

Turbulent Flow and Heat Transport over a Two-dimensional Steep Hill: Wind-tunnel Experiments

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Transport of momentum and scalars in turbulent boundary-layer flows over complex terrain has been of great interest in the atmospheric sciences and wind engineering communities. Applications include but are not limited to weather forecasting, air pollution dispersal, aviation safety control, and wind energy project planning. While linear models have been well accepted to predict boundary-layer flows over topography with gentle slope, modeling flow separation and recirculation induced by topography of sufficiently steep slope has to be achieved through non-linear models, such as Reynolds-averaged Navier-Stokes (RANS) solvers and Large-Eddy Simulations (LES). High-quality measurements in the field and laboratory settings are in high demand for development and validation of such numerical models. Dynamics of the separated boundary-layer flows over steep topography is affected by the shape and size of the topography, surface characteristics (e.g., roughness and temperature distribution) and atmospheric thermal stability. Majorities of wind-tunnel experiments of boundary-layer flows over representative and idealized topography features (e.g. 2-D or 3-D hills, axisymmetric bumps) do not take thermal stability effects into account due to challenges in physical simulation. We conducted experimental investigation of stably-stratified boundary layers over a steep 2-D hill in the thermally-controlled boundary-layer wind tunnel. The 2-D model hill has a steepest slope of 0.73 and its shape follows a cosine square function. High-resolution Particle Image Velocimetry (PIV) provides dynamic information of the separated shear layer, the recirculation zone and flow reattachment. Mean surface shear stress and surface heat flux were directly measured in the wake. Results indicate that suppressed turbulence in the stable boundary layer noticeably alters the topology of the circulation zone. Surface shear stress and surface heat flux downwind of the 2D hill slowly approach the equilibrium values of the non-disturbed boundary layers. This work can improve our understanding of the effects of thermal stability on steep topography, and provide reliable datasets for development and validation of numerical models.

Nomenclature

H = hill crest height
 L = width of the hill model

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θ_0	=	air temperature
θ_s	=	floor temperature
U	=	mean stream-wise velocity in the x direction
u'	=	fluctuated stream-wise velocity in the x direction
W	=	mean vertical velocity in the z direction
w'	=	fluctuated vertical velocity in the z direction
U_0	=	mean stream-wise velocity in the boundary layer flows
I_u	=	stream-wise turbulence intensity
δ	=	boundary layer depth
ω_y	=	spanwise vorticity
τ	=	mean surface shear stress downwind of the hill
τ_0	=	mean surface shear stress of the non-disturbed boundary layer flow
Q_s	=	mean surface heat flux downwind of the hill
Q_{s0}	=	mean surface heat flux of the the non-disturbed boundary layer flow

I. Introduction

Atmospheric boundary layer (ABL) flow over complex terrain is of great practical interest to the atmospheric science and wind engineering communities. The pollutant dispersion and hazardous materials in mountainous or hilly areas, wind-induced loading on civil structures, and prediction of local weather conditions are just a few examples. One of the most significant applications at present is the wind power production. With the exponentially growth of wind energy projects worldwide, installation of wind farms are being spread to complex terrain in addition to relatively smooth and homogeneous land surface. By placing wind turbines along ridges in hilly terrain and even in mountainous areas, complex wind behaviour such as flow separation and recirculation may, however, substantially reduce power production efficiency and increase the structural loads on the wind turbines. Reliable prediction of such wind features is therefore essential for planning wind turbine sitting in complex terrain.

