Handout: Channel Flow - Definitions and Governing Equations

Definition

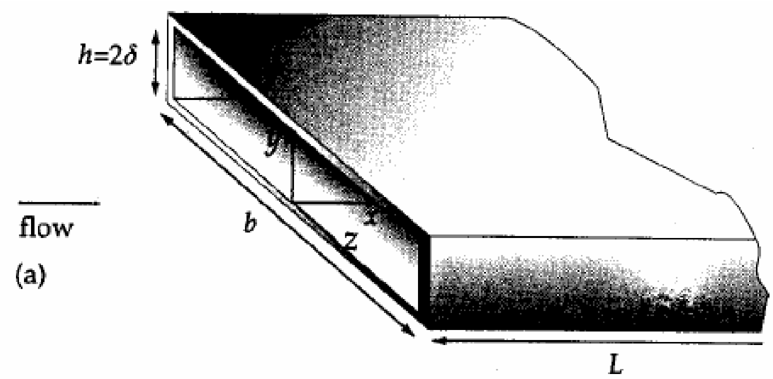

Fig. 1 Sketch of channel flow (Pope's book)

Assumption:

$$L >> \delta$$

$$b >> \delta$$

We focus on the fully developed region (large x) in which velocity statistics no longer vary with x. As the duct has also a large aspect ratio, the turbulence statistics are homogeneous in x and z (away from the lateral walls)

The lower and upper walls are situated at y=0 and y=2δ with the channel centerline situated at y=δ

Reynolds numbers:

$$\mathrm{Re} = \frac{2\delta\overline{U}}{\nu}; \quad \overline{U} \text{ is bulk velocity}$$

$$\mathrm{Re}_0 = \frac{\delta U_0}{\nu}; \quad U_o = \langle U \rangle_{y=\delta} \text{ is the centerline velocity,}$$

<> denotes Reynolds averaged quantity

Laminar flow for Re<1350, Fully developed for $\mathrm{Re} > 1800$, transitional effects present up to Re=3000

Governing Equations

Continuity

$$\frac{\partial\langle U\rangle}{\partial x}+\frac{\partial\langle V\rangle}{\partial y}=0$$

$$\Rightarrow\frac{\partial\langle V\rangle}{\partial y}=0$$

Integrate in y with $\langle V\rangle_{y=0}=0$

$$\Rightarrow\langle V\rangle=0$$

Lateral Momentum Equation

$$\langle U\rangle\frac{\partial\langle V\rangle}{\partial x}+\langle V\rangle\frac{\partial\langle V\rangle}{\partial y}=-\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial y}+\nu\frac{\partial^2\langle V\rangle}{\partial x^2}+\nu\frac{\partial^2\langle V\rangle}{\partial y^2}-\frac{\partial\langle uv\rangle}{\partial x}-\frac{\partial\langle v^2\rangle}{\partial y}$$

$$\Rightarrow\frac{\partial}{\partial y}(\frac{\langle p\rangle}{\rho}+\langle v^2\rangle)=0$$

Integrate from 0 to y

$$\langle v^2\rangle+\frac{\langle p\rangle}{\rho}=\frac{p_w(x)}{\rho}$$

Differentiate with respect to x

$$\frac{\partial\langle p\rangle}{\partial x}=\frac{\partial p_w}{\partial x}$$

$\Rightarrow$ Pressure gradient uniform across flow

Axial Momentum Equation

$$\langle U\rangle\frac{\partial\langle U\rangle}{\partial x}+\langle V\rangle\frac{\partial\langle U\rangle}{\partial y}=-\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial x}+\nu\frac{\partial^2\langle U\rangle}{\partial x^2}+\nu\frac{\partial^2\langle U\rangle}{\partial y^2}-\frac{\partial\langle u^2\rangle}{\partial x}-\frac{\partial\langle uv\rangle}{\partial y}$$

Total shear stress (viscous + Reynolds stress):

$$\tau=\rho\nu\frac{\partial\langle U\rangle}{\partial y}-\rho\langle uv\rangle$$

$$\Rightarrow \frac{\partial \tau}{\partial y} = \frac{\partial p_w}{\partial x}$$

