette and Poiseuille flow will be combined if there exist both induced flow due to relative movement of the boundaries and through flow due to pumping or gravity. Evidently, the net result can be either additive or subtractive, depending upon the relative magnitudes and directions of the two types of flow, even to the extent of producing backflow in part of the cross section. If, now, the originally parallel boundaries are slightly inclined to one another, conditions of continuity will require the two types of flow to be subtractive where the spacing is wider and additive where it is narrower. Since the head must drop in the direction of the Poiseuille type of flow, there will be a pressure build-up between the end sections; in other words, the two boundaries tend to be forced apart by the flow. Such is the

Air-lubricated bearing demonstration

elementary principle of viscous lubrication, as demonstrated by this model Kingsbury thrust bearing. The supporting shoes can tilt as the surface above them turns, the load being maintained by the pressure in the film of lubricant - in this case air. Metal-to-metal contact obviously does not occur till motion nearly ceases.

Fall of sphere Animation



Fall of two spheres, one near wall

Fall of three spheres

Dissolve on

$$v = \frac{\Delta \gamma d^2}{18 \mu} \quad R = \frac{v d}{\mu / \rho} < 1$$

Large bubble rising

As illustrated by this sphere settling in glycerine, fluid deformation is also involved in flow around immersed bodies. In fact, the resistance to such relative motion is called deformation drag, because its effect upon the surrounding fluid extends so far away from the body. As a result, a sphere falling near a boundary will be retarded more than one farther away. Similarly, each of a group of spheres will affect the motion of its neighbors. The density does not appear in the resistance equation known as Stokes' law, because it applies only when inertial effects are negligible in comparison with viscous effects. Since the Reynolds number represents the ratio of inertial and viscous effects, for Stokes' law to be valid the Reynolds number must be very low. At higher values, Stokes' law no longer applies, for inertial effects then become important, as is illustrated to an extreme degree by this slow-motion scene of a bubble in water.

Gravity flow of mercury, water, and gas through U-tubes between containers

In all flows which involve mass action, it is not the dynamic viscosity μ which is significant, but its ratio to the density ρ , called the kinematic viscosity ν . Viscous fluids of different densities flow under gravity through small passages such as these at speeds inversely proportional to their kinematic viscosity. The dynamic viscosity of mercury is greater than that of water, but its kinematic viscosity is evidently smaller. A heavy gas, however, having a dynamic viscosity much smaller than water, has a much greater kinematic viscosity and hence flows far more slowly.

Flow through a tube, hydrogen bubbles

Dissolve on

$$-\gamma \frac{dh}{dx} = 32 \frac{\mu V}{D^2}$$

Flow at tube inlet, low Reynolds number

Inlet flow at higher Reynolds numbers

Dissolve on

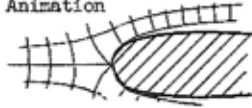
$$\frac{x}{D} = 0.07 R$$

In Poiseuille flow through uniform tubes, the velocity distribution is parabolic, just as in the case of plane Poiseuille flow. The resistance which it encounters is likewise independent of the fluid density, because of the lack of acceleration in the zone of uniformity. At the inlet of such a tube from a reservoir, however, not only does acceleration occur, but the parabolic velocity profile develops only with distance downstream. For the same glycerine solution at higher speed the development is appreciably less rapid than it was at lower speed, while for fast-moving water the parabola develops extremely slowly. Such flows evidently involve inertial effects and hence depend upon the Reynolds number. In fact, the relative distance from the inlet required to establish the parabolic velocity distribution increases directly with the Reynolds number.