Extensive studies on the ABL flows over hills with gentle slope have been conducted in the frame work of linear theory under the conditions that hill-induced perturbation on the flow are small enough^{1,2}. This linear theory has been successfully used to predict flows over gently sloping terrain and low hills in neutral boundary layers, showing favorable comparison to field observation, such as that carried out over Askervein hill. However, linear models are often applied on scenarios away from their design conditions for convenience. Once the hill has a sufficiently steep slope, flow separation is provoked and non-linear effects dominate flow around the hill, of which the linear theories based on small perturbation techniques break down. The calculations over smooth and rough two-dimensional hills suggest that linear models yield unacceptably large error for slopes greater than 0.2³. Reasonable prediction of the onset of flow separation, the extent of the separation bubble and the turbulence production are challenging tasks. For example, unlike flow over bluff bodies with a sharp edge (e.g., building block or cube) where separation always occurs on the edge, the onset of flow separation over a (smooth) curved surface is affected by multiple factors, such as: specific shape/curve of the hill, surface features (roughness and temperature), inflow turbulence levels and atmospheric stability. In addition, when large recirculation regions form downwind of hills, not only the flow in the separated region changes, but significant changes occur in the whole flow field over the hill⁴. It is clear that alternative computational methods must be sought- the use of more advanced computational fluid dynamics (CFD) tools, such as higher-order turbulence closure or Large Eddy Simulation (LES).

Development of appropriate CFD tools calls for high-quality data measured in the field or controlled laboratory settings. Due to the complexity of the field conditions, however, interpretation of the field data itself is a challenging task and need assistance of knowledge based on well-controlled laboratory experiments. In addition, full-scale data hardly achieves the level of detail that is necessary to verify numerical predictions. As an alternative of full-scale field measurements, understanding of flow separation and reattachment can be achieved by scaled laboratory experiments using water channels/towing tanks and wind tunnels. To date, the majority of wind-tunnel simulations of flow over complex terrain have been conducted in neutral stability. For atmospheric flows, recognition of the importance of buoyancy forces has led to the development of facilities for simulating density changes in the flow - thermally stratified wind tunnels and salinity-stratified towing tanks. Hunt and his colleagues conducted extensive theoretical and experimental studies of the stable stratified flow over hills with shallow to moderate slopes⁵⁻⁷. Besides their work, there is not many wind-tunnel simulations because of difficulty to create a thermally-stratified boundary layer. In particular, very few studies carried out to study the flow over steep hills in thermally-stratified boundary layers. Takahashi et al⁸ measured mean velocity and turbulence statistics of the flow over a 3-D hill in stable, neutral and unstable boundary layers ($Ri = 0.008$ and -0.002). Their results show limited difference in the

profiles in the wake just downstream of the hill affected by the thermal stability. This study indicates the difficulty to achieve relatively strong thermal stability condition in wind-tunnel simulations. Ross et al⁹ reported wind-tunnel experiments in stable and neutral boundary layers. They focused on flow dynamics and used the wind-tunnel data as the reference case to evaluate numerical models.

In this study, we quantified turbulent flow and heat transport over an idealized 2D hill with a steep slope in both neutral and stable boundary layers. The thermal stability effects on flow separation, re-circulation zone topology, reattachment location are evaluated. In addition, development of mean surface shear stress and surface heat flux in far wake region is presented.

II. Experimental Facility and Methods

A. Thermally-controlled Boundary-layer Wind Tunnel

Experiments were conducted in the thermally-stratified boundary-layer wind tunnel at the Saint Anthony Falls Laboratory, University of Minnesota. The close-looped boundary-layer wind tunnel has a plan length of 37.5 m with a main test section fetch of 16 m. The contraction upwind of the test section is of 6.6:1 area ratio and flow conditioning consists of a coarse wire mesh and honeycomb flow-straightener. The turbulence intensity in the centre of the wind tunnel is 1–2 % for an approximately 2.0 m/s freestream speed.

The test-section floor consists of a series of 0.3 m long smooth aluminum plates, each of which has a temperature control system to ensure the desired temperature level. To achieve thermal stratification conditions (neutral, stable or unstable), temperature of the test section floor and air flow can be independently controlled in the range of 5 °C -- 80 °C (± 0.25 °C). The stable boundary layer was generated by heating the air to $\theta_0 = 58$ °C and cooling the floor to $\theta_s = 8$ °C. For the neutral boundary layer, both air and the floor temperature were kept at 30 °C. The rough surface was created by placing metallic chains of approximately 5 mm high, covering about 10 m of the tunnel test section. The chains were aligned perpendicular to the flow direction and separated apart from each other by 0.20 m. This roughness setup is ideal to incorporate thermal effects through heating/cooling of the test section.