$\Rightarrow$ Total shear stress compensated by pressure gradient

$$\frac{\partial \tau}{\partial y} \neq f(x) \text{ and } \frac{\partial p_w}{\partial y} \neq f(y)$$

$$\Rightarrow \frac{\partial \tau}{\partial y} = \frac{\partial p_w}{\partial x} = const$$

$$\tau(y) = \tau_w \left(1 - \frac{y}{\delta}\right)$$

Skin friction coefficients

$$c_f \equiv \frac{\tau_w}{1/2 \rho U_0^2}$$

$$C_f \equiv \frac{\tau_w}{1/2 \rho \overline{U}^2}$$

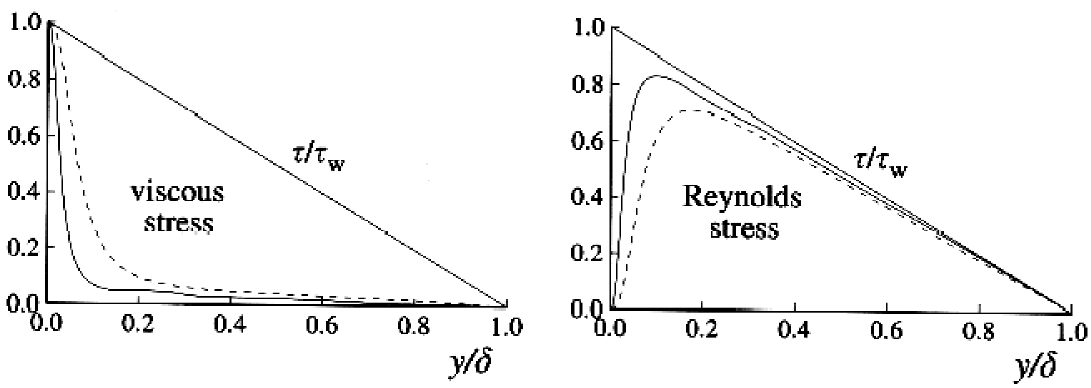

Fig. 7.3. Profiles of the viscous shear stress, and the Reynolds shear stress in turbulent channel flow: DNS data of Kim *et al.* (1987): dashed line, Re = 5,600; solid line, Re = 13,750.

Figure 7.3 from Pope's Turbulent Flow book: Reynolds stress, viscous stress, and total stress (DNS of KIM, Mion, Moser (1987)

Wall Shear Stress

τ_w given only by viscous stress as <uv>=0 at the wall

$$\tau_w = \rho\nu \frac{\partial \langle U \rangle}{\partial y}\Big|_{y=0}$$

Friction velocity:

$$u_\tau \equiv \sqrt{\frac{\tau_w}{\rho}}$$

Viscous length scale:

$$\delta_\nu \equiv \frac{\nu}{u_\tau} = \nu\sqrt{\frac{\rho}{\tau_w}}$$

$$\Rightarrow \mathrm{Re}_\nu \equiv \frac{\delta_\nu u_\tau}{\nu} = 1$$

Friction Reynolds number (Reynolds number defined with the friction velocity):

$$\mathrm{Re}_\tau = \frac{u_\tau \delta}{\nu} = \frac{\delta}{\delta_\nu}$$

Wall units:

$$y^+ = \frac{y}{\delta_\nu} = \frac{u_\tau y}{\nu} \quad \text{(similar to local Reynolds number)}$$

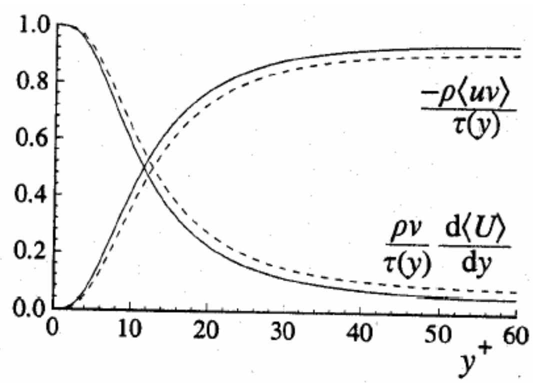

Fig. 7.4. Profiles of the fractional contributions of the viscous and Reynolds stresses to the total stress. DNS data of Kim *et al.* (1987): dashed lines, Re = 5,600; solid lines, Re = 13,750.

- Outer layer: $y^+ > 50$ (viscous stresses can be neglected, direct effect of viscosity is negligible)

- Viscous wall region: $y^+ < 50$
- Viscous sublayer: $y^+ < 5$ (Reynolds stresses are negligible)

As Re increases, the fraction of the channel occupied by the viscous wall region decreases, since $\frac{\delta_\nu}{\delta} \approx \mathrm{Re}_\tau^{-1}$