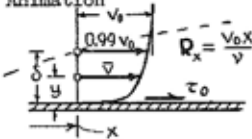
Much the same situation is found in flow past an immersed body. When the liquid is a glycerine solution and the speed is low, the zone of appreciable fluid deformation is quite large, but as the

Motion at higher R

Animation



Animation



Velocity profiles 1, 10, 100, and 1000 plate thicknesses from edge

Dissolve on

$$\frac{\delta}{x} = \frac{5}{R_x^{1/2}}$$

$$\frac{\tau_0}{\rho v_0^{3/2}} = \frac{0.67}{R_x^{1/2}}$$

Boundary layer becomes turbulent

Onset of turbulence in pipe

Change in jet form with turbulence

Eddy formation at unrounded pipe inlet

Lecture

$$v \sim l_m v_m$$

$$\epsilon \sim l_e v_e$$

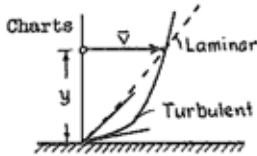
$$\frac{\bar{\epsilon}}{\rho} = \frac{\partial \bar{v}}{\partial y} (v + \epsilon)$$

speed is increased, the zone of deformation becomes more restricted to the vicinity of the body. When the glycerine is replaced by water, the zone becomes still narrower, and at sufficiently high Reynolds numbers it is aptly called a boundary layer. The pressure distribution on the body can then be evaluated according to the approximately irrotational flow outside the boundary layer, and the distribution of shear according to the fluid deformation within the highly rotational region of the boundary layer itself. The latter effect alone can be studied conveniently in the flow along a thin flat plate. The boundary shear causes the velocity to vary from zero to that of the passing flow, δ being the nominal boundary-layer thickness. Now, however, the primary reference dimension is the distance from the leading edge, so that the Reynolds number increases directly with the velocity and with displacement along the boundary. As shown by the hydrogen-bubble technique, close to the leading edge of the plate δ is thus small; 10 times as far downstream it is visibly greater; 100 times as far, the layer has thickened still more; evidently the boundary-layer thickness continues to increase with distance from the leading edge, though at an ever-decreasing rate. Through the momentum principle, the relative thickness of the layer and the relative intensity of the boundary shear can both be expressed as negative powers of the distance Reynolds number.

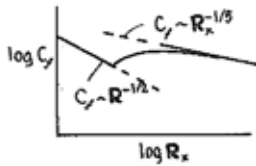
Laminar flow eventually becomes unstable to disturbances of an intermediate size or frequency, the range of which increases with the Reynolds number. If such disturbances are present in the flow, its orderly character will deteriorate rapidly into the irregular state known as fluid turbulence. The careful injection of dye at the rounded inlet of a pipe will reveal similar occurrences. The flow is seen to be stable at low Reynolds numbers even though it is obviously disturbed. If the Reynolds number R_D is sufficiently high, however, even imperceptible disturbances will produce the onset of turbulence in sudden bursts, as can also be observed at the pipe outlet. (The fin seen below the laminar jet is simply low-velocity fluid from the wall region.) Once R exceeds about 2000, existing disturbances will generally lead to turbulence, whereupon the surface of the jet will take on a totally different character. Here the initial disturbance is provided by separation at the edge of an unrounded inlet. Multiple dye jets now permit the formation and lateral spread of the individual eddies to be followed.

Though fluids are treated as homogeneous, it is really the interaction of the individual molecules that produces the viscous effects. For instance, the kinematic viscosity is - at least for a gas - proportional to the product of the molecular spacing and the molecular velocity. Fluid turbulence can be imagined to have somewhat the same properties, but on a molar rather than a molecular scale. Thus, the combination of an eddy size and an eddy velocity is called the kinematic eddy viscosity. The simple expression for the shear in

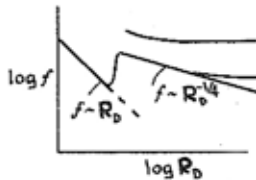
uniform flow in terms of the molecular viscosity can now be extended to include the eddy viscosity as well. It should be noted that, whereas ν is a function of fluid properties, ϵ depends wholly on the state of motion.