The 2-D model hill, made of aluminum, is placed directly on the floor normal to the incoming wind direction. Its shape follows a cosinesquared function:

$$z = H \cos^2\left(\frac{\pi x}{L}\right), \quad \text{for } -L/2 \leq x \leq L/2 \quad (1)$$

where H ($= 7$ cm) is the crest height and L ($= 14.5$ cm) is the characteristic length of the hill representing the distance from the crest to the foot (half-height point). It also has the same shape as that used by Ross et al⁹. This hill model has the maximum slope or steepness of 0.73 (42°) and is fully immersed in the surface layer of the boundary layers ($H/L = 0.12$) in the experiments. The hill model is occupying the whole span of the wind tunnel (1.6 m) to minimize the side wall effects. A small sized chains were attached on the surface of the hill as well as the whole test section to simulate a rough surface. The spacing is 10 cm on the floor and decreased to 2 cm on the hill model. Once an equilibrium state reaches, through conduction between the test-section floor and the bottom of the model, temperature uniformity on the hill surface and the floor is monitored by surface-attached thermal-couples. Fig. 1 shows the experiment set-up, definition of the coordinate system and the geometric features of the 2D hill model. Simulated BL flow characterization, including the mean velocity, temperature and turbulence intensity profiles are summarized in Fig. 2.

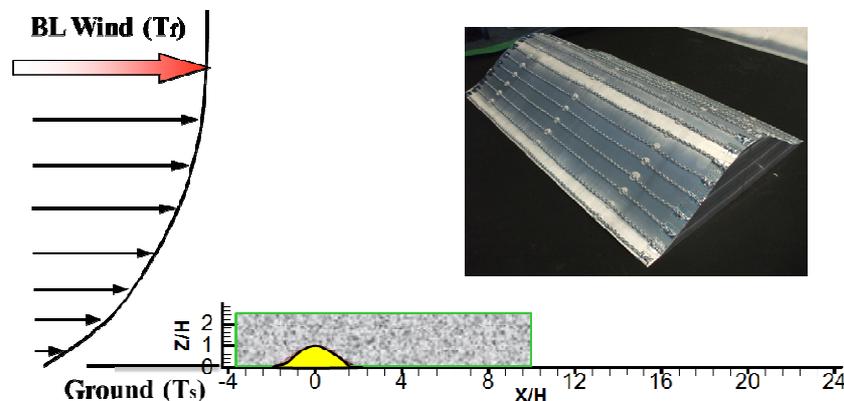


Figure 1: Experiment setup, coordinate system and geometric parameters of the 2D steep hill model.

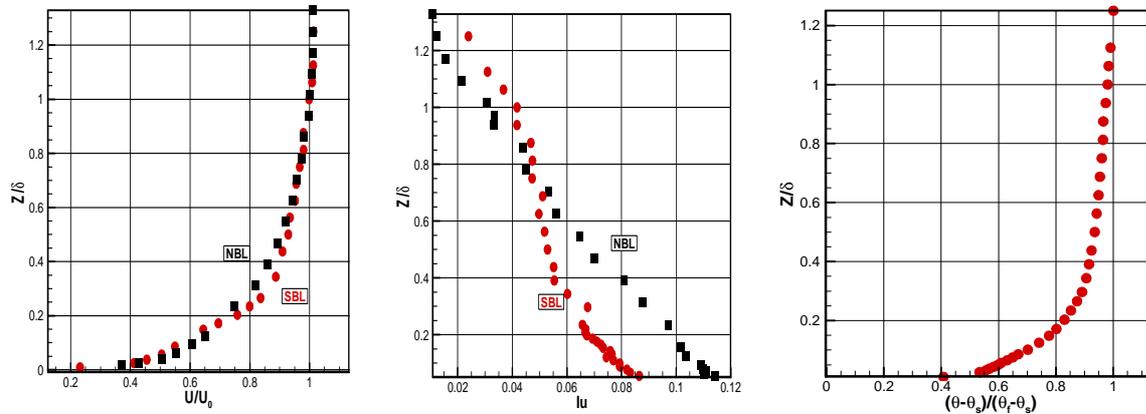


Figure 2: Inflow conditions of the neutral and stable BL. Mean stream-wise velocity (*left*), Stream-wise turbulence intensity (*middle*) and Mean temperature profile (*right*).

B. Measurement Techniques

We used a high-resolution PIV system (TSI, Inc.) to measure the flow field over the 2D steep hill, in the region of $x/H = -4 - 8.5$ and $z/H = 0 - 2$ where flow is highly turbulent and flow separation is expected to be provoked. As a supplement, a customized triplewire (combination of a cross-wire and a cold-wire) is used to characterize the turbulent momentum and heat flux in the far wake and up to the top of the boundary layer. With this approach a complete picture of the turbulent flow and heat transport can be achieved.

The PIV laser light sheet illumination is provided by using a Nd:YAG pulsed laser system (Quantel USA) with $\lambda = 532$ nm and 190 mJ/pulse. Olive oil particles, with a mean diameter between 1 and 3 μm , atomized with an array of six Laskin nozzles was employed to seed the flow. A Powerview Plus 4MP 12-bit CCD camera captured particle images at 1.4 Hz with the resolution of 2048 x 2048 pixels. A 105 mm focal length lens was used to achieve images with the field of view (FOV) of 135 x 135 mm^2 at the largest aperture setting of f#2.8. The corresponding physical resolution is about 2.3 mm x 2.3 mm. Observation was made in the vertical plane, parallel with the incoming flow at the center line of the wind tunnel. The floor surface was painted with a thin layer of flat black paint and fluorescent paint to minimize the reflection of laser light. In order to cover the entire flow region of interest around the hill, particle images of seven FOVs were acquired.

Raw particle images were pre-processed by subtracting of the background noise, to improve the signal to noise ratio. The particle images were evaluated using an iterative multigrid method with second-order accuracy and a final interrogation window of 32×32 pixels with 50% overlapping (INSIGHT3G, TSI). Erroneous vectors, less than 1% of the total calculated vectors, were replaced by vectors interpolated through a Gaussian scheme from valid neighboring vectors. The mean velocity field was obtained by ensemble-averaging 3000 instantaneous velocity fields. All fluctuating velocity fields, derived by subtracting the mean velocity field from the instantaneous velocity fields, were used to compute turbulence intensities and Reynolds shear stresses. The measurement uncertainty level of velocity vectors is estimated to be within 2%.

Mean surface shear stress downwind of the 2D hill was measured using a Preston tube following the procedure described by Patel¹⁰. Only data beyond $x/H = 10$ are shown considering that the Preston tube tends to perform poorly in regions of strong pressure gradients and near flow separation. Measurements of mean surface heat flux were made using flat-plate type heat flux sensors (Captec, Inc.). The thin-foil heat flux sensor consists of a thermoelectric panel laminated between flexible heterogeneous plastic layers. Each heat flux sensor is 1cm by 1cm, with a thickness of 0.4 mm to minimize flow disturbance from mounting them on the surface. A silicone-based heat sink compound was used to ensure good contact between the sensor and the surface. The heat transfer sensitivity of this sensor is 0.6 V/Wm² and the response time is 0.3 s. This method was used in quantifying surface heat flux change induced by a model wind farm by Zhang et al¹¹.