The comparative roles of ϵ and ν can be seen from diagrams of the velocity distribution for laminar and turbulent flow. Because of the mixing process, the velocity in the turbulent zone is much more nearly constant. At the wall, where the turbulence is inhibited, the velocity gradient and hence the intensity of shear is thus many times as great as in purely laminar flow.



This disparity is also evident from a plot of the resistance coefficient against the Reynolds number for the boundary layer. The line for purely turbulent flow lies farther and farther above that for laminar flow as the distance Reynolds number increases. If laminar flow becomes turbulent at some intermediate value, the composite curve will approach the turbulent limit only asymptotically.



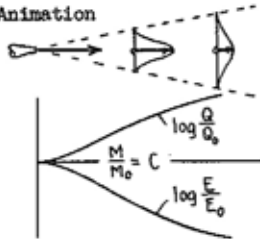
Much the same situation is found in pipe resistance, though now the laminar case is actually independent of the Reynolds number. Again the resistance to turbulent flow is many times as great. If the pipe is rough, it may still behave like a smooth one at intermediate Reynolds numbers, when viscous effects at the wall are still appreciable, but like a fully rough one at high Reynolds numbers. An increase in relative roughness will simply displace the curve in the direction of higher resistance.

Smoke plume rises, breaks, and diffuses

We have dealt so far with the kind of flow in which boundary shear plays an essential role. Another kind is that in which the shear takes place far from a boundary, between neighboring zones of different velocity, called free-shear flow. This is illustrated very effectively by the plume of smoke from burning incense, the plume rising because of the buoyancy of the heated air. As it rises, shear with the surrounding air brings this into motion as well. With increasing distance from the source the flow becomes more and more susceptible to the disturbances present in the air and eventually breaks down into the apparently haphazard motion classed as turbulent.

Smoke jet from nozzle

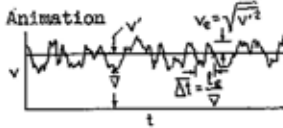
Animation



The characteristics of free turbulence are illustrated even more graphically by a jet produced by submerged efflux from a nozzle rather than by convection from a source of heat. Now no external force like gravity acts upon the flow beyond the nozzle section. The jet expands in the form of a cone, the mean velocity-distribution curve having a similar form at section after section, and the corresponding momentum flux M remaining constant. But the volume flux Q will steadily increase through air entrainment, and the kinetic-energy flux E will rapidly decrease through dissipation.

Whether the fluid is a gas or a liquid, seen at normal speed or in slow motion, turbulence is evidently a heterogeneous eddy pattern

Submerged water jet
Hot-wire anemometer



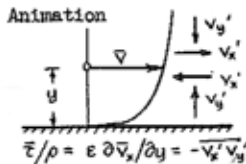
superposed upon the mean flow. Despite its lack of order, two significant features are readily apparent: the average eddy intensity and the average eddy scale. Each can be determined from the velocity fluctuations recorded by the hot-wire anemometer, a fine resistance wire on a forked probe, the electrical signal of which varies with the rate of cooling, which in turn varies with the speed of the flow. On such a record of signal versus time, the mean velocity is given by the average elevation of the curve. The root-mean-square amplitude of the velocity fluctuation then represents the average intensity of the eddy motion, and the mean period of the fluctuation indicates the average scale of the eddies.

Recorder comparisons
for hot-wire
traverses

Moving the hot wire diametrically across a jet shows a typical rise and fall in mean velocity, eddy intensity, and eddy scale. On the other hand, as the wire is moved outward along the jet axis, the mean-velocity signal now being eliminated, the intensity is seen to diminish and the scale to increase with distance from the nozzle. There is no limit, of course, to either the scale or the intensity of turbulence. Such small-scale eddies produced by heat from a gas flame do not differ in kind from dangerously violent boils in a river below a spillway, or from large atmospheric disturbances that require time-lapse photography to make their motion apparent.