III. Results and Discussion

A. Flow Separation and Recirculation

In an attached boundary layer, the span-wise vorticity ω_y near the surface is negative. An adverse pressure gradient along the streamwise direction leads to a flux of positive vorticity. The resultant vorticity at the wall and the consequent possibility of separation are determined by the balance of the generated new vorticity, the advection of vorticity from upwind, and the turbulent diffusion of vorticity to the wall from upper part of the boundary layer⁴. The downwind location where vorticity changes sign at the wall can be indicators of the onset of separation as well as the reattachment point. Figure 3 shows the span-wise vorticity ω_y contours calculated from ensemble-averaged PIV velocity field for the NBL and SBL cases. There is a very thin layer of intensive negative vorticity (*in blue*) over the upslope surface until flow passes the hill crest, where flow remains attached. One can find that flow separates just downwind of the hill crest, at $x_s/H = 0.39$, where surface vorticity changes the sign from negative to positive (*in red*). Separated flow reattaches at $x_r/H = 6.4$ as the vorticity becomes negative again. This yields a separation bubble of $6.0 H$ long in the neutral BL case. In the stable boundary layer, length of the separation bubble is about $6.6 H - 10\%$ longer— estimated from separation at $x_s/H = 0.50$ and reattachment at $x_r/H = 7.1$.

The streamlines of the mean flow and stream-wise velocity contours were shown in Figs. 4-5 for cross-checking the separation zone topology. The results turn out to be consistent with that detected by the vorticity distribution in Fig. 3. As a comparison, Loureiro et al¹² reported that the flow separates at $x/H = 0.5$ and reattaches at $x/H = 6.67$, which gives the separation bubble length of $6.17H$ over a similar hill model under the neutral stability condition. Ross et al⁹ reported that stable stability case shows a weaker reversal flow near the downwind foot of the hill, but did not quantified the size of the separation zone.

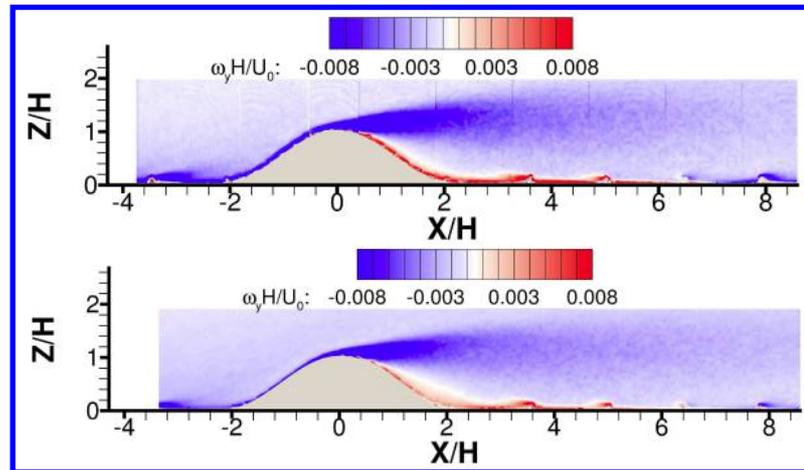


Figure 3: Span-wise vorticity ($\omega_y H / U_0$) contours over the 2-D steep hill for both the neutral (*top*) and stable BL (*bottom*) cases.

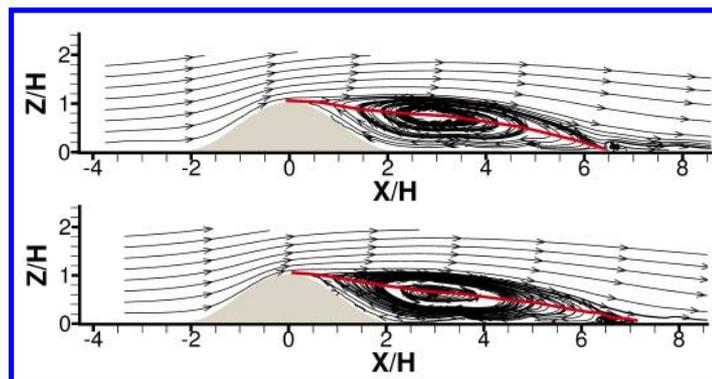


Figure 4: Mean stream-lines over the 2-D steep hill for the neutral (*top*) and the stable BL (*bottom*) cases.

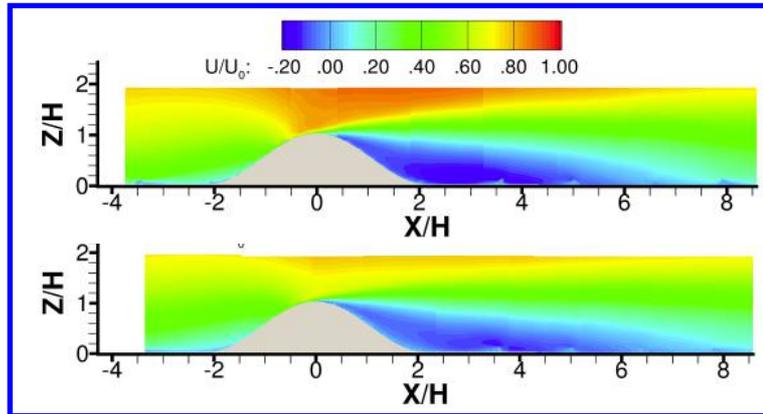


Figure 5: Contours of the mean stream-wise velocity U over the 2-D steep hill. Neutral BL case (*top*) and the stable BL case (*bottom*).

B. Turbulent Shear Layer

Turbulent flow is quantified by the contours of normal and Reynolds shear stresses in Figs. 6 - 8. The separated shear layer downstream of the top of the 2D hill is the most salient feature of this flow, similar to that found in other turbulent flows bounding a separation region. A region of high magnitude of the streamwise and normal turbulence intensities as well as the shear stress, appearing above the $U = 0$ contour, coincident with the separated shear layer. Peak values of normal stresses occur in the zones characterized by the highest mean velocity gradients. The most prominent feature of the streamwise turbulence intensity distribution is the large region encompassing the maximum values. In the neutral BL case, this region begins near the wall, slightly upwind of the hill crest (approximately $x/H = -0.5$) and extends downwind. The maximum values are observed at $x/H = 4 - 5$. Also, the vertical turbulence intensity has the maximum values in the same region.

Figure 8 shows the Reynolds shear stress distribution of the turbulent flow around the model 2D hill in both neutral and stable BL cases. In the neutral BL case, a region of maximum values of the time-average Reynolds shear stress begins to develop near $x/H = 0.5$, $z/H = 1.04$ with a value of approximately 0.5. It continuously increases in extent until reaching a global maximum (1.93) at about $x/H = 4.5$. Further downwind the Reynolds shear stress keeps decreasing. The contours of constant stress are roughly parallel to the surface. The distribution of the Reynolds shear stress, quantifies the turbulent diffusion along the shear layer.

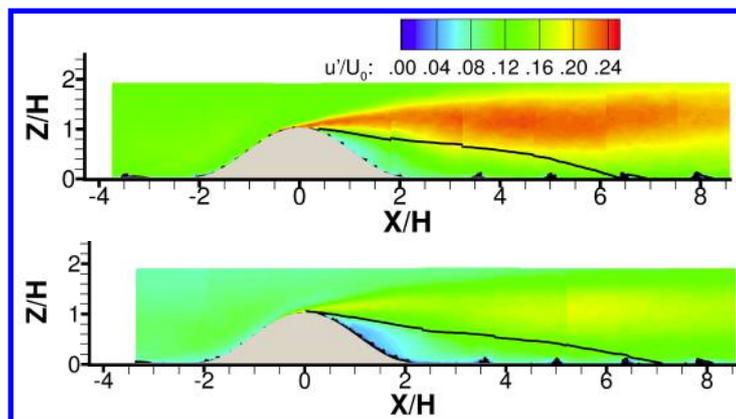


Figure 6: Streamwise velocity fluctuation u'/U_0 over the 2-D steep hill in the neutral (*top*) and the stable BL (*bottom*) cases. The black line indicates $U = 0$ contour.