Eddies in heated tank,
river, and clouds

Flow along conduit wall



In addition to the eddy viscosity, the eddy scale, and the eddy intensity, in flow along a boundary or in a jet one further characteristic of the turbulence should be noted. This is the apparent shear that it produces. Such shear has already been expressed in terms of the eddy viscosity and the mean velocity gradient. It is also given by the mean product of the velocity fluctuations in the two directions. At the axis of a conduit or jet where the shear is zero, the circular pattern of hot-wire signals shows that the fluctuations are randomly distributed, whereas the elliptical pattern of negative correlation corresponds to the zone of intense shear near the edge of the jet or wall of the conduit.

Oscilloscope correlation
patterns

Flow along conduit wall

Animation

$$\rho \sim \rho v_x' v_y' \frac{\partial v}{\partial y}$$

Three essential phenomena remain to be described: turbulence production, turbulent mixing, and turbulence decay. Turbulence production at any point depends primarily upon the intensity of turbulent shear and the gradient of the mean velocity. Eddies are generated, to be sure, by any body causing separation, like the bars in a grid, but such form effects will be discussed in the next film. Well downstream from the grid there is neither a mean velocity gradient nor a state of mean shear, and hence no more turbulence is produced. In a boundary layer or free-shear zone, on the other hand, new turbulent eddies are constantly being formed. This represents either work done by shear, pressure gradient, or weight, or else a loss of kinetic energy by the mean flow.

Flow behind grid
of cylinders

Eddy formation
at edge of jet

Animation

$$M = -\epsilon \frac{\partial m}{\partial y}$$

Turbulent mixing depends upon two factors: the eddy viscosity or diffusion coefficient, and the gradient of the quantity being mixed - such as turbidity, heat, or dye. In the absence of turbulence, the

Mixing of blue jet
with yellow fluid

diffusion can only be molecular; even if turbulence exists, there must also be a difference in concentration from zone to zone of the quantity being diffused - represented here by color - for the mixing to have any effect. So far as the turbulence properties are concerned, the efficiency of mixing will vary directly with both the eddy intensity and the eddy scale.

Single and multiple
ring vortices

Now the core of any isolated eddy, like that of this ring vortex, will tend to thicken with time because of the viscous shear within it. However, the velocity fields of two or more vortices in the same vicinity will each cause the other's core to stretch and thereby grow progressively thinner. As each eddy of an actual turbulence pattern is thus forced to reduce its scale, the viscous dissipation within it becomes steadily more rapid. Thus, whereas it is primarily the large eddies that do the mixing, it is primarily the small ones that accomplish the transformation of mechanical energy into heat. This can be seen by a simple modification of the expression for viscous shear into the power form of laminar dissipation. In terms of turbulence characteristics its counterpart indicates that the dissipation rate is proportional to the viscosity, the square of the eddy intensity, and the inverse square of the eddy size. The smallest eddies thus decay most rapidly, and it is for this reason that the eddy scale increases with distance away from a pipe wall or along the axis of a jet. Here, for example, after passage of the grid the average size of the remaining eddies is seen to increase steadily with time.

Eddy evolution in pipe

Smoke chamber

Animation

$$\tau = \mu \frac{\partial v}{\partial y}$$
$$D_t = \tau \frac{\partial v}{\partial y} = \mu \left(\frac{\partial v}{\partial y} \right)^2$$
$$D_t \sim \mu \frac{v^2}{l^2}$$

Decay of turbulence
in wake of grid

Eddy patterns produced
by off-scene paddles

Through what is obviously a very complex process, turbulence thus drains energy from the mean flow, modifies the mean flow pattern through shear and mixing, and ultimately produces its own decay by the same viscous resistance to deformation discussed at the opening of this film. The whole situation is nicely described by L. F. Richardson's well-known parody on a child's verse: "Big whirls have little whirls, which feed on their velocity; little whirls have smaller whirls, and so on to viscosity."