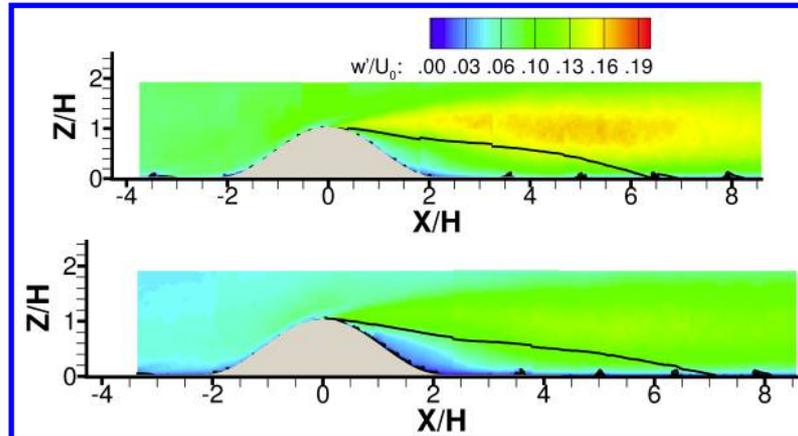


Figure 7: Vertical velocity fluctuation w/U_0 over the 2-D steep hill in the neutral (*top*) and stable BL (*bottom*) cases. The black line indicates $U = 0$ contour.

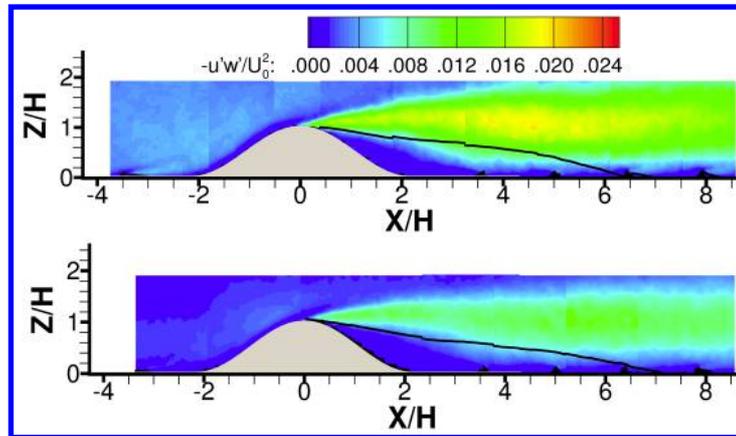


Figure 8: Contours of Reynolds stresses $-u'w'/U_0^2$ of the flow over the 2-D steep hill in the neutral (*top*) and stable BL (*bottom*) cases. The black line indicates $U = 0$ contour.

C. Surface Shear Stress and Surface Heat Flux

Surface shear stress distribution in the far wake of a 2D steep hill in the neutral BL was plotted in Fig. 9. It is expected that the mean surface shear stress is zero at the reattachment location x_r and then approaches that of the incoming boundary layer flow as the wake fully recovers. The recovery trend of the mean surface shear stress have been successfully modelled using exponential and power law functions. The current data are best fitted with a power law function in the following format:

$$\frac{\tau}{\tau_0} = 1 - 15.83 \left(\frac{x}{H} \right)^{-1.47} \quad (2)$$

τ reaches 90% of the initial BL value τ_0 at downwind distance of $x/H = 30$ and 95% at $x/H = 50$. Though mean surface shear stress data is not available for the stable BL case, the recovery length is conjectured to be longer. Compared to the neutral BL, surface cooling may suppress the vertical motion and prevent eddies from transporting momentum from the shear layer above and hence limit the recovery of surface shear stress.

Mean surface heat flux was measured downwind of a 2D smooth hill in the SBL case, up to $x/H = 70$ (in Fig. 10). Similar to the development of the surface shear stress in the wake, the surface heat flux is also expected to finally reach the equilibrium value of the incoming BL flow. Fig. 10 shows two regimes: the rapid recovery prior to the flow reattachment fitted with a linear function, and a gradual decline beyond the reattachment following a power law trend. The peak of the mean surface heat flux (1.62) is approximately at the reattachment point. Using the power law function as following:

$$\frac{Q_s}{Q_{en}} = 2.42 \left(\frac{x}{H} \right)^{-0.18} \quad (3)$$

full recovery of the surface heat flux is predicted to occur at $150H$ downwind of the 2D smooth hill.

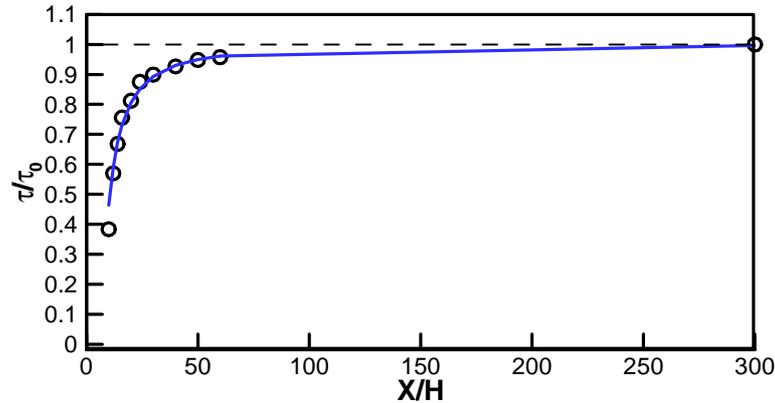


Figure 9: Surface shear stress τ in the wake of a 2D steep hill in the neutral BL case.

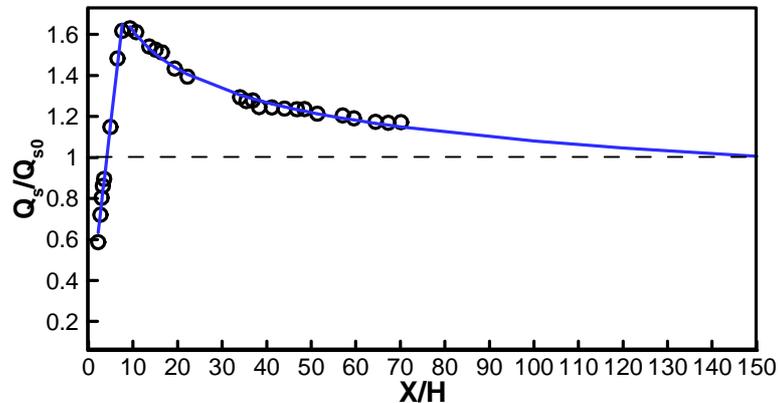


Figure 10: Normalized mean surface heat flux Q_s/Q_{s0} in the wake of a 2D steep hill in the Stable BL case.

IV. Conclusion

Turbulent flow and heat transport over a 2D hill with a steep slope in the lowest neutral and stable boundary layers were studied in the thermally-controlled wind tunnel. The vortical flow around the hill was mapped with high-resolution PIV method. Recovery of surface shear stress and surface heat flux to that of the incoming BL flows was directly measured in the far wake region of the topography. Results indicate that: the separation bubble is elongated by about 10 % in the Stable BL than that in the NBL. High turbulence intensity and Reynolds shear stress are observed in the central region of the shear layer in both cases. However, stable stratification weakens turbulent mixing, thus causes reduced turbulence intensity. The development of the surface shear stress and surface heat flux is useful in developing appropriate schemes to define boundary conditions in numerical simulation. The on-going analysis includes looking into spatial distribution of temperature, momentum and heat fluxes in the near wake region.

Acknowledgments

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